

## Lecture 11

- 11.1. Draw two graphs, each having 6 nodes exactly, where one is a planar adjacency graph, and the other one is a nonplanar graph.
- 11.2. Solve Exercise 9 on page 154 in the textbook.
- 11.3. Why we cannot apply the Euler formula (as defined on page 1, Lecture 11) for nonplanar graphs?
- 11.4. Show that a  $3 \times 3$  square in  $[\mathbb{Z}^2, A_8]$  is planar.
- 11.5. Draw examples of 4-regions and calculate  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$ , and  $\chi^+$  (for exercising the meaning of these parameters).
- 11.6. Draw  $K_{3,3}$ , assume that local circular orders are defined by clockwise order in your drawing, and calculate  $\chi^+$ .
- 11.7. Draw an example of a 4-region which has an 8-hole that consists of at least three 4-holes.
- 11.8. Consider regions in  $[\mathbb{Z}^2, A_4]$ . Is it possible that a pixel in a proper hole of such a region is 8-adjacent to a pixel in the background component? What about this in case of an improper hole?
- 11.9. Revise the figure on page 8 (Lecture 11) using the order "black first".
- 11.10. Draw the region adjacency graphs for the figure on page 8 (Lecture 11), and the revised figure resulting from 11.5. Are these trees?

## Lecture 12

- 12.1. Apply Pick's formula to examples of grid polygons as shown on page 1 (Lecture 12).
- 12.2. Map the picture on page 6 (Lecture 11) into the frontier grid, and apply Pick's formula for area calculation. (Note: in case of a hole we have to subtract the area of a hole.)
- 12.3. Describe the algorithm of discrete column-wise integration by describing the basic idea, and by specifying at least 6 of all 12 local patterns describing one step around the frontier.
- 12.4. Apply discrete column-wise integration to the (black) region on page 1 (Lecture 10) by (a) mapping into the frontier grid, and (b) showing the varying values towards  $\alpha_0$  as in the figure at the bottom of page 5 (Lecture 12).
- 12.5. Specify a method for deciding whether a traced border cycle is either an inner or an outer border cycle.
- 12.6. Is it possible that a simply connected finite set  $M \subseteq \mathbb{Z}^2$  can have (i) a proper hole and/or (ii) an improper hole? If so, then give an example; if not, then explain why.
- 12.7. Consider examples of a simple 4-curves. Apply Theorem 4 (page 11, Lecture 12) for the border cycles of your examples.

## Lecture 13

- 13.1. Complete the sentence: Two sets  $A, B$  are incident iff ...
- 13.2. Assume two different 2-cells  $c_1$  and  $c_2$ . Is it possible that  $c_1$  and  $c_2$  are incident?
- 13.3. How many 0-cells are incident with one 3-cell?
- 13.4. Consider a chessboard (which we consider to be an  $8 \times 8$  binary picture in cell representation) and assume “black ; white” for the Maximum-Value Rule. Assign labels to all the (virtual) 0- and 1-cells of the picture.
- 13.5. Is the union of two closed sets of pixels (in the cell model) again a closed set of pixels? What about the union of open sets? Explain your answers.
- 13.6. Discuss the effects of different total orders of picture values when applying the Maximum-Value Rule to a (small) three-level picture of your choice (similar to the example on page 11, Lecture 13).
- 13.7. Draw the s-adjacency graphs which are equivalent (with respect to pixel connectedness) to topological situations specified in 13.6. Note: there will be one s-adjacency graph for any of the assumed total orders of picture values.

## Lecture 14

14.1. In the frontier grid we always represent one border cycle at a time. Why we cannot represent all border cycles at once?

14.2. The frontier tracing algorithm contains the same basic steps as the border tracing algorithm (page 9, Lecture 10) besides an additional test at “flip-flop locations” (i.e., whether we can follow the 4-border cycle here as in border tracing, or not). Explain in which case we cannot follow the 4-border cycle, but have to take another turn (hint: see page 10, Lecture 14).

14.3. Specify all invalid  $m$ -cells in both figures on page 7 (Lecture 13), for  $m = 0$ ,  $m = 1$ , and  $m = 2$ .

14.4. Provide boundary counts  $b_{ij}^M(c)$  for selected cells  $c$  in these two examples considered in 14.3.

14.5. What are the total boundary counts  $b_{ij}^M$  of these two examples?

14.6. Describe how these boundary counts can be calculated when tracing a frontier. Can this still be a linear online algorithm, doing tracing as well as calculating all total boundary counts?

## Lecture 15

15.1. What is a *rational digital ray*, and what is an *irrational digital ray*?

15.2. Construct (by drawing) an example of an initial (finite) segment of a digital ray (simulating grid-intersection digitization), and represent it by a word of directional codes.

15.3. In the text of Theorem 1 (page 4, Lecture 15) change “less than 1” into “less than, or equal to 1”. Would that be still a true statement in general? Explain your answer.

15.4. Similar to 15.3, consider “less than, or equal to  $\sqrt{2}$ ” for the text of Theorem 2 (page 4, Lecture 15).

15.5. Consider two parallel lines  $y = -0.6x$  and  $y = -0.6x + 1$ . What is the main diagonal for this pair of lines?

15.6. Is the following

07000700070000700070007000700007000700007000

a DSS or not (explain)?

15.7. Is the following

0000700070007000070007000700070000700070000700

a DSS or not (explain)?

15.8. Illustrate by an example (different to page 10, Lecture 15) that the choice of the start point (when segmenting an 8-path into subsequent DSSs of maximum length) may change the number of resulting DSSs.

## Lecture 16

16.1. Segment the frontier of set  $M$  on page 6 (Lecture 12) into subsequent 4-DSSs of maximum length, by applying Theorem 2 (page 4, Lecture 15). Note that this is equivalent to applying algorithm **K1990**.

16.2. The 4-path on page 10 (Lecture 16) can be represented by the following word of directional codes:

020002000200200002

Discuss the application of algorithm **K1990** to a 4-path represented by

4464446464

16.3. Would it be possible that a clockwise traversal of the frontier of a digital region  $M$  produces a segmentation into  $m$  subsequent maximum-length 4-DSSs, and an anticlockwise traversal produces  $m + 2$  subsequent maximum-length 4-DSSs? What about  $m$  and  $m + 1$ ? If “yes”, then provide an example; if “no”, then explain.

16.4. Specify the asymptotic time complexity of algorithm **K1990** when segmenting a 4-curve consisting of  $n$  pixels. Explain.

## Lecture 17

17.1. The figure on page 3 (lecture 17) shows a DSS. Specify  $a$ ,  $b$  and  $\mu$  such that the shown DSS is a subset of  $D_{a,b,\mu,\max\{|a|,|b|\}}$ .

17.2. Is it possible that  $D_{a,b,\mu,\max\{|a|,|b|\}} = D_{b,a,\mu,\max\{|a|,|b|\}}$ , for  $a \neq b$ ? Explain.

17.3. What is  $D_{a,b,\mu,0}$ ?

17.4. What is the meaning of sets  $D_{a,b,\mu,\max\{|a|,|b|\}}$  for the justification of algorithm **DR1995**? What are *supporting* (or *leaning*) lines?

17.5. Discuss similarities and differences between basic ideas implemented by algorithms **K1990** and **DR1995**.

17.6. The example on page 9 (lecture 17) is given by the word

1010010101

of directional codes. Discuss the application of algorithm **DR1995** instead for an 8-path defined by the word

01010001

17.7. What is the asymptotic time complexity of algorithm **DR1995** when segmenting an 8-curve consisting of  $n$  pixels. Explain.

## Lecture 18

18.1. For warming up: Draw examples of finite sets of points and their convex hulls.

18.2. What is a *simple polygon*?

18.3. Explain the basic steps in Graham's scan. Consider the following two examples of inputs: (i) a set  $S$  of  $n$  points, all sampled along one straight line, and (ii) a set  $S$  of  $n$  points, all sampled on a circle.

18.4. Explain why the convex hull of a set  $A$  is in general different to the relative convex hull of  $A$  with respect to a set  $B$  which contains set  $A$ .

18.5. Specify a way to decide whether three subsequent points  $v_1, v_2, v_3$  define a *convex vertex*  $v_2$ , a *concave vertex*  $v_2$ , or are all three *collinear*.

18.6. Discuss (e.g., by means of an example) the impact of changing your  $xy$ -coordinate system from a right-hand to a left-hand system when using your criterion defined in 18.5.

18.7. How are MLPs and relative convex hulls (for sets in  $\mathbb{R}^2$ ) related to one-another?



## Lecture 19

- 19.1. Discuss the relationship between local length estimators and chamfer metrics in the 2D grid.
- 19.2. The *most probable original length estimator* delivers always a value which is slightly larger than the Euclidean distance between the two endpoints of a DSS. Why?
- 19.3. How is the DSS estimator (for the length of a digitized curve) defined?
- 19.4. Specify a way how to approximate a tangent (or a normal) at a pixel  $p$  of an 8-path.
- 19.5. When is a length estimator multigrid convergent for a given class of curves in the real plane? What is the role of function  $dig_h$  in this definition? What is the role of function  $\kappa(h)$  in this definition?
- 19.6. What is the asymptotic speed of multigrid convergence of the DSS estimator for frontiers of convex polygons which always digitize into an 8-connected set, for all  $h \geq h_0$ .
- 19.7. Is it possible that a convex polygon does not digitize into 8-connected sets, for all  $h \geq h_0$ ? Explain.

## Lecture 20

20.1. A digitized curve  $\gamma$  results into

- (i) an 8-path, which has a length (i.e., number of pixels on it), which can also be understood as being
- (ii) a polygonal curve with isothetic steps of length 1 and diagonal steps of length  $\sqrt{2}$  (and the length of this polygonal curve is then a sum of 1s and of  $\sqrt{2}s$ ), and based on the 8-path we can also estimate
- (iii) the length of  $\gamma$  (e.g., based on DSS approximations).

Explain why the length estimation (of  $\gamma$ ) should be based on methods such as DSS approximation or MLP calculation if we have high-resolution pictures, and why local length estimators are acceptable for low-resolution pictures, or “short curves”.

20.2. Define three measures for evaluating length estimators.

20.3. Why it is recommended to use a sliding mean when evaluating values calculated for all grid resolutions  $h$  in an interval such as, for example, from 30 to 1000?

20.4. Low-price digital cameras captured beginning of 2005 images at a size of about 10 megapixels. What is the size (number of columns and rows) of these pictures assuming a ratio of 4 : 3. Professional cameras (e.g., in photogrammetry) captured beginning of 2005 images at a size of about 40 megapixels. What is the size of these pictures?