Lecture 01

1.1. Assume a color picture in RGB format (i.e., three channels R, G, and B). How to map this picture into (a) a single-channel, or (b) a three channel gray-level picture?

1.2. Assume a gray-level picture with $G_{max} = 255$. What is the general definition of a gray-level *histogram* of such a picture?

1.3. What is a *binary picture*?

1.4. Define *pixel* and *voxel* (as used in these lectures) in the (a) grid point and (b) in the grid cell model.

1.5. Specify *0-cell*, *1-cell*, *2-cell*, and *3-cell* in the 3D cell model.

1.6. Define 4-adjacency and 8-adjacency between pixels in the 2D grid point model.

1.7. Define 1-adjacency and 0-adjacency between pixels in the 2D grid cell model.

1.8. What is the difference between adjacency set $A_0(c)$ and neighborhood $N_0(c)$?

1.9. Define 6-adjacency and 26-adjacency between voxels in the 3D grid point model.

1.10. Define 2-adjacency and 0-adjacency between voxels in the 3D grid cell model.

Lecture 02

2.1. *dpi* stands for what?

2.2. Assume we are using 4-adjacency for all pixels (either black or white) in a binary picture. What kind of situation has been identified by Rosenfeld and Pfaltz as 'paradox' for this case?

2.3. Assume we are using 8-adjacency for all pixels (either black or white) in a binary picture. What kind of situation has been identified by Rosenfeld and Pfaltz as 'paradox' for this case?

2.4. What is a *4-path* of pixels in the 2D grid point model?

2.5. What is a *1-path* of pixels in the 2D grid cell model?

2.6. What is an *8-path* of pixels in the 2D grid point model?

2.7. What is a *0-path* of pixels in the 2D grid cell model?

2.8. What is an α-connected set of pixels or voxels with respect to α-adjacency in the 2D or 3D grid point model?

2.9. What is a *1-component* of pixels in the 2D grid cell model?

2.10. Would it be possible that two different 2-components of a 3D binary picture have a non-empty intersection?

Lecture 03

3.1. What is an *equivalence relation*, and how does an equivalence relation defined on a set M partition this set M into pairwise disjoint *equivalence classes*?

3.2. Assume a gray-level or color picture P defined on a rectangular $m \times n$ grid $\mathbb{G}$. For $p, q \in \mathbb{G}$ let pRq iff $P(p) = P(q)$. Show that R is an equivalence relation on $\mathbb{G}$.

3.3. Characterize the equivalence classes defined by the equivalence relation in 3.2. Is it true that these equivalence classes of pixels are not necessarily α-connected, for $\alpha = 4$ or $\alpha = 8$?

3.4. Describe the FILL procedure assuming that 4-adjacency is used for all non-object pixels, and 8-adjacency for all object pixels in a binary picture? (hint: as in box on page 4, but be more specific)

3.5. What is *standard scan*, and what is *reverse standard scan* for a 2D picture?

3.6. Consider the 8 chessboard as on page 10. How many 4-components of white pixels, and how many 8-components of black pixels do we have in this case?

Lecture 04

4.1. Assume a curve γ in the real plane. What is the resulting sequence $\rho(\gamma)$ of pixels if we assume that γ has been digitized by grid-intersection digitization?

4.2. Consider as γ the ray $y = x$ in the real plane, starting at the origin $(0, 0)$. What is the resulting sequence $\rho(\gamma)$ of grid points?

4.3. Assume the directional encoding scheme as on page 3, and assume we are encoding a ray digitized by grid-intersection digitization. How many different code numbers may occur in the resulting sequence of codes?

4.4. What is a *singular* code in such a sequence as asked for in Exercise 4.3?

4.5. DSS stands for what?

4.6. Bresenham's DSS algorithm for the first octant is based on comparing two values h_1 and h_2. Explain how these two values decide about the next step, from x_i to x_{i+1}.

4.7. The length of a chain (i.e., of an 8-arc) is the number of code numbers in it, and the length of the polygonal arc defined by such a chain is given by using weight 1 for isothetic steps, and weight $\sqrt{2}$ for diagonal steps, assuming grid resolution $\theta = 1$. Calculate both values for a few examples of DSSs, and compare these values with the length of a straight segment possibly generating your DSS by grid-intersection digitization (hint: see page 9).

Lecture 05

5.1. What is a *regular* n*-gon*?

5.2. How was π estimated by Archimedes? Describe briefly his basic approach.

5.3. Assume a real circle of diameter 1 (i.e., a *unit circle*) embedded into the square $(0,0)$, $(0,1)$, $(1,1)$, $(1,0)$, and assume grids of grid resolution $1/h$, for $h = 4$ and $h = 8$. Digitize the circle by grid-intersection digitization in these two grids, and calculate the length of resulting 8-curves, and the length of resulting polygonal curves (see Exercise 4.7). Compare these values with the true perimeter of the unit circle.

5.4. Illustrate the "staircase effect" (i.e., the length of a 4-arc or 4-curve is not suitable for estimating the length of a digitized arc or curve) by an example of your choice.

5.5. Now assume that we use the length of polygonal curves defined by 8-arcs or 8-curves (i.e., applying weights $1/h$ for isothetic steps, and weight $\sqrt{2}/h$ for diagonal steps, in a grid of resolution $1/h$). Show an example of a real arc or curve where the resulting length values, if increasing h, will not converge to the true length of the given arc or curve (hint: assume grid-intersection digitization).

Lecture 06

6.1. Would you apply the model of Gauss digitization for digitizing curves?

6.2. Specify the *relative deviation* of an observed property from its true value.

6.3. We can use pixel counts for estimating the area of a region. Which argument can be used to justify this approach?

6.4. Multigrid convergence has been defined for an estimator used to calculate a particular property of digitized sets being elements in a family of sets. Specify three examples of estimators (with naming property, family of sets, and digitization model) and classify them into *multigrid convergent*, or into *not multigrid convergent*.

6.5. In case of a multigrid convergent estimator, we also define the *speed of convergence*. Specify one example of an estimator with linear speed of convergence.

6.6. Consider a square (of fixed size in the real plane) and a picture (of fixed resolution). Assume we use Gauss digitization after rotating the square by some angle. Does the multigrid convergence of the estimator of area by number of grid points in a set (see Exercise 6.3) require that the number of grid points of the digital square cannot change for different rotation angles? Explain your answer.

Lecture 07

7.1. Specify the three axioms defining a metric d on a set S.

7.2. Show that the function d_b defined on page 3 is a metric.

7.3. Show that $[d_e]$ is not a metric on $\mathbb{Z}^2$.

7.4. Define the *Manhattan metric* and the *chessboard metric* for points in the 2D grid $\mathbb{Z}^2$.

7.5. Assume two grid points p and q in the 2D grid $\mathbb{Z}^2$ and the union of all shortest α-paths from p to q. Which set results for $\alpha = 4$, and which for $\alpha = 8$?

7.6. Show that $d_8(p,q) \leq d_e(p,q) \leq d_4(p,q) \leq 2 \cdot d_8(p,q)$, for all $p, q \in \mathbb{Z}^2$.

7.7. The intrinsic α-diameter of a set S of grid points is always lower bounded by the α-diameter of S. Why?

7.8. Assume an $n \times n$ square of grid points. What is its 4-center in case of (a) n is odd, and (b) n is even.

7.9. Assume an $n \times n$ square of grid points. How is its centroid positioned compared to its 4-center?

7.10. What is the 8-center of a simple 8-path?

Lecture 08

8.1. Assume a picture with an 7×7 square of object grid points. Show the d_4 distance transform of this object.

8.2. Assume a picture with an 7×7 square of object grid points. Show the d_8 distance transform of this object.

8.3. Describe the Two-Pass Algorithm. Show the intermediate results after the first scan for d_4 and d_8 and the 7×7 square of object grid points.

8.4. Specify values a and b such that $d_4 = d_{a,b}$ (i.e., a chamfer distance). Do the same for d_8.

8.5. Assume set $A = \{(i,j) : \ i,j \in \mathbb{Z} \wedge 1 \leq i,j \leq 5\}$ and set $B = \{(i,j) : \ i,j \in \mathbb{Z} \wedge 3 \leq i,j \leq 10\}$. Calculate the Hausdorff distances $d_4(A,B)$, $d_8(A,B)$, and $d_e(A,B)$.

8.6. Assume the same sets A, B as in Exercise 8.5. Calculate the distances $d_{sym}(A,B)$ and $d'_{sym}(A,B)$ as defined by the symmetric difference of these two sets.

8.7. Now calculate the centroids of both sets, translate them into sets A' and B' such that both centroids coincides with the origin $(0,0)$, and then calculate these five distance measures $d_4(A',B')$, $d_8(A',B')$, $d_e(A',B')$, $d_{sym}(A',B')$ and $d'_{sym}(A',B')$ again. Note: the centroid is not necessarily a grid point; some rounding will be necessary.

Lecture 09

9.1. Define the d_α MAT of a 2D binary picture P.

9.2. Assume a picture with an 7×7 square of object grid points. Show the d_4 MAT of this object.

9.3. Assume a picture with an 7×7 square of object grid points. Show the d_8 MAT of this object.

9.4. Explain why a d_4 medial axis of a 4-region (i.e., a finite 4-connected set) is not necessarily 4- or 8-connected.

9.5. Give an example of a 4-region where the d_4 medial axis forms a 2×5 rectangle of grid points.

9.6. Specify the independent row and column scans for calculating the Euclidean distance transform of a 2D picture.

9.7. Illustrate the result of Step 1 of the independent row and column scans for calculating the Euclidean distance transform for a 2D binary picture of your choice, showing two 4-regions containing more than 10 pixels each.

9.8. Assume you swap (Rowscan 1) with (Rowscan 2) in Step 1; would that effect the final outcome of the algorithm?

9.9. Explain three ways for improving the efficiency of the general independent row and column scans for calculating the Euclidean distance transform for a 2D binary picture.

Lecture 10

10.1. Show that the 4-border of a 4-region is not necessarily 4-connected.

10.2. Give two sets of pixels (at least 10 pixels each) where every pixel is (a) a 4-border pixel, or (b) an 8-border pixel.

10.3. Give an example of a 2D binary picture and the resulting s-adjacency graph, where the example contains at least 3 flip-flop cases. Which order of picture values do you assume?

10.4. Assume 8-adjacency in the 2D grid and clockwise local circular order. Illustrate the generation of an atomic cycle in this grid, by showing the sequence of individual steps, each adding one directed edge to the atomic cycle.

10.5. What is an *atomic 4-cycle*, and what is a *4-border cycle* in a 2D picture? Describe these notions informally in your own words.

10.6. Assume that we trace a 4-border cycle. Can we stop tracing when reaching the same pixel for a second time? Explain your answer.

10.7. What is the shape factor of a simply-connected region? (hint: see page 12)