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The University of Auckland
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"Curvature Maps and the Digital Michelangelo Project"

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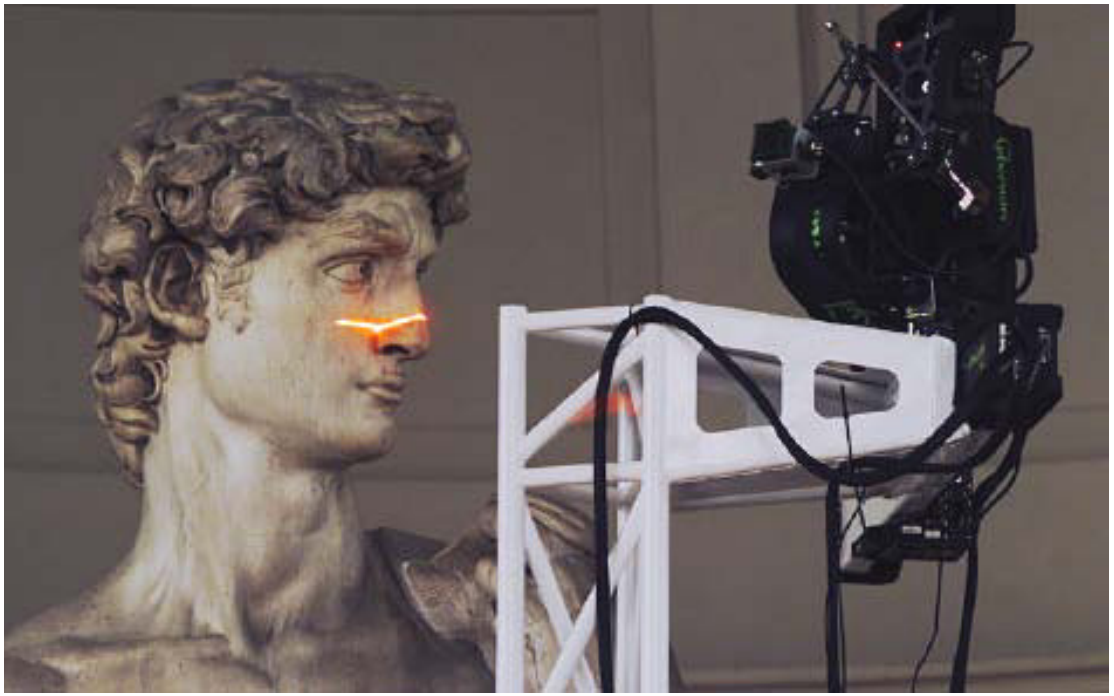
ELECTRICAL & COMPUTER ENGINEERING

MANUKAU INSTITUTE OF TECHNOLOGY
Te Whare Takiura o Manukau

Michelangelo's "David"

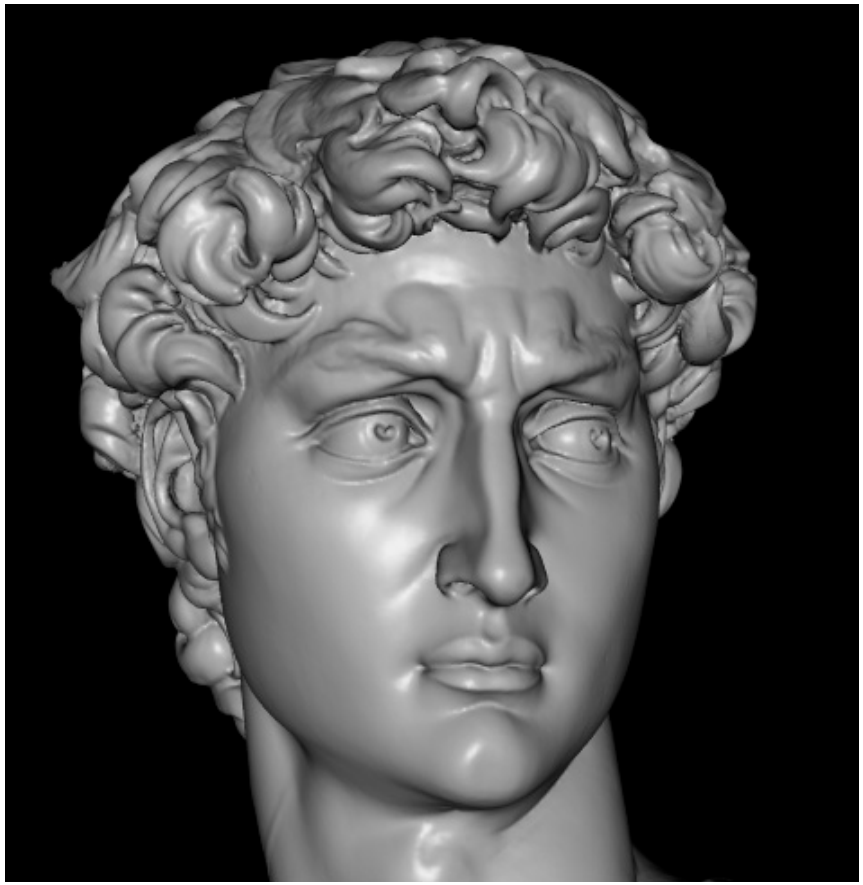
This historically significant, hand-carved, solid marble statue, completed in 1504, was located outside for nearly 400 years, subject to dirt and weathering. Today it is housed inside the Galleria dell'Accademia, Florence, Italy.

Digitized in 1998-99 by a team of 30 faculty, staff, and students from the Computer Science Departments at the University of Stanford and the University of Washington using a custom designed laser triangulation scanner.

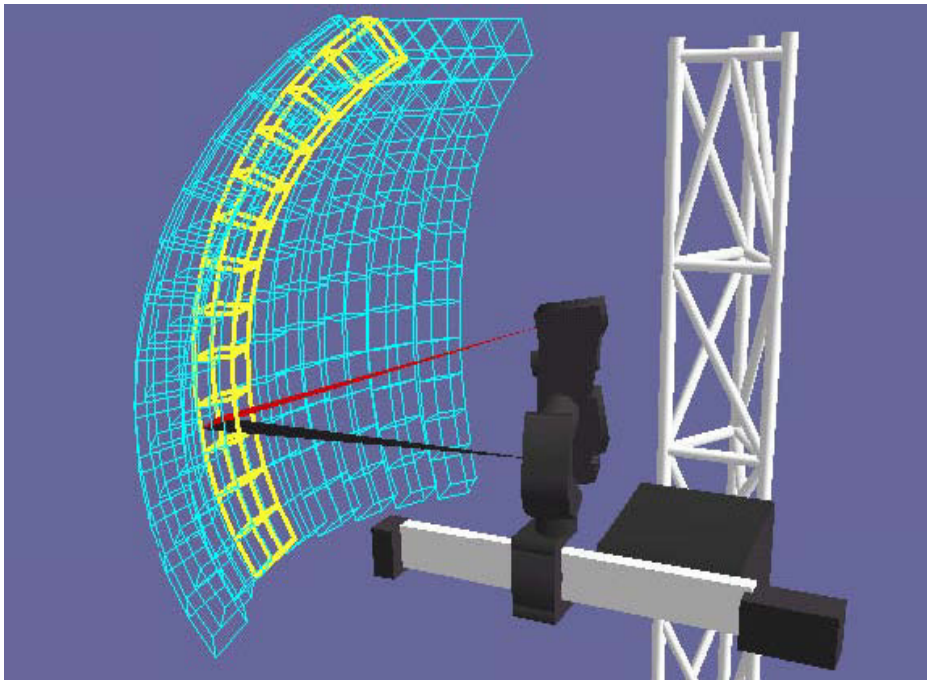


1.93 giga-bytes data, 6540 scan files, ~1.1 billion 3D points!

Reference rendering of a resultant mesh model.



The scanner geometry.



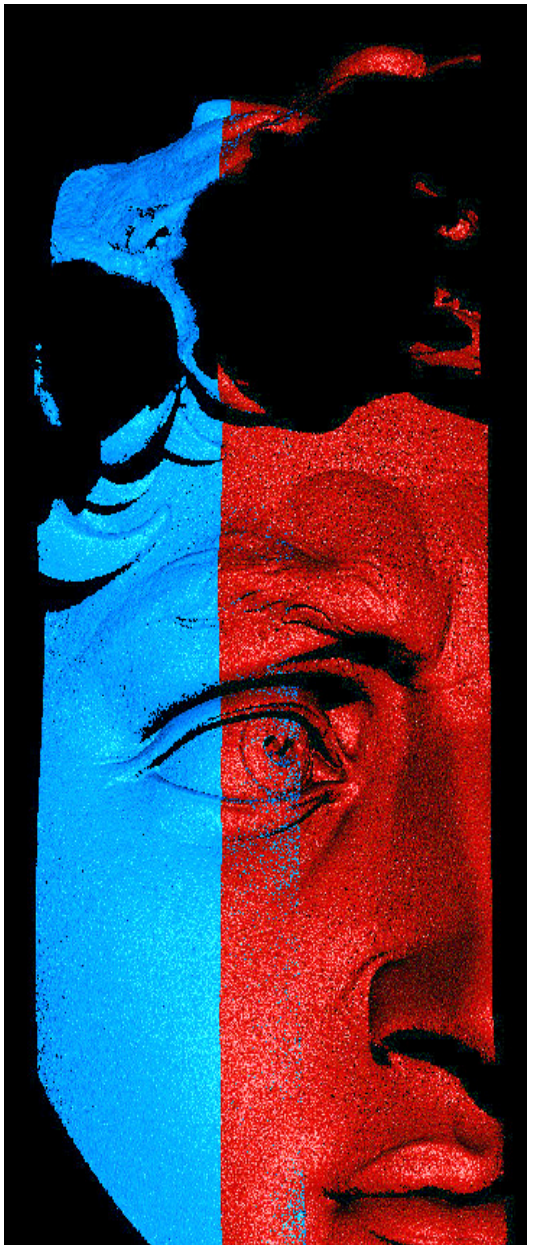
Stanford software converts depth values into 3D points using a concatenated sequence of transformation matrices modeled on the system geometry.

The C++ source code contains class definitions that define data structures. Consider the following code fragment:

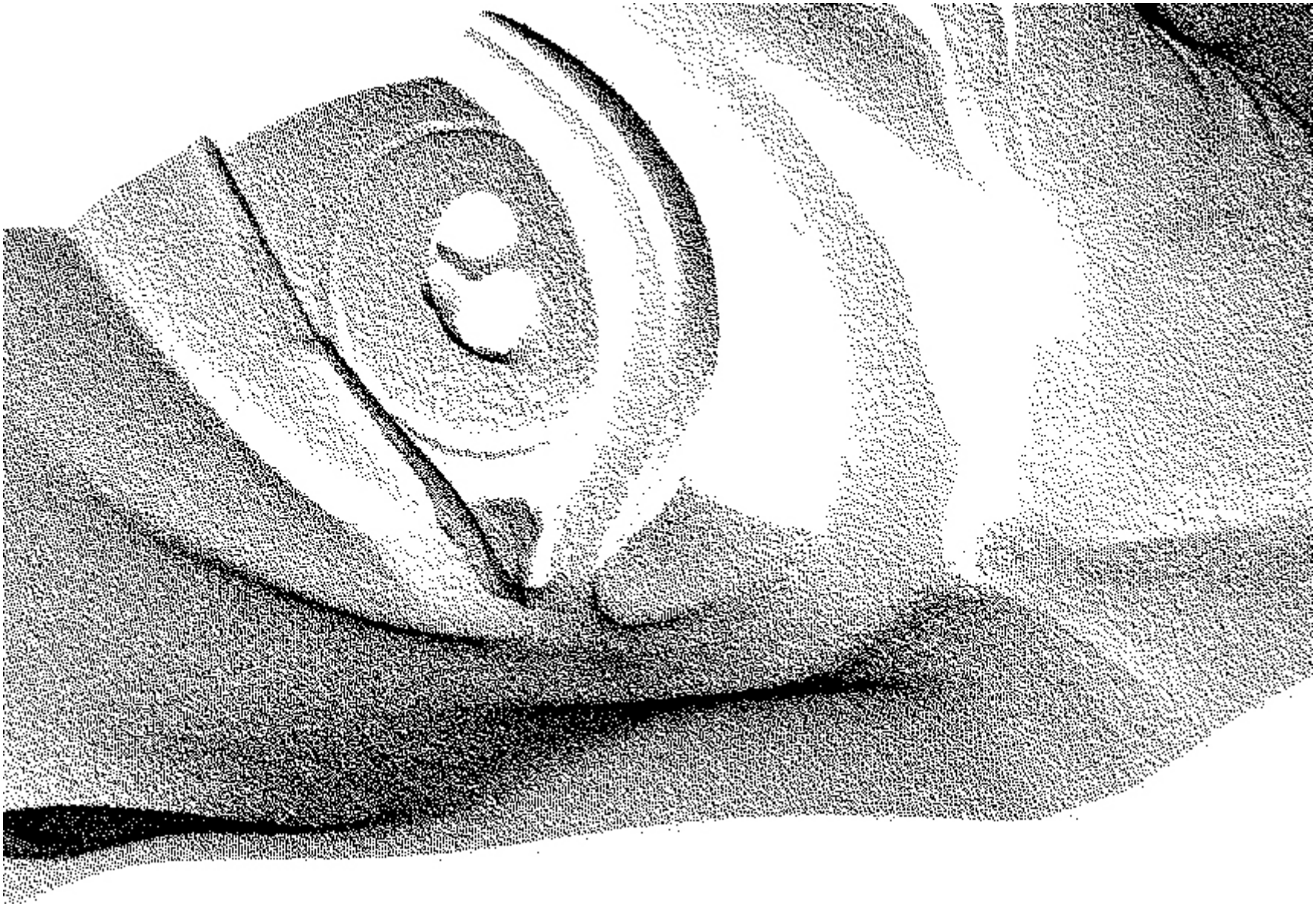
```
class SDfile{
public:
     unsigned int pts per frame;
     unsigned int n frames;
     unsigned short *z data;
};
```

Scan files 13 and 14 from the scan group Face1, for example, both have 486 points per frame and have 1480 frames and 1515 frames respectively.

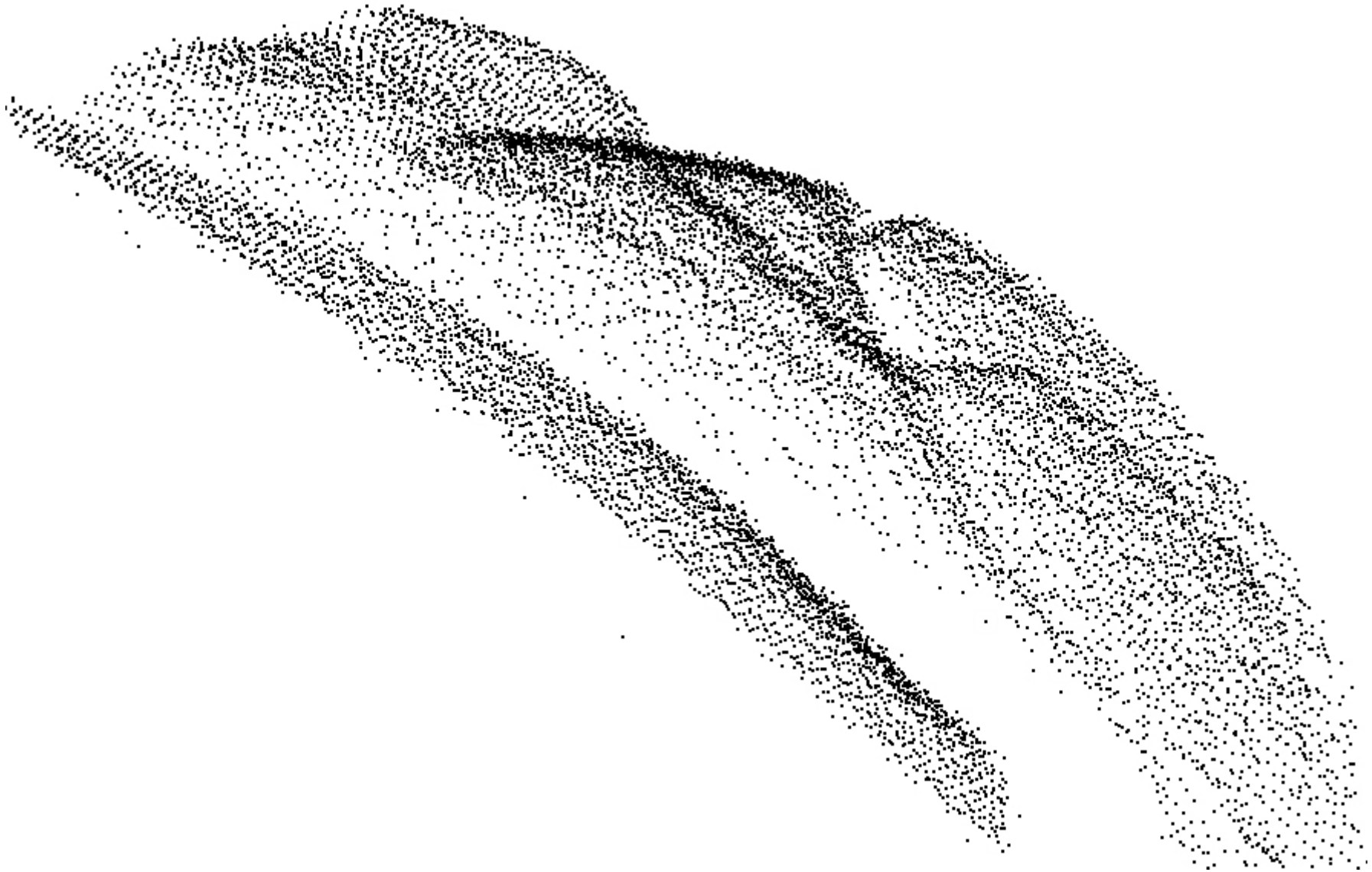
Each scan was acquired over a fixed width of approximately 140mm and a variable height.



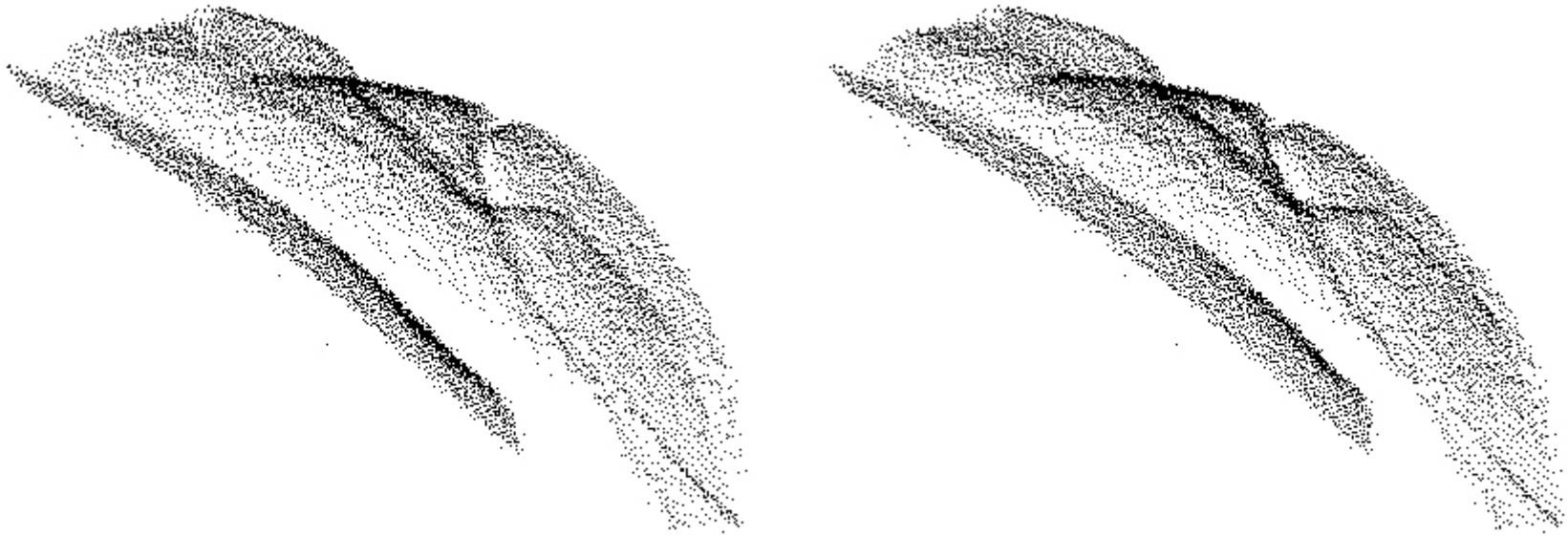
Scan point visualization using C++ and OpenGL.



Zoom in.



Stereo pair for 3D visualization!



Curvature:

Planar curves.

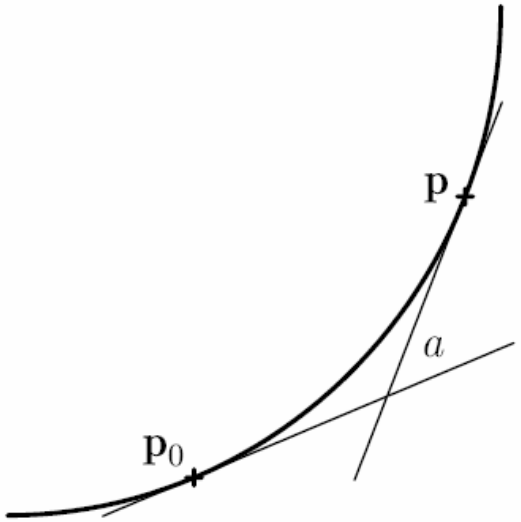


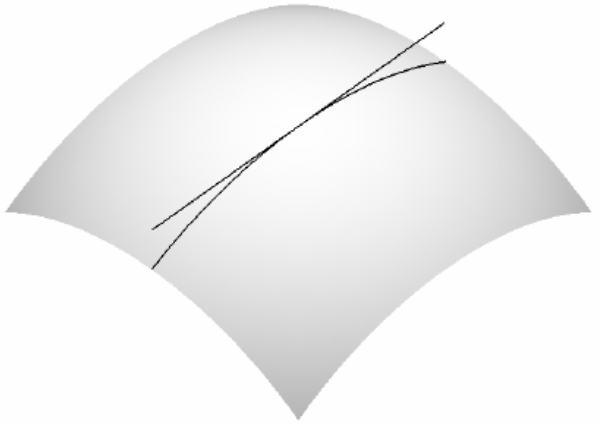
FIGURE 1. Curvature.

Consider a planar curve with arc-length l from the point $\mathbf{p}_0$ to the point $\mathbf{p}$, and the counter-clockwise angular advance a between the tangents at $\mathbf{p}_0$ and $\mathbf{p}$ as illustrated, for example, in figure (1). Then the *curvature* of the curve at the point $\mathbf{p}_0$ is defined to be

$$k = \lim_{\mathbf{p} \to \mathbf{p}_0} \frac{a}{l} = \frac{da}{dl}$$

Surfaces.

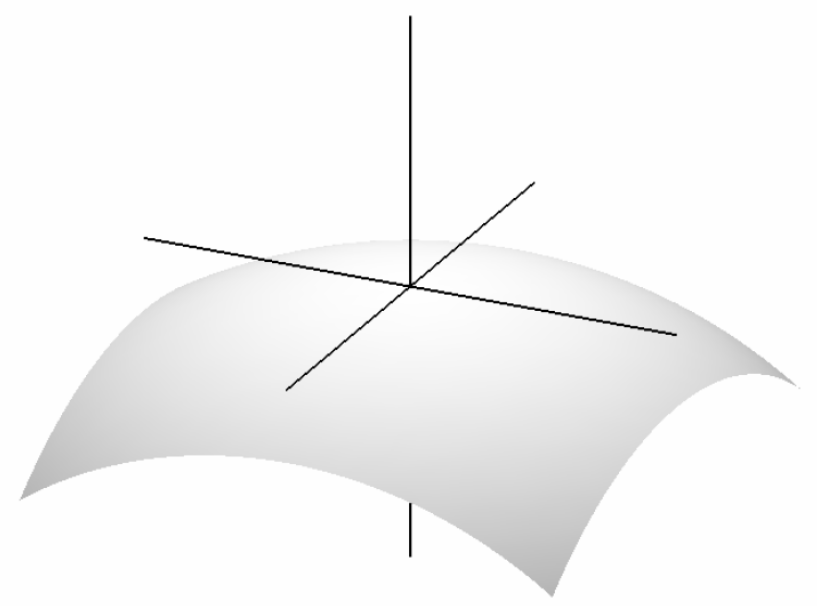
The intersection of surface and plane is a plane curve in the surface.



Tangent & curvature.

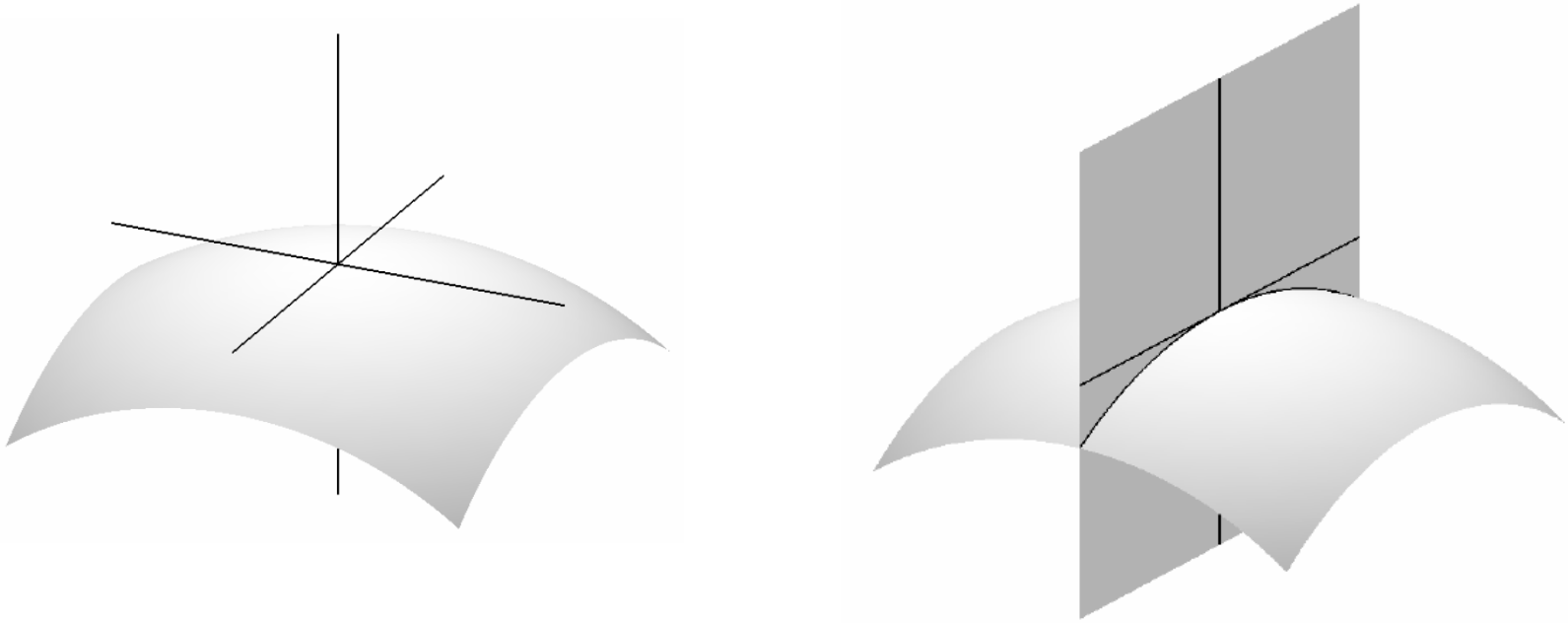
Surfaces.

Two tangents & the normal.



Surfaces.

Normal curvature is unique for each tangent direction.



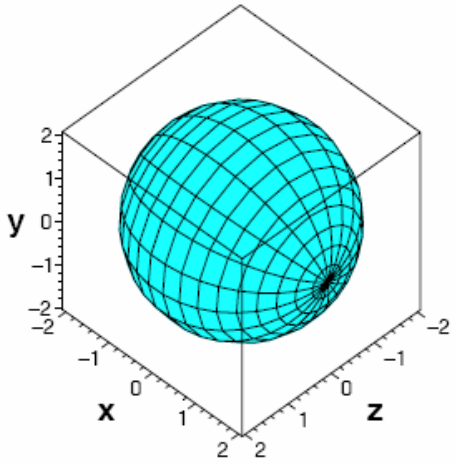
Surfaces.

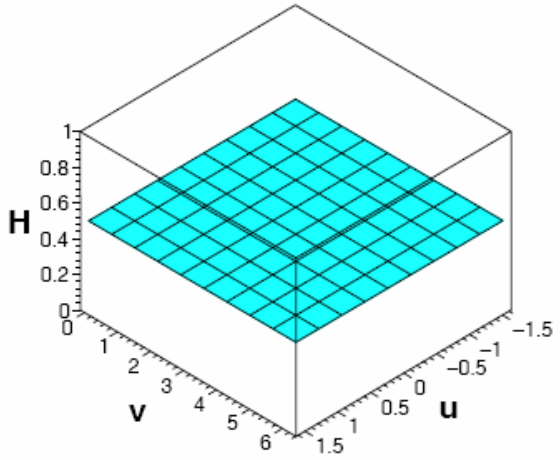
The two *principle curvatures* k_1 and k_2 are defined as the minimum and maximum normal curvatures.

The *mean curvature* is defined as: $\dfrac{k_1 + k_2}{2}$

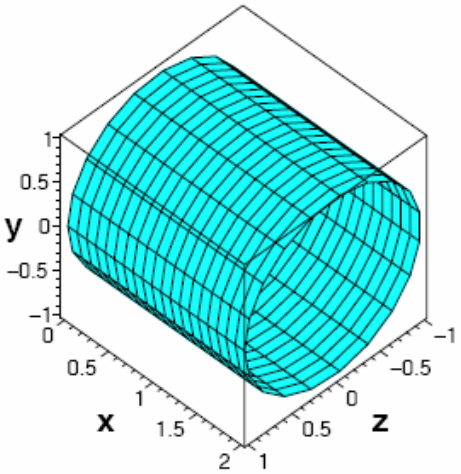
Mean Curvature.

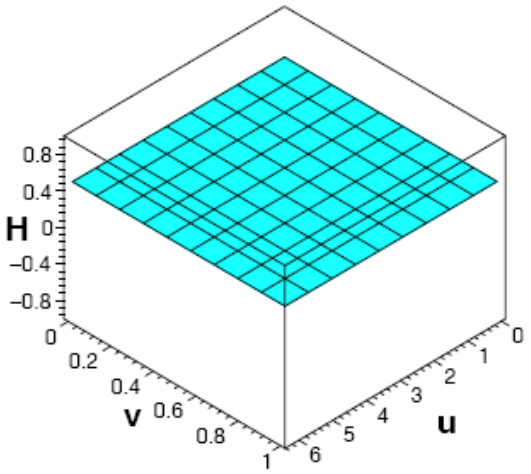
Sphere.





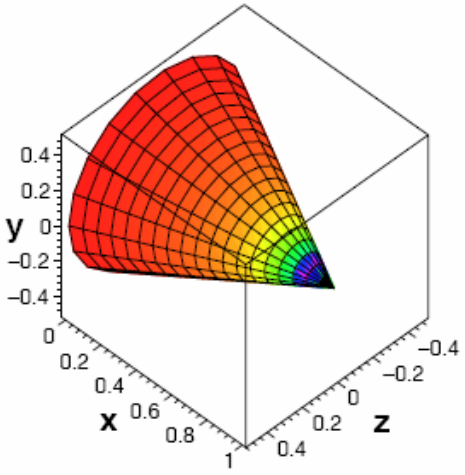
Cylinder.

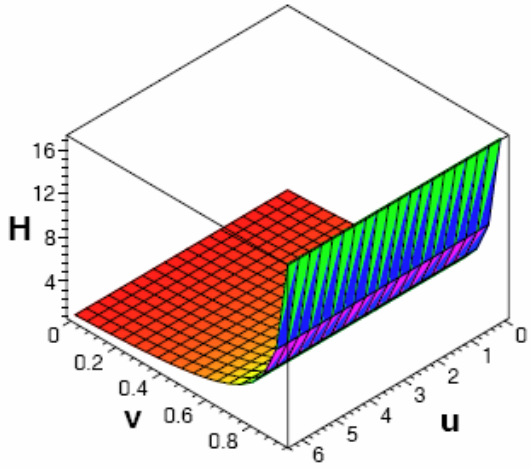




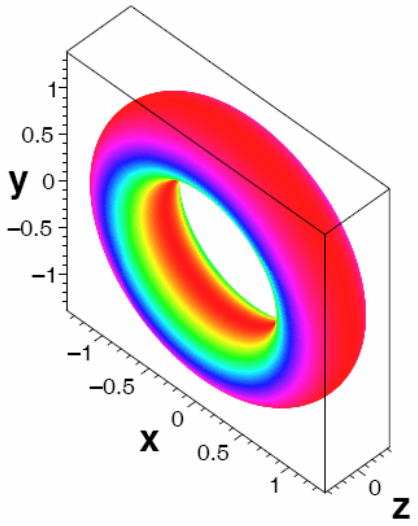
Mean Curvature.

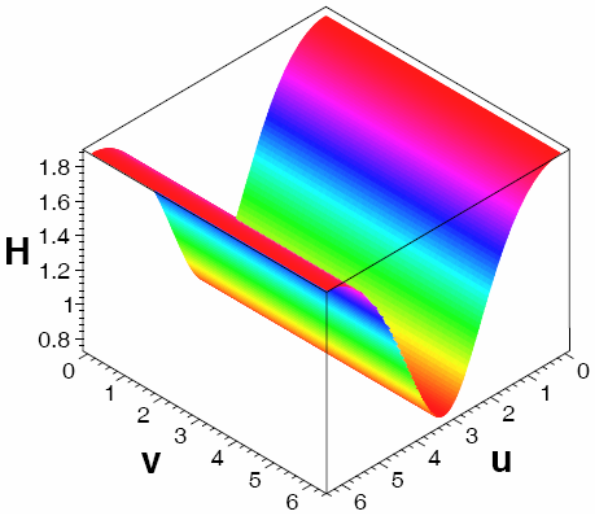
Cone.





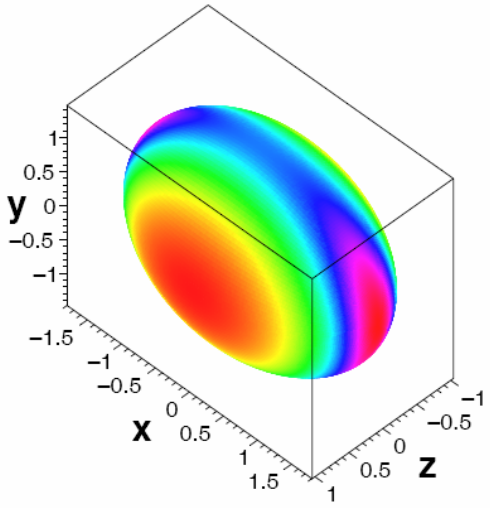
Torus.

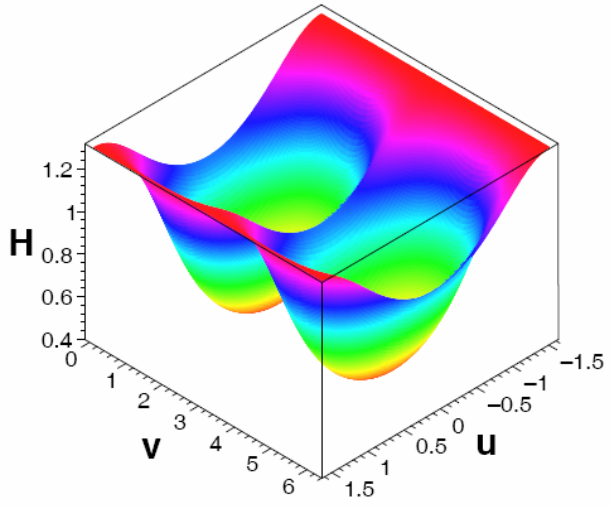




Mean Curvature.

Ellipsoid.



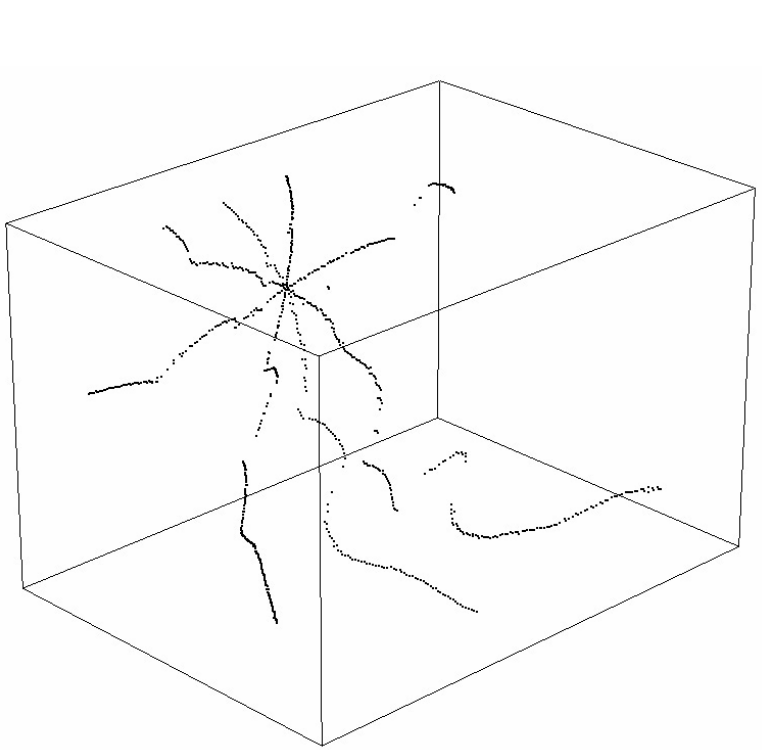


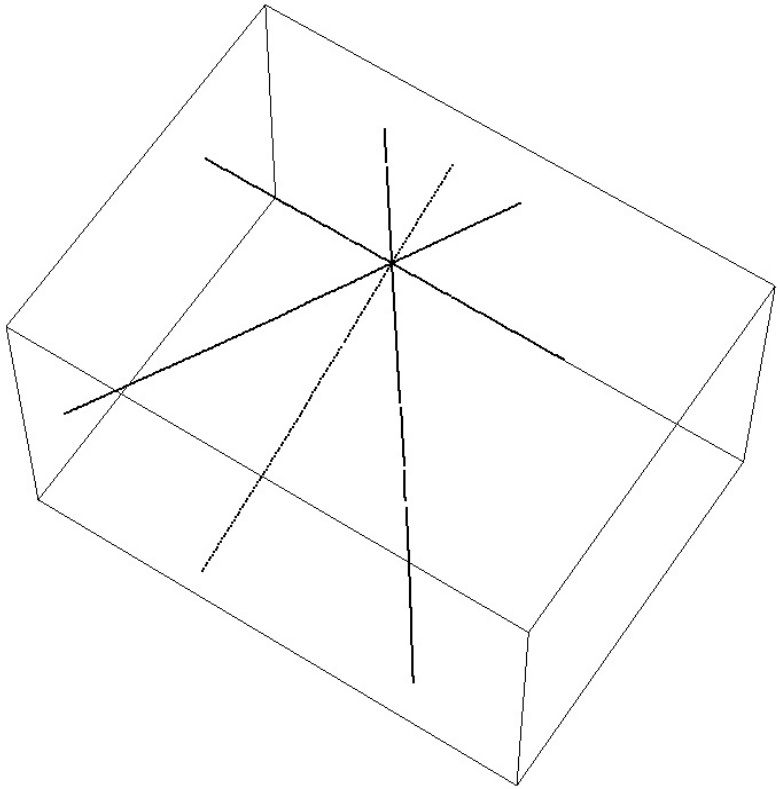
Surfaces.

For mean curvature, the normal curvatures associated with any two orthogonal *cuts* will work!

$$H = \frac{k_1 + k_2}{2}$$

The David statue:
Two different views of four cuts through the same point in scan file 14.

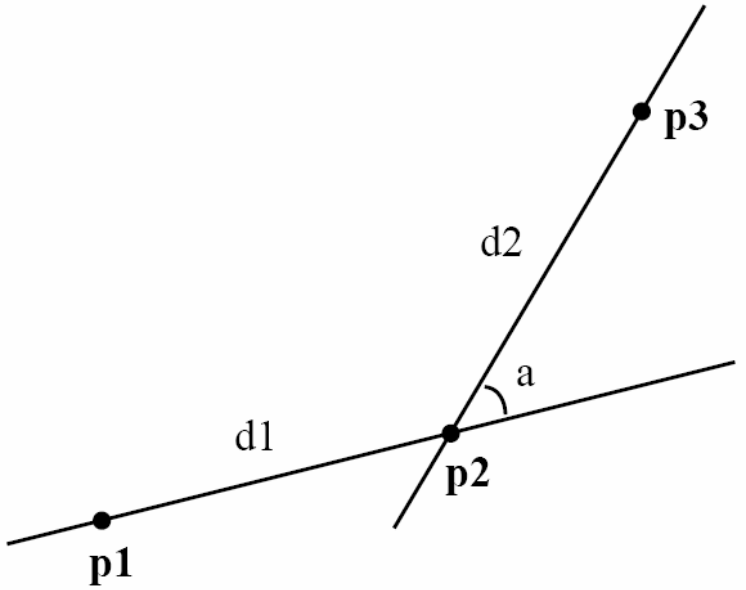




Curvature *estimation* from scan data.

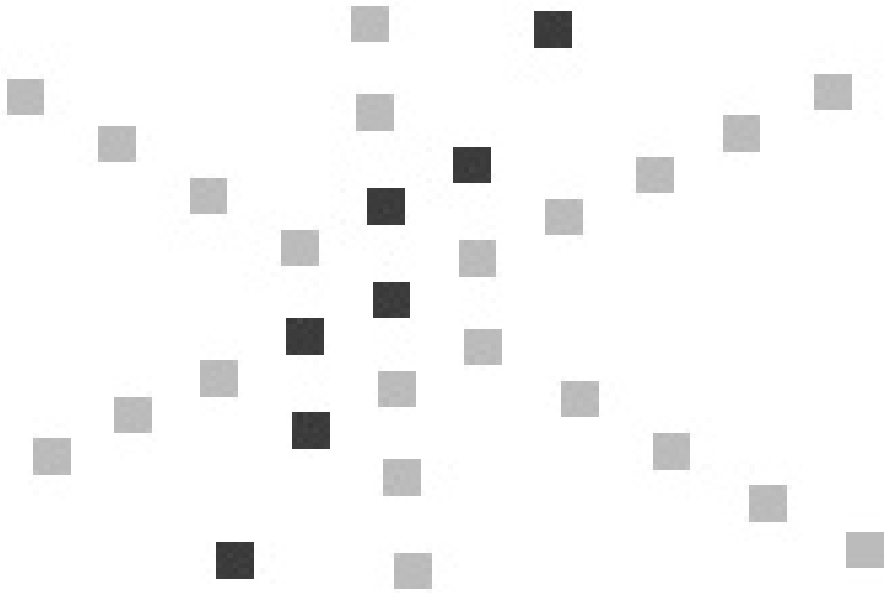
Planar curvature:

$$k = \frac{a}{(d1 + d2)/2}$$



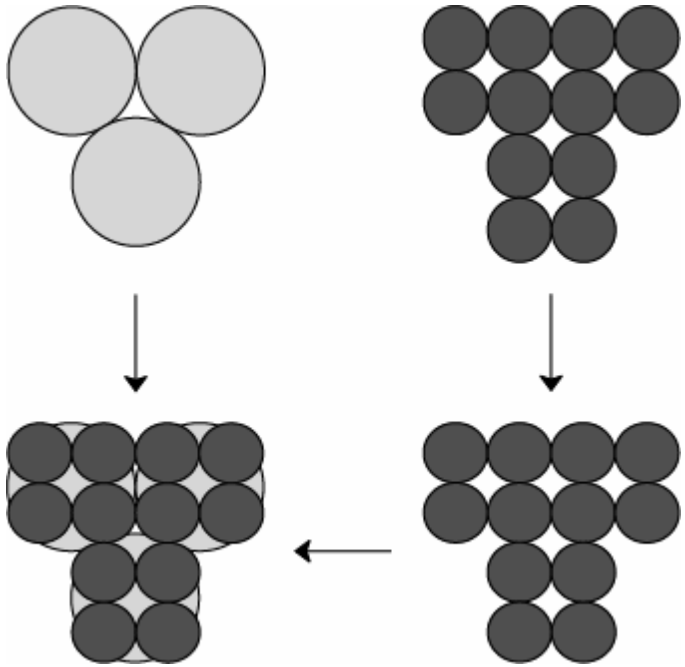
Hexagonal adjacency pattern and interleaved acquisition.

Cut selection for curvature calculations?



Display results on a 2D rectangular grid?

Squashed dot mapping.

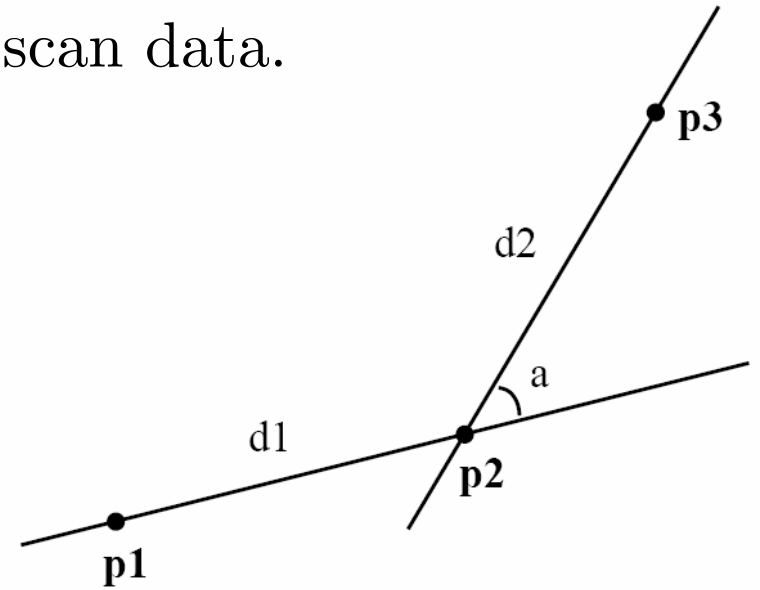


Results:
Depth maps of scans 13 and 14.

Points closest to the laser source are shading coded as white, and points further away are encoded with decreasing intensity towards black.

Out of range points (either too close, too far away, or beyond angular limit) as well as non-reflective points are all coded as black.

Curvature calculation from scan data.



$$k = \left(\frac{2}{||\mathbf{v1}|| + ||\mathbf{v2}||} \right) \cos^{-1}\left(\frac{\mathbf{v1} \cdot \mathbf{v2}}{||\mathbf{v1}||\,||\mathbf{v2}||} \right)$$

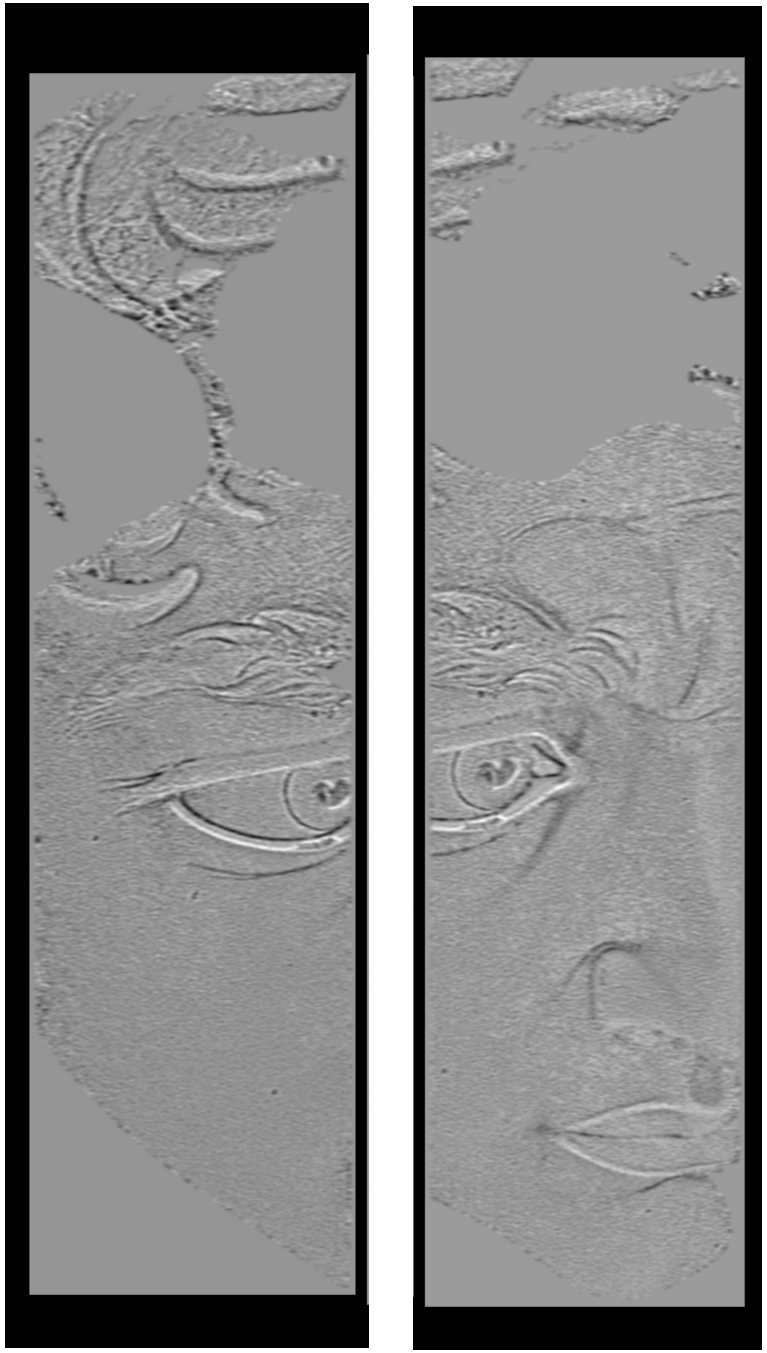
where $\mathbf{v1} = \mathbf{p2} - \mathbf{p1}$ and $\mathbf{v2} = \mathbf{p3} - \mathbf{p2}$

Results:
***Curvature maps* of scans 13 and 14.**

Maximum positive curvature is shading coded as white.

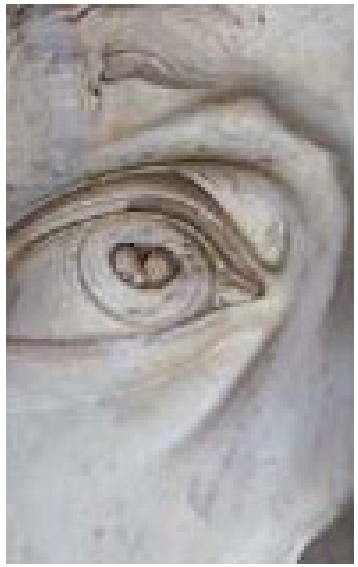
Zero curvature is shading coded as medium grey and maximum negative curvature is encoded as black.

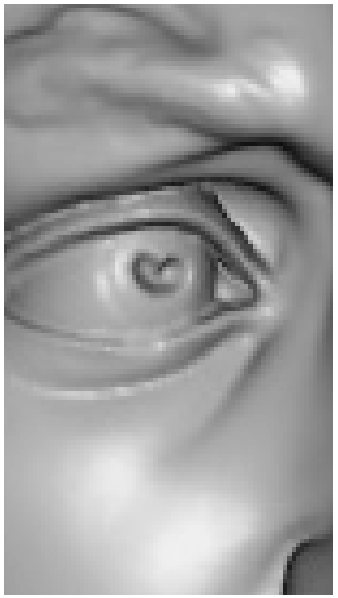
Out of range points are encoded as medium grey.

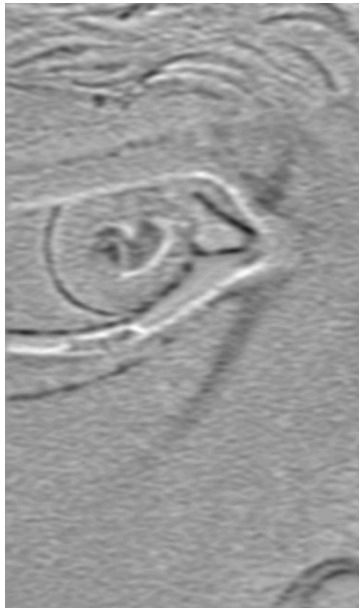


Conclusion:

The curvature maps extract surface detail (such as the chip in the lower eyelid of David's right eye!) that is not as readily apparent in the photographic image or in the reference rendering.







Acknowledgements:

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References:

[1] M. Levoy, K. Pulli, B. Curless, S. Rusinkiewicz, D. Koller, L. Pereira, M. Ginzton, S. Anderson, J. Davis, J. Ginsberg, J. Shade, and D. Fulk, "The digital Michelangelo project: 3D scanning of large statues," in *Proc. SIGGRAPH*, pages 131–144, 2000.

[2] M. Levoy, "The digital Michelangelo project." presentation slides, SIGGRAPH, 2000.

[3] K. Pulli, B. Curless, M. Ginzton, S. Rusinkiewicz, L. Pereira, and D. Wood, *Scanalyze v1.0.3: A computer program for aligning and merging range data*. Stanford Computer Graphics Laboratory, 2002.

[4] A. Davies and P. Samuels, *An Introducion to Computational Geometry for Curves and Surfaces*. Oxford University Press, Oxford, 1996.

[5] R. Klette and A. Rosenfeld, *Digital Geometry*. Morgan Kaufmann, San Francisco, 2004.

[6] S. Hermann and R. Klette, "Multigrid analysis of curvature estimators," CITR-TR-129, Centre for Image Technology and Robotics, University of Auckland, 2003.

[7] K. Pulli, "Multiview registration for large data sets," in *Proc. Int. Conf. 3D Digital Imaging and Modeling*, pages 160–168, 1999.