

## Research Retreat, February 5-9, 2014, South Africa

**Location:** [Intundla Game Lodge](#) near Pretoria

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[ARA web site:](#)

<http://dl.dropbox.com/u/370127/ARA2014SA/ARA2014SA.html>

Organized by Willem Fouche and Andre Nies

We appreciate the help by the CEMS staff, especially by Cecile vd Merwe.

*Participants:*

George Davie (Unisa)	Kolmogorov complexity, Brownian motion, ergodic theory
Richard de Beer (Unisa)	Analysis, dynamical systems, C*-algebras
Cameron Freer (MIT)	Random structures, probability theory, computable analysis
Willem Fouche (Unisa)	Dynamical systems, Ramsey theory, randomness
Safari Mukeru (Unisa)	Brownian motion, Ito calculus
Andre Nies (Auckland U)	Computability, randomness, descriptive set theory
Arno Pauly (U Cambridge)	Computable analysis, descriptive set theory
Paul Potgieter (Unisa)	Nonstandard analysis, Kolmogorov complexity, Fourier analysis, Brownian motion
Philipp Schlicht (Bonn University)	Set theory and its connections to computability
Dieter Spreen (Unisa and U Siegen)	Effective topology, domain theory



## Feb 5

14:00 Short introductions to research area and statement of an open problem.  
(De Beer, Davie, Mukeru, Potgieter, Spreen)

*Davie*: A problem related to Birkhoff's theorem for algorithmically random points, as studied in the paper by Bienvenu.

Reference: *Ergodic-type\_characterizations\_of\_algorithmic\_randomness-Submitted.pdf*

*Spreen*: A problem related to the computable isomorphism result in my paper in my folder. Also, a problem related to a representation of the reals based on Stone duality, as studied in the MSc thesis of my student Jaya, also in my folder. (*Obs.* The thesis still contains small mathematical errors.)

References: *Spreen\_isom-numb-top-final.pdf*, *MSc-Jaya.pdf*

*Potgieter*: I will be discussing ongoing work in Salem sets in the context of the integers. There are some correspondences between results in the continuum and in the whole numbers, and my current work attempts to make this explicit.

Reference: *Potgieter\_Arithmetic\_progressions.pdf*

*Mukeru*: I will discuss an extension of Frostman's Lemma from compact sets to general subsets of the real line. This can possibly be extended to higher dimensions and more general metric spaces.

Reference: *Mukeru\_Frostman\_Lemma.pdf*

*de Beer*: I will discuss some basic constructions in measure theoretic and topological dynamics, especially those relating to problems of effective computation.

Reference: *de Beer\_1309.0125.pdf*, *Keane\_Peterson\_Birkhoff\_Ergodic.pdf*, *Counterexamples in ergodic theory and number theory.pdf*

17:00 Gromov-Hausdorff distance and isometry (Philipp Schlicht)

The Gromov-Hausdorff distance measures how closely two metric space can be embedded into a third metric space. For compact metric spaces, Gromov-Hausdorff distance 0 implies isometry, but this is false for Polish spaces.

From the viewpoint of Borel reducibility, the equivalence relation  $E_{GH}$  between Polish spaces defined by having Gromov-Hausdorff distance 0 is at least as complicated as isometry of Polish spaces with arbitrary diameter. However, the analogous question for Polish spaces of bounded diameter is open. In order to compare  $E_{GH}$  with isometry, we study the complexity of isometry on single  $E_{GH}$  classes and show how to realize  $E_0$  and the iterated Friedman-Stanley jumps of the equality relation.

8:00pm Workshop on refereeing

## Feb 6

10:00 Algorithmic randomness, Ramsey theory and amenable non-archimedean groups (Willem Fouche, Andre Nies, Cameron Freer)

*Nies:* Let  $F$  be a countable Fraïssé structure. Let  $\text{Age}(F)$  denote its finite substructures. A Polish group  $G$  is called extremely amenable if every continuous action on a compact space has a fixed point.

An influential theorem of Kechris, Pestov, and Todorcevic (2006) says that  $\text{Aut}(F)$  is extremely amenable iff  $\text{Age}(F)$  has the Ramsey property and consists only of rigid structures. For instance, this reproves the result of Truss that the automorphism group of  $\mathbb{Q}$  as a linear order is extremely amenable.

Reference: file *Lionel\_Nguyen\_von\_The\_KPT\_Fraïssé.pdf*

*Fouche:* A Polish group is called non-archimedean if it has a system of neighborhoods of the unity consisting of open subgroups. Examples are the additive  $p$ -adic groups (and more generally all profinite groups), and the group  $S$  of permutations of the natural numbers. Becker, Kechris, Rosendal (1996) showed that up to continuous isomorphism, these groups are exactly the closed subgroups of  $S$ .

We will also discuss the Ramsey degree and its use in characterizing the extremely amenable closed subgroups of  $S$ . Further, we will look at Blass' recent result that these subgroups of  $S$  are exactly the ones making the Boolean prime ideal theorem hold in the corresponding permutation model.

*Freer:* A viewpoint based on measures.

8:00pm Workshop on writing and publishing. If you want a paper of yours discussed please put it into the relevant folder.

## Feb 7

10:00 Calibrating the strength of Frostman's Lemma by its position in the Weihrauch lattice (*Arno Pauly*).

10:30 Tutorial on Brownian motion, based on slides of Krzysztof Burdzy (Safari Mukeru, A. Nies, Paul Potgieter). Reference: file *Brownian\_Tutorial.pdf*

17:00 Excursion to see the giraffe



8:00pm Workshop on getting grants

## **Feb 8**

10:00 Open session

13:00 Braai

8:00pm Open workshop

Possible topics: necessity of research institutions, conference participation

### **Notes from writing workshop**

*Q1. What makes a good paper?*

a) Content. Each of the following can contribute.

Solve an open problem

Make connections

Pose good questions that lead to new research

New useful technique/objects

Depth/elegance

Surprise element

Centered on one idea

b) Writing.

It should be easy to figure out for an expert what is in the paper by looking at the first pages

Give motivation

*Q2. How to motivate?* Here are some suggestions what to say.

We make new connections between fields

We fill gaps/ improve current situation/improve understanding of concepts

We give new examples of...

You can also motivate from history or from the paper where the question you solve was asked. This in effect defers the job of motivating.

You can motivate from applications. “Wouldn’t it be nice if we had a result that led to the following? ...”

*Q3. How do I write better?*

Don’t overload intro with citations. If citing, include the location within the source. “By Lemma 3.4 in [9], ...”

Credit carefully, esp. your potential referee. Also credit recent important definitions.

Avoid (...), footnotes (some contrary opinions here)<sup>1</sup>

Use short sentences. Avoid forward references, such as using a term in a sentence and the defining it. “We show that all weakly quasiregular maps are strongly inobtrusive. Here, a weakly quasiregular map is...” Better, exchange these two sentences.

Also, use descriptive terminology

Long proofs: Make structure of proof clear, give informal outline of the argument. Consider putting proofs of lemmas at the end if they distract from the main flow of the argument.

*Q4 Structure of the paper.*

*Intro.* Gives background motivation, states main concepts and results, but not in full technical detail, esp. for longer works

*Prelims.* Gives technical defs that are needed in several places.

*Sections.* Each has short intro, with more detail than in the paper intro.

*Conclusion* (possibly)

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<sup>1</sup> For instance, by Arno