Surface Curvature Maps and Michelangelo's David

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Abstract

In 1998 - 1999 a team of researchers from the Computer Science Departments at the University of Stanford and the University of Washington digitized a number of Michelangelo's sculptures, including the David statue, using a custom designed laser triangulation scanner. The resultant data has been made available to the research community. This paper explores the data structures and the inherent geometry associated with the David data set. An estimation of surface curvature that exploits the structure and geometry of the data set is described. Finally surface curvature maps are defined and several curvature maps of David are presented.

Keywords: surface curvature, laser scan, Michelangelo

1 Introduction

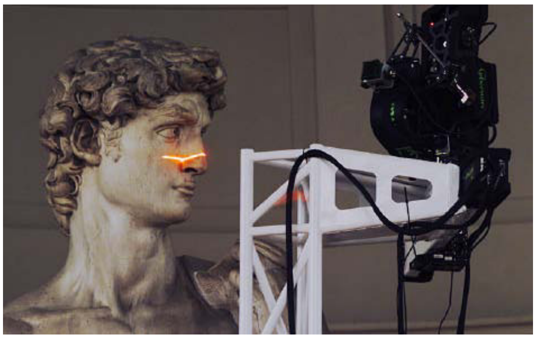

Figure 1: The large statue scanner in action [2].

Marc Levoy, *et al.* [1] have reported the massive effort that went into the Digital Michelangelo Project. They developed both significant customized hardware and software to achieve their goal and they reported opportunities for further work. The resultant data set has historical, cultural and technical significance. Their large statue scanner is shown acquiring data in Figure 1.

A photograph of David's face is shown in Figure 2 and a rendering (by the author of this paper) of a 2mm resolution composite triangle mesh model from the Stanford archive is shown in Figure 3. Surface curvature detail is clearly visible in Figures 2 and 3. Note however that the visual cues in photographs and rendered images from which apparent depth and surface curvature detail are discerned are highly dependant on lighting intensity, lighting position as well as the reflectance properties of materials.

Figure 2: A photograph of David's face [2].

Figure 3: A reference rendering.

2 The David data set

The height of the David statue is 5.17 meters and the scanning was done with a sample spacing of

0.29mm. The result is 1.93 giga-bytes of data which has been made available in nine compressed files. The uncompressed data set represents approximately 1.1 billion 3D space points. There are a total of 6540 raw scan files collected into 515 groupings.

Each scan was acquired over a fixed width of approximately 140mm and a variable height. A rendered image of the points from two adjacent overlapping scans is shown in Figure 4.

Figure 4: Points from two scans rendered with lighting.

2.1 Data structures

C++ program code [3] developed by Stanford for the project includes class definitions that define data structures. Consider the following code fragment:

```
class SDfile {
public:
    unsigned int pts_per_frame;
    unsigned int n_frames;
    unsigned short *z_data;
};
```

Each scan line of acquired data is referred to as a frame. Scan files 13 and 14 from the scan group Face1, for example, both have 486 points per frame and have 1480 frames 1515 frames respectively. The z_data array holds raw depth values.

Because the scanner uses an interlaced CCD sensor, the values in each frame actually represent a zig-zag acquisition pattern of effectively two scan lines. This pattern and its implications are explored in the next section.

2.2 Data geometry

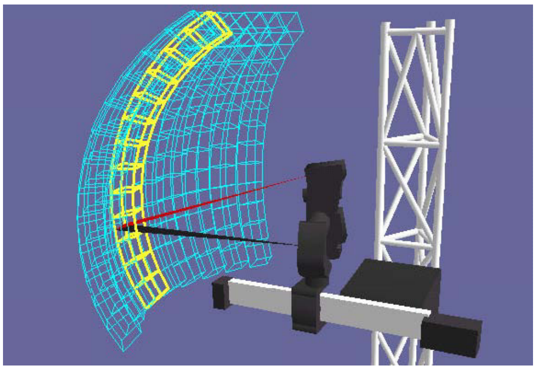

Figure 5: The scanner's imaging volume [2].

The scanning system's physical geometry is illustrated in Figure 5. The Stanford software converts z_data depth values into 3D points using a concatenated sequence of transformation matrices modeled on the system geometry.

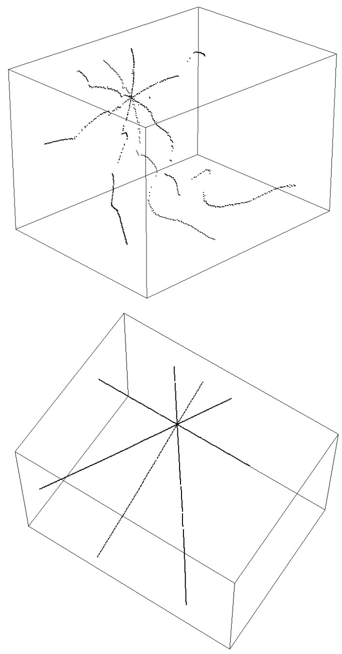

Figure 6: Two different 3D views of four cuts through the same point.

Careful 3D visual exploration of the 3D points appears to reveal alignment with four cutting

Figure 7: The hexagonal adjacency pattern.

planes through each point. Four cuts through the same point in Scan 14 are shown in Figure 10.

A close up view is shown in Figure 7. The dark black points are those associated with the frame that includes the center point. The zig-zag pattern of frame acquisition results in hexagonal point adjacency. This hexagonal pattern is somewhat problematic in that it can not be directly displayed on a square grid. This problem can be overcome by using a slightly squashed array of smaller dots as shown in Figure 8.

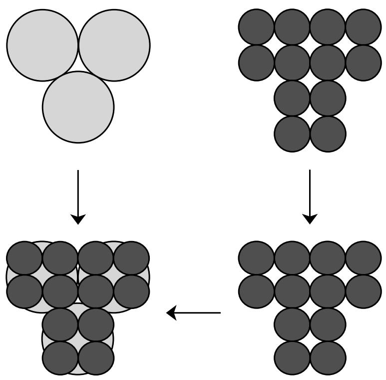

Figure 8: A squashed dot mapping.

This squashed dot mapping can be used to correctly generate displayable 2D arrays of the raw z_data depth values. *Depth maps* of Scans 13 and 14 are shown in Figure 9. Points closest to the laser source are shading coded as white, and points further away are encoded with decreasing intensity towards black. Out of range points (either too close, too far away, or beyond angular limit) as well as non-reflective points are all coded as black.

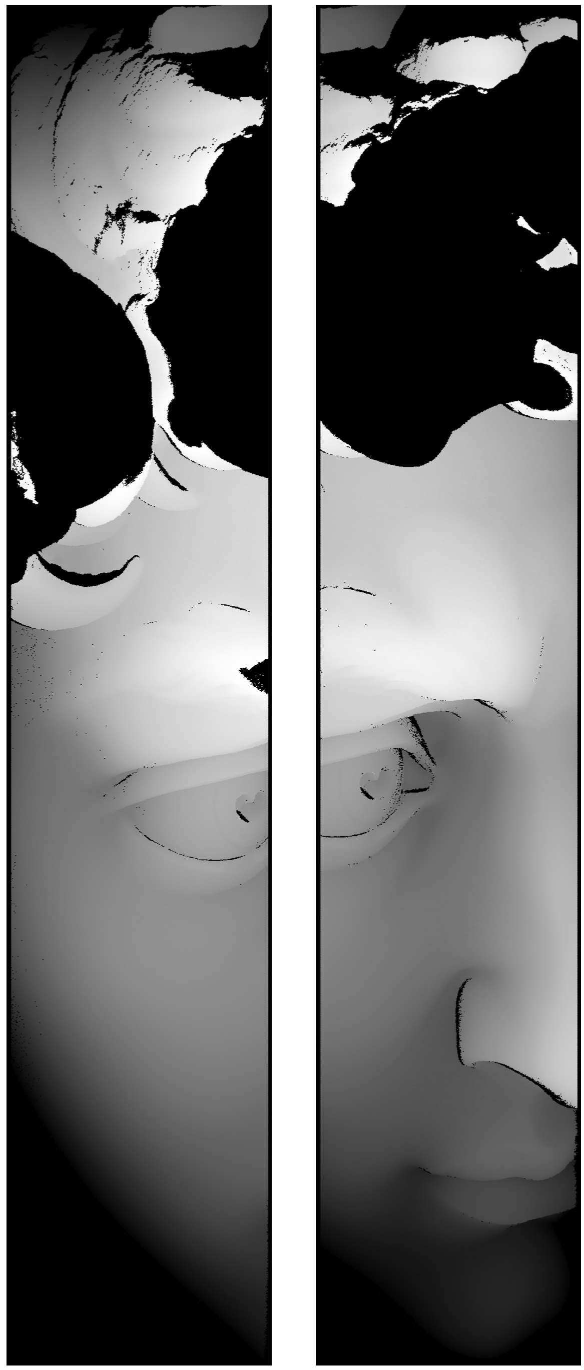

Figure 9: Depth maps of scans 13 and 14.

3 Curvature maps

3.1 Surface curvature

Surface curvature is a well-defined property for continuous smooth surfaces [4]. When working with point set data the surface curvature can only be estimated [5], [6].

The curvature estimation approach used in this paper is as follows: For each point,
1) select points associated with two orthogonal cuts as illustrated in Figure 10;
2) for each cut calculate an estimated signed planar line curvature; with reference to Figure 11, the planar line curvature is estimated as the incremental angular advance divided by the incremental change in length

$$k = \frac{a}{(d1 + d2)/2} \tag{1}$$

and calculated as

$$k = \left(\frac{2}{||\mathbf{v1}|| + ||\mathbf{v2}||} \right) \cos^{-1}\left(\frac{\mathbf{v1} \cdot \mathbf{v2}}{||\mathbf{v1}|| \, ||\mathbf{v2}||} \right) \tag{2}$$

where $\mathbf{v1} = \mathbf{p2} - \mathbf{p1}$ and $\mathbf{v2} = \mathbf{p3} - \mathbf{p2}$;

Figure 10: Orthogonal cut points.

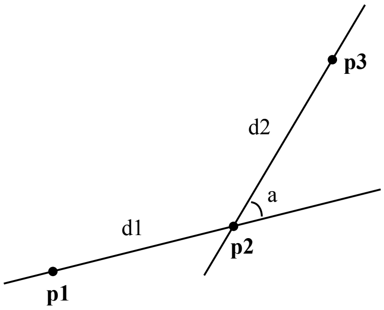

Figure 11: Planar line curvature estimation.

3) take the mean of these curvatures as an estimation of the mean surface curvature[1];
4) then, using squashed dot mapping, convert the results into a displayable 2D *curvature map*.

3.2 Results

Estimated mean surface curvature for Scans 13 and 14 is shown in the curvature maps in Figure 12. Positive surface curvature is defined as that which bends away from the laser source or, equivalently in this case, as that curvature associated with viewing a convex hull from the outside. Maximum positive curvature is shading coded as white. Zero curvature is shading coded as medium grey and maximum negative curvature is encoded as black. Out of range points are encoded as medium grey.

The curvature maps extract topological detail (such as the chip in the lower eyelid of David's right eye!) that is not as readily apparent in either of the other images shown in this paper.

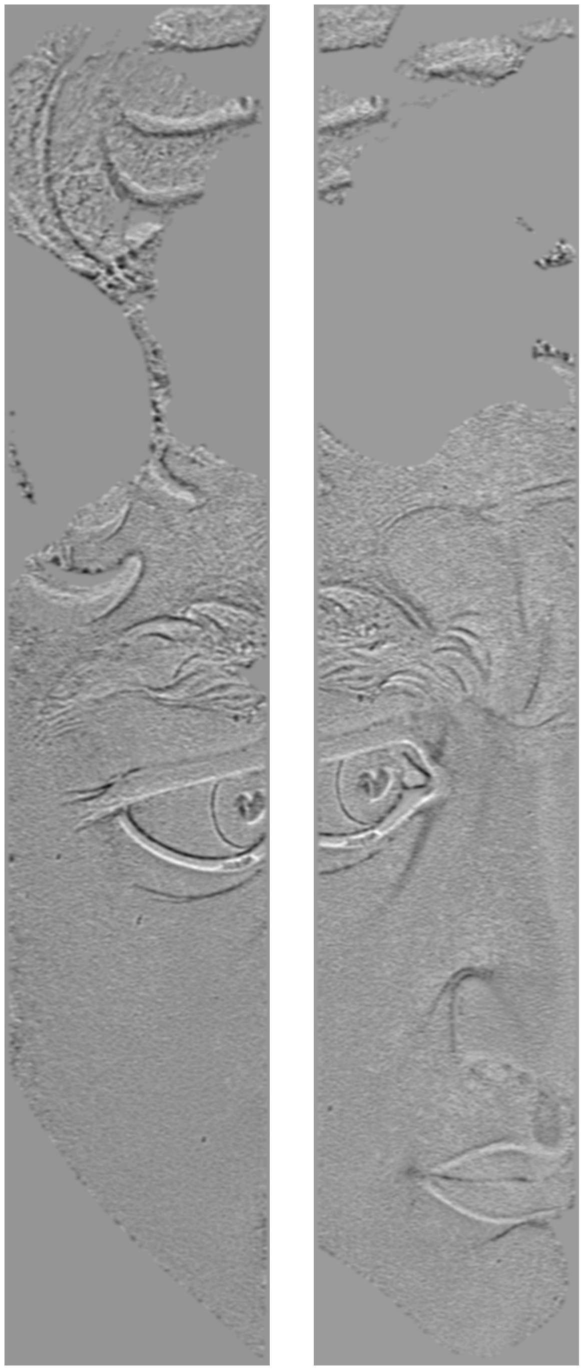

Figure 12: Curvature maps of scans 13 and 14.

4 Further work

Further work is anticipated to include using curvature maps to automatically align diverse overlapping scans of the same object [7].

5 Acknowledgements

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[1] In this first estimation off-axis correction has not been applied. This correction can be done using Meusnier's theorem [4].

References

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