DIGITAL GEOMETRY

Geometric Methods for Digital Picture Analysis

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Preface

Digital geometry deals with the geometric properties of subsets of digital pictures and with the approximation of geometric properties of objects by making use of the properties of the digital picture subsets that represent the objects. It emerged in the second half of the 20th century with the initiation of research in the fields of computer graphics and digital image analysis. It has its mathematical roots in graph theory and discrete topology; it deals with sets of grid points which are also studied in number theory (since C.F. Gauss) and the geometry of numbers, or with cell complexes (which have been studied in topology since the middle of the 19th century). Studies of gridding techniques, such as those by Gauss, Dirichlet, or Jordan (for measuring the content of a set), also provide historic context for digital geometry. Digitizations on regular grids are also frequently used in numeric computation in science and engineering.

This book uses the term "picture" rather than "image," because pictures can be the result of drawing, painting, stitching, or other technologies that do not involve imaging processes. The book deals with digital geometry in the context of picture analysis. The medium on which digital pictures reside is called a *grid* which is a finite set of grid points, grid cells, or other types of discrete elements; the book discusses the geometric and topologic properties of subsets of grids.

Digital geometry can be viewed as a special branch of discrete geometry that deals with graph-theoretical or combinatorial concepts. It can also be viewed as approximate Euclidean geometry on the basis of the fact that picture analysis generally makes use of ideas about Euclidean space. However, digital geometry differs from approximation theory in its use of digitized input data (grid points that are not necessarily on the original curve) rather than sampled input data (sample points that are on the curve but that are not necessarily grid points) and in its focus on understanding the data in digital terms rather than approximating the data with the use of polynomials. Digital geometry also differs from computational geometry, which deals with finite sets of geometric objects in Euclidean space.

The book is intended to be a text that can be used in advanced undergraduate or graduate courses about image analysis in fields such as computer science or engineering. Selections from the material in this book should be sufficient to fill a one-semester course; see the course proposals in the section called "Structure of this Book" for suggested selections. Prerequisites to the use of this book are a basic knowledge of set theory and graph theory and programming experience for the suggested experimental exercises (course assignments). It should be pointed out that some of the exercises are quite difficult; see the references provided in the Commented Bibliography sections at the ends of the chapters for additional information.

The book is also designed to be a comprehensive review of research in digital geometry. The authors have chosen a mathematic viewpoint rather than a practitioner's viewpoint. However, the fundamentals of digital geometry are also of value to those who work on applications of image analysis or computer graphics, especially if they are concerned with theoretical foundations. Each chapter concludes with exercises and has references to related or more advanced work. When proofs are not given, references to the relevant literature are provided.

This book provides discussions and citations of important mathematic ideas and methodologies that are important to digital geometry and date back, in some cases, to previous centuries or even to ancient times. This information should give students and researchers a better understanding of where the subject fits into a long-term historic process of knowledge acquisition, which began long before their own work or that of their supervisors.

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Reinhard Klette and *Azriel Rosenfeld*
Auckland, New Zealand and Baltimore,
Maryland, USA
October, 2003

I greatly regret that Professor Rosenfeld did not live to see our book published in final form. I have lost not just a friend, but an outstanding teacher and scientist colleague. I shall miss him.

Reinhard Klette
May, 2004

To Gisela, Kristian, and Alexander Klette, and to Abraham Rosenfeld and his family

Professor Azriel Rosenfeld
19 February 1931 - 22 February 2004

Structure of this Book

Chapters 2 through 8 provide foundations for digital geometry; they discuss grids, metrics, graphs, topology, and geometry and introduce concepts and methods used in digital geometry that are related to these subjects.

This book is organized as shown below.

Basics

Chapter 1: Introduction
Chapters 2–8: Grids, Metrics, Graphs, Topology, Geometry

Selected topics

Chapters 9–12: Straight Lines, Curves, Planes, Surfaces
Chapters 13–16: Hulls and Diagrams, Transformations
(Geometrical, Morphological, Deformations)
Chapter 17: Other Properties and Relations

Chapters 9 through 13 discuss topics in digital geometry: digital "straightness" in Chapter 9; length and curvature of arcs and curves in Chapter 10; 3D straightness and planarity in Chapter 11; area and curvature of surfaces in Chapter 12; and hulls and diagrams in Chapter 13.

Chapter 14 discusses geometric operations on pictures; Chapter 15 discusses the application of operations of mathematic morphology to pictures; Chapter 16 discusses deformations of pictures; and Chapter 17 discusses picture properties and spatial relations.

Chapter 1 provides a general introduction and should be read first. Depending on the background of the reader, the different chapters may allow more or less independent reading. However, there are some obvious "clusters", such as Chapters 4 and Chapter 5 (graph-theoretical models of pictures), Chapters 6 and 7 (basics of topology in the context of picture analysis), Chapters 8, 9, 11, 13, 14, and 17 (basics of geometry in the context of picture analysis), and Chapter 10 and 12 (performance evaluation of algorithms in digital geometry).

A third year undergraduate course about algorithms for digital pictures (in a program in electrical engineering, computer science or mathematics involving picture analysis or computer graphics) could focus on selected algorithms (see the List of Algorithms at the end of the book) and on the fundamentals that underlie these algorithms. The students would have the benefit of related mathematical topics and material for additional reading being provided in the same textbook. For example, the course could be structured as follows:

1. (1-2 lectures) Start with Section 1.1
2. (3-5 lectures) Follow this with Chapter 2, possibly shortening Section 2.3 and adding the example from Section 1.2.7 to the presentation of Section 2.4.
3. (3 lectures) Follow this with metrics and distance transforms (Chapter 3).
4. (2-3 lectures) Continue with the border tracing algorithm of Chapter 4 (with related property calculations; see, e.g., Section 8.1.6).
5. (2 lectures) Cover the frontier tracing algorithm of Chapter 5.
6. (2-3 lectures) Follow this with one or two DSS approximation algorithms (**K1990** in Chapter 9, related to frontier tracing, or **DR1995**, related to border tracing of planar regions).
7. (3-8 lectures) Facilitate an extensive discussion about methods, algorithms, and performance evaluation for different arc length and curvature estimators (see Chapter 10).
8. (3-8 lectures) If time allows, algorithms for 3D region analysis could be added. This would include surface scanning from Section 8.4 (with related property calculations; see, e.g., Section 8.3.7), DPS approximation from Chapter 11, and surface area and curvature estimation with comparative performance evaluation from Chapter 12.
9. (3-6 lectures) Algorithms from Chapter 13 (hulls and diagrams; see also Section 1.2.9) or from Chapter 15 (morphologic operations) could also be added to the course.

Note that the exercises in this book are of varying complexity and should be selected carefully for such a course; however, all of the experimental exercises can be recommended for course work (assignments). The course could also cover other algorithms from the List of Algorithms (e.g., geometric transforms, which are not difficult to implement, or simple deformations, which require a good understanding of the "more challenging" concepts discussed in Chapter 16).

Graduate courses could focus on more specific topics clustered around selected sections in the book, such as the following:

(i) *Picture Analysis and Topology* (Chapter 2 as an introduction, then Chapters 4 through 7 and Chapter 16).

(ii) *Multigrid Analysis of Property Estimators in Picture Analysis* (basics from Chapter 2 and Chapter 3, including the example from Section 1.2.7, followed by multigrid subjects in Chapters 10, 12, and 17).

(iii) *Combinatorial Picture Analysis* (combinatorial subjects in Chapters 1 and 2 as an introduction, then Chapters 4 and 5, combinatorial subjects in Chapters 9 and 11, digital tomography in Chapter 14, and digital moments in Chapter 17).

(iv) The axiomatic approaches to different subdisciplines, especially the axiomatic theory of digital geometry in Chapter 14, could provide material for a graduate research seminar about *Mathematical Fundamentals of Picture Analysis* (see also the List of Axioms at the end of this book).

The extensive bibliography, with commented bibliography sections at the ends of the chapter, should also provide support for designing graduate student research seminars based on selected readings.