Coordination via Polling in Plurality Voting Games under Inertia

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COMSOC, 2012-09-13

Papers referred to below

- Annemieke Reijngoud and Ulle Endriss. *Voter Response to Iterated Poll Information*. In Proceedings of AAMAS-2012.
- Jean-François Laslier. *The Leader Rule*. Journal of Theoretical Politics 2009.
- M. Messner and M. K. Polborn. Miscounts, Duverger's law and Duverger's hypothesis. Preprint, 2011.

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- How do you know that a has no chance in a mass election? Usually, through opinion polls.
- We aim to study the dynamics of repeated polling combined with compromising.

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- In each round, voters are polled.
- The total number K of rounds is not known to the voters.
- Voters do not communicate with each other and have no knowledge of preferences or any other voter characteristics, apart from what is reported in the polls.

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- We assume that after each poll, the scores of each candidate are reported. Nothing is known about further preferences of voters. This fits into the framework of Reijngoud & Endriss.

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- If $\theta = 0$ then the voter has complete confidence in the announced scores. If $\theta = 1$ then the voter ignores the announced information completely.
- Example: if the announced scores of a, b, c are $35\%, 30\%, 25\%$ then a voter with $\theta < 1/13$ believes that a is leading. If $\theta > 1/6$ it is consistent for the voter to believe that c is leading.

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  - it allows for more complicated strategic behaviour depending on the exact values of utilities and probabilities.

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- She votes as though the current poll is the actual election, with no attempt to mislead other voters. Note that the lack of information on lower preferences of other voters is crucial here.
- For artificial agents, we could simply decree that they vote in this way, and consider this model as describing an algorithm for reaching consensus, rather than a realistic attempt to model human elections.

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  - (upward closure) if I believe that x is a potential winner and I believe that y has higher score than x, then I must believe that y is a potential winner;
  - (overtaking) a necessary condition for me to believe that x is a potential winner is that x can eventually overtake a higher candidate whom I also believe to be a potential winner, by attracting support from lower-scoring candidates.

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- Now W can be computed algorithmically by dealing with candidates in decreasing order of reported score.
- Note that the assumption that K is unknown is important. Otherwise, for example, a candidate may not have enough time left to attract enough support to win.

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- Note that $29 + 21 + 10 > 40$ and $21 + 10 > 29$. Thus in the best case where all voters have low inertia and preferences allow it, b can overtake a, and c can overtake b. Thus v believes $b, c \in W$.
- However d is not a potential winner according to v. Any voter who would consider switching to d from a, b, or c would only do that by abandoning that candidate; upward closure of the potential winner set makes this impossible.

Results

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- For general inertia distributions, we have analytical results only in special cases for 3 candidates, and experimental results otherwise. Results are highly dependent on the shape of the inertia distribution and the voters' lower preferences.

Example: uniform inertia distribution

Suppose that the distribution of inertia is uniform over voters and suppose that the initial scores for a, b, c are $A\%, 35\%, (65 - A)\%$ where $50 > A > 35$.

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  - When $A = 45$, convergence is subexponential and depends on tiebreaking.
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- Such results illuminate "Duverger's law" of plurality-based parliamentary systems.

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- Introduce the concept of inertia into other strategic voting models of voter behaviour and see what happens.
- Validation of the model using empirical/experimental work for human elections is desirable. Can we measure inertia? It may have other behavioral interpretations.
- Relaxing the informational assumptions we have made often leads to much more complicated models often involving many equilibria - repeated games of incomplete information. Some relaxations will be more tractable than others.