## The 2's complement representation



The 2's representation is a variation on 1's complement which does not have 2 representations for 0 .
This makes the hardware that does arithmetic (addition, really) faster than for the other representations.

A 3-bit example:

| bit pattern: | 100 | 101 | 110 | 111 | 000 | 001 | 010 | 011 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1's comp: | -3 | -2 | -1 | 0 | 0 | 1 | 2 | 3 |
| 2's comp: | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |

The negative values are all "slid" down by one, eliminating the extra zero representation.

To add, subtract, or multiply two complement numbers, just perform the operation as if the numbers are unsigned, and throw away any additional bits generated. To perform negation, form the ones complement (subtract from all ones (the representation of -1 ), then add 1 .

How to get an integer in 2's comp. representation:
The positive values are the same as for sign mag. and 1's comp.
They will have a 0 in the MSB (but it is NOT a sign bit!)

Positive: just write down the value as before
Negative: use the positive value $00101(+5)$
take the 1's comp.
11010 (-5 in 1's comp)
add 1
$+\quad 1$
11011 (-5 in 2 's comp)

For example - 30 is represented as $11111111-00011110+1=11100001$ $+1=11100010$

To get the additive inverse of a 2's comp integer,

1. Take the 1's comp.
2. Add 1

## Exercise

What is the bit pattern for decimal 42 and -42 as 8 bit two's complement numbers?

To add 1 without really knowing how to add:
Start at LSB, for each bit (working right to left)

1. While the bit is a 1 , change it to a 0 .
2. When a 0 is encountered, change it to a 1 and stop.
3. All other remaining bits are the same as before.

## Example:

What decimal value does the two's complement 110011 represent?
It must be a negative number, since the most significant bit (msb) is a 1. Therefore, find the additive inverse:

$$
\begin{gathered}
110011 \text { (2's comp. -?) } \\
001100 \text { (1's complement) } \\
+1 \\
-----1 \\
001101 \text { ( } 2 \text { 's comp. }+13 \text { ) }
\end{gathered}
$$

## A LITTLE BIT ON ADDING

| carry in | a | b | sum | carry out |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$$
\begin{array}{rr}
\mathrm{a} & 0011 \\
+\mathrm{b} & +0001 \\
-- & ---- \\
\text { sum } & 0100
\end{array}
$$

Same as for decimal!
$0+0=0$
$1+0=1$
$1+1=2-->10$ in binary, sum is 0 and carry out is 1 .
$1+1+1-->3$ sum is 1 , and carry out is 1 .

So you all understand it perfectly?
Find the 1 's complement of $-0 x f c=1,1111,1111,1111,1011,1111$
2 's complement of $-0 x f c=1,1111,1111,1111,1100,0000$

