Topological isomorphism for classes of closed subgroups of S_∞

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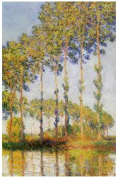

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The setting

- S_∞ is the topological group of permutations of $\mathbb{N}$.
- $\mathcal{C}$ is a Borel class of closed subgroups of S_∞.

We study the complexity of the isomorphism problem for $\mathcal{C}$:

Given groups G, H in $\mathcal{C}$,
how hard is it to determine whether $G \cong H$?

All isomorphisms of groups will be topological isomorphisms.

- Closed subgroups of S_∞ are exactly the automorphism groups of structures with domain $\mathbb{N}$.
- isomorphism of the structures (in the same language) implies conjugacy of the automorphism groups.

Borel reducibility $\leq_B$

A standard Borel space is a space of the form $(Z, \mathcal{B})$, where $\mathcal{B}$ is the σ-algebra generated by the open sets of a Polish topology on Z. Sets in $\mathcal{B}$ are called Borel sets of Z.

- Let X, Y be standard Borel spaces. A function $g\colon X \to Y$ is Borel if the preimage of each Borel set in Y is Borel in X.
- Let E, F be equivalence relations on X, Y respectively. We write $E \leq_B F$, and say that E is Borel below F, if there is a Borel function $g\colon X \to Y$ such that

  $$uEv \leftrightarrow g(u)Fg(v)$$

  for each $u, v \in X$.

An equivalence relation is called smooth if it is Borel-below $\mathtt{id}_Y$, the identity relation on some uncountable Polish space Y (say, $\mathbb{R}$).

The Borel space of closed subgroups of S_∞

For a 1-1 map $\sigma\colon \{0, \ldots, n-1\} \to \mathbb{N}$ let

$$N_\sigma = \{\alpha \in S_\infty \colon \forall i < n\, [\sigma(i) = \alpha(i)]\}$$

The closed subgroups of S_∞ can be seen as points in a standard Borel space. To define the Borel sets, we start with sets of the form

$$\{G \leq_c S_\infty \colon G \cap N_\sigma \neq \emptyset\},$$

where $G \leq_c S_\infty$ means that G is a closed subgroup of S_∞.

The Borel sets are generated from these basic sets by complementation and countable union.

Example: for every $\alpha \in S_\infty$, the set $\bigcap_k \{H \colon H \cap N_{\alpha\restriction k} \neq \emptyset\}$ is Borel. It expresses that a closed subgroup of S_∞ contains α.

Two antipodal classes

We focus on two Borel classes of closed subgroups of S_∞:

- Oligomorphic groups:
  for each k, the natural action on $\mathbb{N}^k$ has only finitely many orbits. These are the automorphism groups of ω-categorical structures with domain $\mathbb{N}$.
  Examples: S_∞, $\mathrm{Aut}(\text{random graph})$, $\mathrm{Aut}(\mathbb{Q}, <)$
- Profinite groups: each orbit of the natural action on $\mathbb{N}$ is finite.
  These are up to isomorphism the Galois groups of Galois extensions of countable fields.
  Examples: $\mathbb{Z}_p$, $GL_n(\mathbb{Z}_p)$, profinite completion of $\mathbb{Z}$

A Borel superclass of those two classes

A closed subgroup G of S_∞ is Roelcke precompact if for each open subgroup U of G there is finite set $F \subseteq G$ such that $UFU = G$.
That is, there are only finitely many double cosets UxU.
Each open subgroup contains a subgroup G_Y (pointwise stabiliser) where $Y \subseteq \mathbb{N}$ is finite. So Roelcke precompactness implies that there are only countably many open subgroups. Also: R.P is Borel.

$$\begin{tikzcd}
& \text{Roelcke precompact} & \\
\text{Oligomorphic} \arrow[ur, Rightarrow] & & \text{Profinite} \arrow[ul, Rightarrow]
\end{tikzcd}$$

Theorem (Kechris, N., Tent, JSL 2018)[1]

$$
\begin{tikzcd}
& \mathrm{GI} & \\
& \cong_{\text{Roelcke precompact}} \arrow[u, rightsquigarrow, "\leq_B"] & \\
\cong_{\text{Oligomorphic}} \arrow[ur, rightsquigarrow, "\leq_B"] & & \cong_{\text{Profinite}} \arrow[ul, rightsquigarrow, "\leq_B"']
\end{tikzcd}
$$

Isomorphism of Roelcke precompact groups is Borel below GI.

Note that graph isomorphism (GI) is universal for S_∞ orbit equivalence relations.

[1] Result independently by Rosendal and Zielinski, JSL 2018

Theorem (N., Schlicht, Tent, 2018)

$$
\begin{tikzcd}
 & & \mathrm{GI} & & \\
 E_\infty & & \cong_{\text{Roelcke precompact}} \arrow[u, rightsquigarrow, "\leq_B"] & & \\
 \cong_{\text{Oligomorphic}} \arrow[u, rightsquigarrow, "\leq_B"] \arrow[urr, rightsquigarrow, "\leq_B"] & & & & \cong_{\text{Profinite}} \arrow[ull, rightsquigarrow, "\leq_B"']
\end{tikzcd}
$$

Isomorphism of oligomorphic groups is Borel below E_∞.

A Borel equivalence relation is called countable if each equivalence class is countable.

E_∞ denotes an equivalence relation that is $\leq_B$-complete for countable equivalence relations; e.g., isomorphism of f.g. groups.

Theorem (Kechris, N., Tent, JSL 2018)

$$
\begin{tikzcd}
 & \mathrm{GI} & \\
E_\infty & \cong_{\text{Roelcke precompact}} \arrow[u, squiggly, "\leq_B"] & \\
\cong_{\text{Oligomorphic}} \arrow[u, squiggly, "\leq_B"] \arrow[ur, squiggly, "\leq_B"] & & \cong_{\text{Profinite}} \arrow[ul, squiggly, "\leq_B"'] \\
 & & \mathrm{GI} \arrow[u, squiggly, "\leq_B"]
\end{tikzcd}
$$

Graph isomorphism is Borel below isomorphism of profinite groups.

Since E_∞ is Borel but graph isom. is not, $\cong_{\text{Profinite}} >_B \cong_{\text{Oligomorphic}}$.

Complexity of the isomorphism relation for Roelcke precompact groups

Roelcke precompactness

Recall: A closed subgroup G of S_∞ is called Roelcke precompact if for each open subgroup U of G

there is a finite set $F \subseteq G$ such that $UFU = G$.

This condition is Borel because

- it suffices to check it for the basic open subgroups $G_Y = \{\rho \in G \colon \forall i \in Y\, [\rho(i) = i]\}$, where Y is finite;
- furthermore, if F exists for U, we can pick it from a countable dense set predetermined from G in a Borel way.

In fact, from such a G we can in a Borel way determine a listing $A_0, A_1, \ldots$. without repetition of all the open cosets.

Theorem (Kechris, N, Tent, JSL, 2018)

Isomorphism of Roelcke precompact groups is Borel reducible to graph isomorphism.

Note that for each closed subgroup of S_∞, the open subgroups form a basis of nbhds of 1, and hence the open cosets form a basis of the topology. For R.p. G we will now define a countable structure with domain this basis. This structure could be called a "coarse group".

Proof.

- For Roelcke precompact G, let $\mathcal{M}(G)$ be the structure with domain the open cosets. Via the listing $A_0, A_1, \ldots$ above, we can identify its domain with ω.
- The ternary predicate $R(A, B, C)$ holds in $\mathcal{M}(G)$ if $AB \subseteq C$.
- The main work is to show that for Roelcke precompact G, H,

$$G \cong H \iff \mathcal{M}(G) \cong \mathcal{M}(H).$$

Complexity of the isomorphism relation between compact closed subgroups of S_∞

Definition

A topological group G is called profinite if one of the following equivalent conditions holds.

(a) G is compact, and the clopen sets form a basis for the topology (i.e., G is totally disconnected).

(b) G is the inverse limit of a system of finite groups carrying the discrete topology.

We assume all topological groups are separable. In this case, we have a further equivalent condition

(c) G is isomorphic to a compact subgroup of S_∞.

Topologically finitely generated profinite groups

The isomorphism relation $P_{f.g.}$ between finitely generated profinite groups G is Borel bi-reducible with $\mathtt{id}_{\mathbb{R}}$.

There are uncountably many non-isomorphic such G. So by Silver's theorem it suffices to show $P_{f.g.} \leq_B \mathtt{id}_{\mathbb{R}}$.

- If two finitely generated profinite groups G have the same finite quotients up to isomorphism, they are isomorphic.
- Let $q(G)$ be the set of isomorphism types of finite quotients, written in some fixed way as an infinite bit sequence. This map is Borel because from G one can "determine" its finite quotients[2].
- Then $G \cong H \iff q(G) = q(H)$. So $P_{f.g.}$ is smooth.

[2] see e.g. Fried and Jarden, Field arithmetic, 16.10.7

Graph isomorphism $\leq_B$ isom. of pro-p groups

A group G is nilpotent-2 if it satisfies the law $[[x, y], z] = 1$.

Let $\mathcal{N}_2^p$ denote the variety of nilpotent-2 groups of exponent p.

Theorem (Kechris, N. and Tent, JSL 2018)

Let $p \geq 3$ be prime. Graph isomorphism can be Borel reduced to isomorphism between profinite groups in $\mathcal{N}_2^p$.

Proof: A result of Alan Mekler (1981) implies the theorem for countable abstract groups. We adapt his construction to the profinite setting.

A symmetric and irreflexive countable graph is called nice if it has no triangles, no squares, and for each pair of distinct vertices x, y, there is a vertex z joined to x and not to y.

Easy fact: Graph isomorphism $\leq_B$ nice graph isomorphism.

Mekler's construction

isomorphism of nice graphs $\leq_B$

isomorphism of countable groups in $\mathcal{N}_2^p$.

- Let F be the free $\mathcal{N}_2^p$ group on free generators $x_0, x_1, \ldots$.
- For $r \neq s$ we write $x_{r,s} = [x_r, x_s]$.
- Given a graph with domain $\mathbb{N}$ and edge relation A, let
$$G(A) = F/\langle x_{r,s} \colon\ rAs\rangle_{\text{normal closure}}.$$
- The centre of $G(A)$ is abelian of exponent p with a basis consisting of the $x_{r,s}$ such that $\neg rAs$.

The tricky part is to show that A can be reconstructed from $G(A)$. Therefore:

if A, B are nice graphs, then $A \cong B$ iff $G(A) \cong G(B)$.

Topological version of Mekler's construction

- Let R_n be the normal subgroup of $G(A)$ generated by the x_i, $i \geq n$. Let $\overline{G}(A)$ be the completion of $G(A)$ w.r.t. the R_n, i.e.,
$$\overline{G}(A) = \varprojlim_n G(A)/R_n.$$
- Each $G(A)/R_n$ is finite, so this is a profinite group.

Again it can be shown that A can be reconstructed from $\overline{G}(A)$:

Let A, B be a nice graphs. Then $A \cong B$ iff $\overline{G}(A) \cong \overline{G}(B)$.

(This only works because of particular properties of the construction; usually, taking completions loses information.)
The map $A \to \overline{G}(A)$ is Borel. So

$$\text{graph isomorphism} \leq_B \text{isomorphism of profinite } \mathcal{N}_2^p \text{ groups.}$$

Complexity of the isomorphism relation for oligomorphic subgroups of S_∞

Oligomorphic groups

- A closed subgroup G of S_∞ is called oligomorphic (Cameron, 1980s) if for each k, the action of G on $\mathbb{N}^k$ has only finitely many orbits.
- For instance, S_∞, Aut(random graph) and $\mathrm{Aut}(\mathbb{Q}, <)$ are oligomorphic.
- whether G is oligomorphic depends on the way G is embedded into S_∞.
- An oligomorphic group cannot be locally compact, let alone countable.

Oligomorphic groups as automorphism groups

Fact

$G \leq_c S_\infty$ is oligomorphic $\iff$ G is the automorphism group of an ω-categorical structure S with domain $\mathbb{N}$.

Proof.

$\Leftarrow$: this follows from the Ryll-Nardzewski Theorem.

$\Rightarrow$:

- For each $G \leq_c S_\infty$ we have $G = \mathrm{Aut}(S)$ where S is the structure with a k-ary relation symbol for each orbit of G on $\mathbb{N}^k$.
- If G is oligomorphic then S is ω-categorical.

$\square$

Conjugacy of oligomorphic groups

The conjugacy relation for oligomorphic groups is smooth, i.e. Borel reducible to $id_{\mathbb{R}}$.

To see this,

- Given a closed subgroup G of S_∞, let V_G be the corresponding orbit equivalence structure: for each $k > 0$ introduce a $2k$-ary relation that holds for two k-tuples if they are in the same orbit of $\mathbb{N}^k$.
- V_G is ω-categorical.
- One checks that for oligomorphic groups G, H

$$G \text{ and } H \text{ are conjugate in } S_\infty \iff V_G \cong V_H.$$

- Isomorphism of ω-categorical structures M, N for the same language is smooth, because $M \cong N \iff \mathtt{Th}(M) = \mathtt{Th}(N)$.

Bi-interpretability

Structures A, B are bi-interpretable if there are first-order interpretations Γ, Δ such that

- $A \cong \Gamma(B)$, $B \cong \Delta(A)$
- some isomorphism $\gamma : A \cong \widehat{A} = \Gamma(\Delta(A))$ is definable in A, and similarly for $B \cong \Delta(\Gamma(B))$.

(Note that $\widehat{A}$ consists of equivalence classes of tuples from A.)
Coquand[3] showed that for ω-categorical A, B we have

$$\mathrm{Aut}(A) \cong \mathrm{Aut}(B) \iff A \text{ and } B \text{ are bi-interpretable.}$$

[3] see Ahlbrandt/Ziegler 1986, or Evans' 2013 Hausdorff Institute notes

The space of theories

- Theories in a countable language can be seen as infinite bit sequences: for each sentence there is an entry whether the sentence is in the theory or not.
- Hence the set of theories inherits a topology from $\{0, 1\}^{\mathbb{N}}$.
- The complete theories form a closed set.
- To be ω-categorical is a Π^0_3 property of a theory T, because by Ryll-Nardzewski this property is equivalent to saying that for each n, the Boolean algebra of formulas with at most n free variables modulo T-equivalence is finite.

Bi-interpretability of structures via their theories

We can express bi-interpretability of ω-categorical structures A, B in terms of their theories:

- $A \cong \Gamma(B)$ means that $\mathtt{Th}(B)$ says
  "the structure interpreted in B via Γ satisfies $\mathtt{Th}(A)$"
- similar for $B \cong \Delta(A)$
- also express that some $\gamma : A \cong \Gamma(\Delta(A))$ is defined by a particular first order formula.

Bi-interpretability of ω-categorical theories

Theorem (N., Schlicht and Tent, on Logic Blog 2018)

There is a Σ_2^0 relation which coincides with bi-interpretability on the Π_3^0 set of ω-categorical theories.

Given ω-categorical theories S, T. We have an initial block of existential quantifiers fixing the dimensions of the interpretations and asserting the existence of the definable isomorphism γ.

- The rest is easy if the signature if finite
- In general, we have to express that a certain tree computed from S, T is infinite. The tree matches types of S and types of T in a way consistent with γ being an isomorphism.
- The branching of the tree is bounded depending on S, T and γ, because the dimensions are fixed, and for each arity there are only so many types. So it is Π_1^0 in S, T to say that the tree is infinite.

Bi-interpretability of ω-categorical theories

Corollary

Bi-interpretability on the set of ω-categorical theories is Borel bi-reducible with a $\mathbf{\Sigma}^0_2$-equivalence relation on a Polish space.

Proof of Corollary.

- There is a finer Polish topology with the same Borel sets in which the set of ω-categorical theories is closed.
- Then the Σ^0_2 relation above yields a $\mathbf{\Sigma}^0_2$ description of bi-interpretability on this closed set.

$\square$

Oligomorphic groups can be seen as countable structures

Theorem

Isomorphism of oligomorphic groups is Borel bi-reducible with the orbit equivalence relation of a Borel action $S_\infty \curvearrowright \mathcal{B}$; where

- $\mathcal{B}$ is an invariant Borel set of structures with domain $\mathbb{N}$ for the language with one ternary relation symbol,
- the action of S_∞ is the natural one.

A Borel equivalence relation on a Polish space is called countable if every equivalence class is countable.

Theorem (The upper bound)

Isomorphism of oligomorphic groups is Borel reducible to a countable Borel equivalence relation.

Proof.

- Above, we proved that isomorphism of oligomorphic groups is Borel reducible to a $\mathbf{\Sigma}^0_2$ equivalence relation on a Polish space.
- So the isomorphism relation on the Borel set $\mathcal{B}$ of structures in the foregoing Theorem is Borel reducible to a $\mathbf{\Sigma}^0_2$ equivalence relation.
- Hjorth and Kechris (1995; Theorem 3.8):

  Suppose the orbit equivalence relation E given by a Borel action of S_∞ is Borel reducible to a $\mathbf{\Sigma}^0_2$ equivalence relation. Then E is Borel reducible to a countable equivalence relation.

Theorem (to discuss)

Isomorphism of oligomorphic groups is Borel bi-reducible with the orbit equivalence relation of the natural action of S_∞ on an isomorphism invariant Borel set $\mathcal{B}$ of structures with domain $\mathbb{N}$.

For Roelcke precompact G, we defined a structure $\mathcal{M}(G)$ with domain consisting of the cosets of open subgroups. We can in a Borel way find a bijection of these cosets with $\mathbb{N}$. We showed

$$G \cong H \iff \mathcal{M}(G) \cong \mathcal{M}(H).$$

We will define an "inverse" operation $\mathcal{G}$ of the operation $\mathcal{M}$ on a Borel set $\mathcal{B}$ of models. For oligomorphic G and $M \in \mathcal{B}$ we will have

$$\mathcal{G}(\mathcal{M}(G)) \cong G \text{ and } \mathcal{M}(\mathcal{G}(M)) \cong M$$

This suffices because it implies the converse reduction

$$\mathcal{G}(M) \cong \mathcal{G}(N) \iff M \cong N.$$

Axiomatizing the range of the map $\mathcal{M}$

- We first define the map $\mathcal{G}$ on an invariant co-analytic set $\mathcal{B}$ of L-structures that contains range($\mathcal{M}$).
- Since $\mathcal{M}(\mathcal{G}(M)) \cong M$ for each $M \in \mathcal{B}$, actually $\mathcal{B}$ equals the closure of range($\mathcal{M}$) under isomorphism, so $\mathcal{B}$ is also analytic, and hence Borel.
- We will observe a number of properties, called axioms, of all the structures of the form $\mathcal{M}(G)$. They can be expressed in Π^1_1 form.
- $\mathcal{B}$ is the set of structures satisfying all the axioms.

Definable relations in $\mathcal{M}(G)$

Recall that our language L only has one ternary relation $R(A, B, C)$ (which is interpreted by $AB \subseteq C$ for cosets A, B, C).

- The property of A to be a subgroup is definable in $\mathcal{M}(G)$ by the formula $AA \subseteq A$. That a subgroup A is contained in a subgroup B is definable by the formula $AB \subseteq B$.
- A is a left coset of a subgroup U if and only if U is the maximum subgroup with $AU \subseteq A$; similarly for A being a right coset of U.
- $A \subseteq B \iff AU \subseteq B$ in case A is a left coset of U.

The first few axioms posit for a general L-structure M that the formulas behave reasonably. E.g., $\subseteq$ is transitive. We use terms like "*subgroup*", "*left coset* of" to refer to elements satisfying them.

The filter group $\mathcal{F}(M)$

Given a structure M, denote by $\mathcal{F}(M)$ the set of filters (for $\subseteq$) that contain a left and a right coset of each subgroup. (These cosets are unique because axioms require that distinct left cosets are disjoint etc.) We use letters x, y, z for such filters.

$A \in x$ means intuitively that A is an open neighbourhood of the group element x.

With this intuition in mind we define an operation on $\mathcal{F}(M)$:

$$x \cdot y = \{C \in M \mid \exists A \in x \exists B \in y \ AB \subseteq C\}.$$

For A a *right coset* of V and B a *left coset* of V, let $A^* = B$ if $AB \subseteq V$. Let $x^{-1} = \{A^* \colon A \in x\}$.
The filter of *subgroups* is the identity 1.

The filter group $\mathcal{F}(M)$

We can express by Π^1_1 axioms that these operations behave as a group: the operation $\cdot$ is associative, and $\forall x\,[x \cdot x^{-1} = 1]$.

The sets $\{x\colon U \in x\}$, where $U \in M$ is a *subgroup*, are declared to be a basis of neighbourhoods for the identity. Positing the right axioms, we ensure that $\mathcal{F}(M)$ is a Polish group.

For each *subgroup* $V \in M$, $LC(V)$ denotes the set of *left cosets* of V. There is an action $\mathcal{F}(M) \curvearrowright LC(V)$ given by

$$x \cdot A = B \text{ iff } \exists S \in x\,[SA \subseteq B].$$

A faithful subgroup

- Each oligomorphic G has an open subgroup V such that the natural translation action $G \curvearrowright LC(V)$, given by $g \cdot (aV) = (ga)V$, is oligomorphic:
  e.g. let $V = G_{\{n_1,\ldots,n_k\}}$ where the n_i represent the k many 1-orbits.
- Call such a V a faithful subgroup.
- By a further axiom for an abstract L-structure M, we require the existence of such V, and that the embedding of $\mathcal{F}(M)$ into S_∞ given by the action $G \curvearrowright LC(V)$ is topological (these axioms are Π^1_1 but not first-order).
- Then $\mathcal{F}(M)$ is oligomorphic and hence Roelcke precompact.

Showing that the coset structure of $\mathcal{F}(M)$ is isomorphic to M

Mainly, we have to show that each open subgroup $\mathcal{U}$ of $\mathcal{F}(M)$ has the form $\mathcal{U} = \{x : U \in x\}$ for some *subgroup* U in M.

- By definition of the topology, $\mathcal{U}$ contains a basic open subgroup $\widehat{W} = \{x : W \in x\}$, for some *subgroup* $W \in M$.
- Since $\mathcal{F}(M)$ is Roelcke precompact, $\mathcal{U}$ is a finite union of double cosets of $\widehat{W}$.
- We require as an axiom for M that each such finite union that is closed under the group operations corresponds to an actual *subgroup* in M.

Turning $\mathcal{F}(M)$ into closed subgroup $\mathcal{G}(M)$ of S_∞

- By Π^1_1 uniformization (Addison/Kondo), from $M \in \mathcal{B}$ we can in a Borel way determine a faithful subgroup V.
- Let $A_0, A_1, \ldots$ list $LC(V)$ in the natural order.
- Then the action $\mathcal{F}(M) \curvearrowright LC(V)$ yields a topological embedding of $\mathcal{F}(M)$ into S_∞.
- Its range is the desired closed subgroup $\mathcal{G}(M)$.

By the arguments above we have $\mathcal{G}(\mathcal{M}(G)) \cong G$ for each oligomorphic G, and $\mathcal{M}(\mathcal{G}(M)) \cong M$ for each $M \in \mathcal{B}$.

Theorem (Finished)

Isomorphism of oligomorphic groups is Borel bi-reducible with the isomorphism relation on an invariant Borel set $\mathcal{B}$ of structures with domain $\mathbb{N}$.

Theorem (N., Tent, Schlicht '18, Recall)

Isomorphism of oligomorphic groups is essentially countable, i.e. Borel below a Borel equivalence relation with all classes countable.

- The latest version of our proof doesn't need bi-interpretability. Rather, it uses a different result by Hjorth/Kechris 95 that characterizes essential countability of the isomorphism relation on a Borel class of structures by model theory with an infinitary language. We have to adapt some of our axioms to accomplish this, so that they can be expressed in $L_{\omega_1,\omega}$.
- A closed subgroup of S_∞ is called quasi-oligomorphic if it is isomorphic to an oligomorphic group.
  Using the methods above, we also show that this larger class is Borel, and we extend the upper bound in the theorem to this class.

A fake reduction from $\cong_{Profinite}$ to $\cong_{Oligomorphic}$

- Evans and Hewitt (1990): every (separable) profinite group is a topological quotient of an oligomorphic group.
- Given oligomorphic G, let $D(G)$ be the intersection of all open subgroups with finite index. Note that $D(G)$ is closed and invariant.
- They show that up to isomorphism, $G/D(G)$ ranges through all profinite groups P.

However, from profinite $P \leq_c S_\infty$, we cannot Borel determine oligomorphic G_P such that $G_P/D(G_P) \cong P$ in such a way that $P \cong Q$ implies $G_P \cong G_Q$. In their construction, one can see how G_P depends on the way P is presented as a subgroup of S_∞.

Some open problems

- How complex is isomorphism of arbitrary closed subgroups of S_∞? Is it $\leq_B$-complete for analytic equivalence relations?
- What is a lower bound for the complexity of isomorphism for oligomorphic groups?
- How about if the signature of an ultrahomogeneous structure N with $\mathrm{Aut}(N) = G$ can be made finite?

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