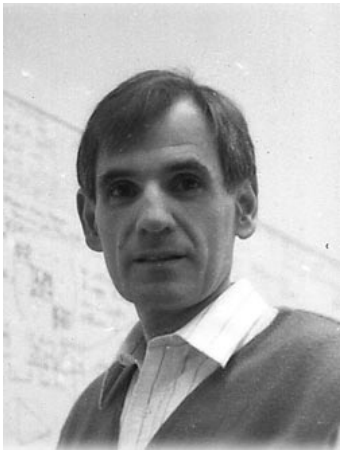
László Babai



László Babai is a professor in the Departments of Computer Science and Mathematics at the University of Chicago. Born in Budapest in 1950, Professor Babai received his degrees in mathematics from Eötvös University, Budapest, and the Hungarian Academy of Science, and he taught at Eötvos University for nearly two decades. He joined the University of Chicago in 1984 but maintained his affiliation with Eötvös University until 1994, splitting his time between the two continents. Professor Babai has served as an editor of 11 periodicals, evenly split between mathematics and computer science. He has written over 150 research papers in combinatorics, group theory, algorithms, and complexity theory, forging many links between these areas. As a "mathematical grandchild of Paul Erdős," he has used probabilistic considerations in all these contexts. He was one of the recipients of the first ACM SIGACT – EATCS Gödel Prize (1993). We quote the citation:
"The 1993 Gödel Prize is shared by two papers (Babai and Moran [BM], Goldwasser, Micali and Rackoff [GMR]). These papers introduced the concept of interactive proof systems, which provides a rich new framework for addressing the question of what constitutes a mathematical proof. The invention of this framework has already led to some of the most exciting developments in complexity theory in recent years, including the discovery of close connections between interactive proof systems and classical complexity classes, and the resolution of several major open problems in combinatorial optimization."
Professor Babai received the Erdős Prize of the Hungarian Academy of Science in 1983. His numerous plenary lectures include the European Congress of Mathematics (Paris, 1992) and the International Congress of Mathematics (Zürich, 1994). In 1990, he was elected a member of the Hungarian Academy of Science. In the past five years he gave lecture series in Canberra, Paris, Rome, Montreal, Bath (U. K.), Jerusalem, and at Penn State on a diverse set of topics, including algorithms in finite groups, interactive proofs, communication complexity, vertex-transitive graphs, and the life and work of Paul Erdős. Babai held the André-Aisenstadt Chair at the Université de Montréal in Autumn 1996. He is the 1996-98 Pólya lecturer of the Mathematical Association of America. He is a cofounder of the highly acclaimed study-abroad program "Budapest Semesters in Mathematics."

The Forbidden Sidetrip

A mathematician by background, I became interested in the theory of computing in the late 70s, several years after completing my doctorate. A key episode of the eventful period of transition, against the background of a bygone bipolar world, is described in this story.

Leningradskiĭ Vokzal[0]

I entered the huge, dimly lit hall with a definite sense of adventure. It was 9:40 p. m. on Friday, November 3, 1978. The place was Moscow, capital of the Soviet empire. I had the feeling that the events of the next day might accelerate a change in the direction of my career, a change that in turn would profoundly affect my future.

After a brisk walk in the autumn chill and drizzle, the dreary hall offered a welcome refuge. I joined one of the three unpromising lines leading to small windows at the distant end of the hall, gateways to joy and fulfillment. My supreme desire, a trip to Leningrad that very night, was apparently shared by hundreds of aspiring souls neatly lined up before me.

My hopes were pinned on the slim chance that I would ever advance to the coveted goal line, and that, once there, I would not be *exposed*.

I was not sure how well I would pass for a Russian. While merely standing in the line, I thought I should be all right. My shoes were arguably more comfortable than what the Brezhnev bureaucracy thought fit for the feet of their faithful citizens, but who would notice? My cap was actually inferior to that of the average local, at least in its capacity to protect against the elements that had beaten Napoleon. I admit my overcoat wasn't exactly from GUM[1], but it was still the product of a "socialist" economy. It was made in my native Hungary, then a "fraternal country" in the Soviet bloc, whose central planners were hardly fettered by an overwhelming sense of duty to heed the consumer's whimsical concerns such as fashion, comfort, or utility. I did not wear my jeans[2], or anything stylish. So I felt quite safe.

My case would become more tenuous, however, right at the *okoshko*, the legendary little window where fates are decided.

My Russian was good enough to conduct a conversation and even to give lectures, thanks to ten years of Russian language classes I had to take at school

[0] The Leningrad Railway Station: located in Moscow, this was the station of departure of trains to Leningrad. After the fall of Communism, that destination regained its old name, Saint Petersburg.

[1] Central shopping mall at the Red Square in Moscow

[2] I bought my first pair of jeans in Vienna the previous summer. Jeans, the products of Western decadence, were not available in the Soviet bloc, except on the black market.

(a splendid demonstration of how effectively guns can spread culture[3]) followed by an adventure-filled semester of study in mathematics in Leningrad, complete with visits to ten out of the fifteen Soviet republics.

Years of Russian studies notwithstanding, my Hungarian accent would give me away the instant I uttered a word. But this, I was hoping, would not necessarily be fatal. Among the many nations that had merrily joined the Soviet family under Stalin's guns (or the Russian empire under the Czar's), some, especially in the Baltic, so detested the Russification of their land and culture that they refused to learn Russian to any decent degree. I knew from experience that my Russian, however poor, was still competitive with that of the average Estonian. And all I needed was to pass for a Soviet citizen, not necessarily a Russian.

But being a Soviet citizen was a necessary pretense. Whether from fraternal Hungary or from the imperialist West, as a foreigner I was not supposed to stand in that line, to mix with the natives. Not in the ticket lines, and not on the train. The attention of the Soviet government to foreigners was painstaking. The offices of the Intourist were there specifically to serve our needs. Hotel? Airline or train ticket? Just go to the Intourist. No lines, no waiting, you are comfortably seated while an agent talks to you in any of several foreign languages, and you get tickets even while Soviet citizens are turned away under the pretext that the flight or the hotel has been fully booked. You will not need to share the crowded railway cars with the locals; this in turn also serves the noble purpose of protecting the virtuous workers of the worker-state from possibly harmful encounters with Western decadence.

There was one little thing, though, that you needed to show at the Intourist office: your passport. I had one, of course, and I also had a permit to be in Moscow. What I did not have, however, was a permit to visit Leningrad, my unforeseen destination. Alas, I was not able to take advantage of this superb, if compulsory, convenience.

Planned research in a planned economy

While waiting in the dull grey line at the "Leningrad" railway station in Moscow, I had ample time to reflect on how easy it would have been, with some foresight, to have a permit for Leningrad in my wallet.

Employed as an assistant professor at Eötvös University, Budapest, I was in Moscow on official scientific exchange pursuant to the scientific cooperation treaty between the Soviet and the Hungarian Ministries of Education. Notably, this fact alone restricted the choice of my host institutions to university departments and excluded the important mathematical research institutes of the USSR because those did not fall under the jurisdiction of the Ministry of Education.

I had applied for the one-month visit 18 months earlier, specifying the institutions to be visited and the names of the host scientists, as well as the scientific subject matters to be explored. Obviously, a responsible scientist would plan

[3] The same teaching tool may have played some role along the way as Shakespeare's tongue gained its universal status...

such things well in advance, rather than haphazardly following a random, if brilliant, new idea that might call for a collaboration with a different group of people at entirely different locations and possibly lead to dramatic breakthroughs extraneous to the meticulously designed multi-year plans.

I thought that this kind of clay-brained bureaucratism was a trademark of the Communist system. How enormous was my astonishment when I learned in recent years, that, following the strict guidelines of the European Union[4], mathematicians all over free Europe, on both sides of the former Iron Curtain, were fervently writing proposals for long-term collaboration between small groups of institutions! If person A has a mathematical problem to discuss with person B, then the colleague next door to A should have a project with the colleague next door to B, right? Post-cold-war Brussels has surely caught up with cold-war-time Moscow's oxymoron of "planned mathematical research[5]."

Having by nature been endowed with the requisite foresight, in May 1977 I set forth a plan calling for spending two weeks at Minsk State University in Belorussia, as Belarus used to be called, and another two at Moscow State University (MGU). At the university in Minsk I had a friend working in graph theory, and more importantly, this plan would give me the opportunity to meet several colleagues working on the interfaces of graph theory and algebra at the Mathematical Institute of the Belorussian Academy of Science, although I could not name that institution among my hosts for the aforementioned reason. As to Moscow, the city used to be a mathematician's Mecca. I had many colleagues at various institutions with whom I had desired to consult; none of them, unfortunately, affiliated with MGU and some of them at institutions altogether forbidden to foreigners.

For the proposal, however, I needed a host at MGU. Through a colleague in Budapest working in universal algebra I contacted professor Skornyakov of MGU, and he generously agreed to be my nominal host. For his kindness he would later be subjected to some unpleasantness. At this point, however, his consent cleared the way for the proposal which was subsequently approved, and

[4] Seated in Brussels, the European Union (European super-government) has recently emerged as a major sponsor of European research cooperation. If you want EU money for a "European research conference," EU bureacrats will tell you where you can hold it, they will grade your speakers according to their "degree of Europeanness" (if you invite too many Russians, Israelis, or Americans, your conference is canceled – even Poles, Czechs, Hungarians are not counted favorably, they are on the wrong side of the *EU curtain*), you even have to keep a balance between various West European countries (too many Britons may be a problem), and pay attention to "less developed regions" of Western Europe (Mainz is apparently less developed than Frankfurt). That you would prefer having the 12 best speakers you could get? Sorry, if their geographic distribution is inappropriate, please go elsewhere for funding.

[5] For an insider's view on this subject, I highly recommend "Lemma One," the captivating and hilarious story of a brilliant female math graduate student in Socialist East Germany, embroiled in a controversy over having dangerously *exceeded* the plan of mathematical production at her department with the proof of a famous conjecture ... [Kö]. The author is Helga Königsdorf, herself a bearer of a degree in mathematics.

I was instructed to wait until the host institutions would report their readiness to receive me. How this readiness was being determined remained a mystery; my friends at those places were certainly ready to see me any time.

While waiting for the green light which was supposed to arrive through the Ministry of Education, I received a letter from the New World; professor Bjarne Jónsson, one of the great innovators of universal algebra, invited me to spend a term at Vanderbilt University (Nashville). Anxious to make my debut in the U. S., I accepted, although universal algebra was not my primary interest, and the salary, to be covered by professor Jónsson's grant, was quite meager, considerably less than what I had been offered two years earlier, in 1976, at Syracuse (N. Y.).

The Syracuse visit fell through because, at the last minute, the Hungarian Ministry of Labor vetoed my employment abroad, and, as a consequence, my exit visa was denied[6] by the Ministry of Interior. I never got an official explanation, but it has been suggested that, at 26, I was not deemed mature enough to navigate the ideologically treacherous American terrain.

Thus denied a new learning experience, I nevertheless did have the pleasure of enjoying a decisively new, albeit somewhat different, and ideologically certainly less onerous, course of studies that same autumn, courtesy of another department of the government: in November 1976 I was drafted to the army as a "student" toward the degree of "reserve officer, rank sub-lieutenant." The memorable curriculum included everything from 2 a. m. alarm drills to spending a week inside a cozy A. P. C.[7] I took several unfinished mathematical manuscripts to the barracks in the expectation that there would be ample leisure time in which to complete the papers. My peers would have tolerated such diligence, everyone in our "class" had a university degree. And there was indeed no shortage of leisure time. My morale as a captive, however, was so low I was unable to make better use of the time than to play chess in the beginning and cards near the end. Mercifully, this interlude in my life lasted only five months. I celebrated my successful "graduation" from that institution of higher learning by completing five research papers in a single week, a feat never again to be repeated.

My newly acquired "degree" did little to quell my desire for an extended visit overseas. Professor Jónsson's plan called for a start in Nashville in January 1979. Graph theorist Pavol Hell, then at Rutgers, helped preface the journey by a month in New Jersey, so I had to leave Budapest on November 30, 1978. I thought nothing would ever come of the Minsk–Moscow visit and was busy with the arrangements for my long awaited first trip to the United States when I got the urgent message from the Ministry of Education that Minsk and Moscow

[6] Exit visa? Those who grew up in a free country might wonder what on earth that might be. Yes, without an exit visa, we could not *leave* our country. So, taking up U. S. employment *without* Hungarian authorization was not an option, I could not even cross the Hungarian border. Found that border on the map? The country within, indeed a wonderful country with rich history and vibrant culture, occupies an area of the *size of Indiana*. Just a bit claustrophobic for a scientist without an exit visa.

[7] Armored Personnel Carrier.

were ready to receive me. This happened at the end of September, and two weeks later my plane landed at Moscow's Sheremetyevo International Airport.

After my return to Budapest from the USSR, I would have three weeks to ready myself for what turned out to be a 16-month journey which included visits to about two dozen universities in the U. S. and Canada. I was aware that any irregularity during my USSR trip, such as an unauthorized side-trip, if noticed by the Soviet authorities, would cancel my exit visa to the West.

Strangely enough, my visa to the USSR contained very little information other than that I was on scientific exchange. The passport control officer at Sheremetyevo Airport completed my permit to stay in the USSR; he asked me the cities to be visited and the names of the host institutions, and did not require any official document to support my statements. I honestly gave him Minsk and Moscow as my destinations, which he dutifully entered on my permit, certifying the new entries with his rubber stamp.

Little did I know that the most important target of this trip should have been Leningrad. Had I had the presence of mind, I could have gained the option to visit Leningrad by simply naming that city, too, to the Sheremetyevo passport officer. As far as I can tell, there was no risk involved. However, what I had told him was now carved in stone, to change the permissions was virtually impossible.

I had seven hours before I would need to board my flight to Minsk[8] at the domestic Domodedovo airport. I used the time to pay a brief visit to my friend Imre Bárány, a discrete geometer from Budapest, who was working in Moscow at the time. This small detour would later acquire vital significance.

Minsk: a place for joy and sorrow

In Minsk I felt like I was coming home. I made new friends and renewed friendships dating back to my undergraduate semester spent in the Soviet Union. I gave a series of lectures (in Russian) to an interested audience; this was the place where my early work, on automorphisms of graphs, was most appreciated. A combination of algebra and combinatorics was a hallmark of a research group striving under the motherly direction of Regina Iosifovna Tyshkevich, a leading mathematician, the wife of algebraist Dmitri Suprunenko, member of the Belorussian Academy of Science, and mother of group theorist Ira Suprunenko.

I learned a thing or two about freedom of speech, too. Friends would take me to the middle of a great park in town, and when at safe distance from everyone else, they would pour out their souls. The talk was about life in the Soviet Union, in general as well as in mathematics, under the oppressive climate of the progressively rigid Brezhnev regime.

[8] Minsk lies slightly off the straight line connecting Budapest to Moscow; the distance of Budapest to Minsk is less than a third of the combined distances of Budapest–Moscow and Moscow–Minsk. Yet, although Minsk was my first destination, I had to enter, as well as leave, the USSR via Moscow. This instance of wastefulness turned out to be unexpectedly helpful a little later.

A few months earlier I had attended the International Congress of Mathematicians in Helsinki. Half of the 30 Soviet invited speakers were "unable to attend." (Eminent Soviet mathematicians apparently tended to have family problems, especially in the month of August. High teaching loads during the summer break may also have been a factor...). Congress participants used a session fully composed by absentee Soviet speakers as a forum of discussion and protest. "Free Sharanski and Massera" buttons were distributed to bring attention to the plight of two scientists who were prisoners of conscience, Sharanski (now a member of the Israeli government) in the Soviet Union, and Massera, a Communist, in South America.

In the hierarchically organized Communist society, every discipline or activity had to have a supreme leader, and the branches of mathematics were no exception. Since combinatorics was a relatively new discipline, not represented by any member of the Academy of Sciences, it was extremely difficult to organize conferences in the area.

L. S. Pontryagin, I. M. Vinogradov, and S. V. Yablonskiĭ had gained supreme power in Soviet mathematical life. Led by the virulently anti-semitic Pontryagin, the troika introduced a crippling bias across the entire spectrum of their activities, ranging from admissions exams at Moscow State University, the leading university of the country, to the awarding of higher degrees in mathematics throughout the USSR, publishing in prestigious journals, and the hiring policy at the Steklov Institute, the elite research institute of mathematics in Moscow then headed by Vinogradov. Emigration to Israel was virtually halted by the government, removing that glimmer of hope for escape.

Pontryagin was instrumental in preventing half the Soviet invited speakers from attending the Helsinki congress. Pontryagin's attention extended to the "struggle against Jewish influence" in secondary school curricula. It is tragic that Lev Semyonovich Pontryagin (1908–1988), the blind genius of 20th century mathematics, should have harbored such abysmal bigotry.

On my first weekend in Minsk, my hosts took me to a gripping World War II memorial. It had been the tactic of the Nazis to retaliate for a guerilla attack in the occupied parts of the USSR by murdering an entire village. They would round up all residents, lock them up in a barn, and then blow up the barn or set it on fire. The symbolic cemetery, set up in 1969 in the village Khatyn'[9] (near Minsk) had hundreds of burial marks, each representing not an individual but a village.

But Minsk was not all sorrow. Curiously, it was Minsk where I first heard "The Köln Concert" by Keith Jarrett and a record by Chico Hamilton. The records that somehow made it across the border were played at the home of a mathematician, jazz fan, and amateur acoustical engineer, played on his homemade stereo.

[9] Not to be confused with Katyn (Katin), a small town near Smolensk, the location of the massacre of thousands of Polish army officers at the hands of Stalin's NKVD (Soviet secret police) in 1939–1940. This event is not remembered in Khatyn'; until Gorbachev, the Russian public was not allowed to know about it.

One evening we went to a baroque concert. Another night, Regina Iosifovna took me to a performance of Macbeth which remains memorable for more than the merits of the play. We agreed to meet in the foyer of the theater. While I considered my jeans to be my regular academic attire, I thought it would not be appropriate for the evening, especially in the company of a lady I so respected, so I put on my slacks, a white shirt, and reasonable sweaters. Fortuitously it was my view that wearing a necktie would have been going too far. Mighty was my surprise when I saw Regina under the chandeliers – all dressed in blue denim[10]! Until then, she had not seen me wear anything but jeans, so she thought she would match my dress code....

The performance was splendid, and my unpredictability remained an anecdote Regina would retell even years later.

The mystery theorem

Amidst all the fun and camaraderie, mathematics was the main subject nevertheless. I learned about counting subgroups of the free group, association schemes, ultrametric spaces, automorphisms of trees, and approaches to the traveling salesman problem, to mention a few of the subjects. The Graph Isomorphism Problem, the focus of my interest at the time, was attractive to several colleagues whose work combined algebra, combinatorics, and algorithms.

I reported my unpublished result that the isomorphism problem can be solved in polynomial time for (directed) graphs with simple eigenvalues, linking linear algebra to the problem. The next question would be to extend the result to graphs with bounded multiplicity of eigenvalues, a problem I had no success with as yet. A colleague, who had attended a recent meeting in Odessa, replied that he had heard that a colleague from Leningrad by the name of Dmitriĭ Grigoryev had solved just that problem, at least for *vertex-symmetrical graphs.*

I found the claim somehow incongruous. Symmetry makes isomorphism testing only more difficult, so if someone could solve the problem under such a symmetry assumption, they should also be able to solve it without that assumption. I expressed my doubts about the accuracy of the report and my informer backed down, not being certain of the precise side condition. But he was adamant that the result pertained to the isomorphism of graphs with bounded multiplicity of eigenvalues.

It was clear that in spite of the perplexing side condition, this was an announcement I could not ignore. However, no one in Minsk was sure about the exact claim, not to mention how to prove it. I was beginning to ponder how I could get in touch with Grigoryev. My hosts knew that he worked at LOMI, the Leningrad branch of the Mathematical Institute of the Soviet Academy of Science. To this date, I cannot tell why did we not consider trying to make a telephone call. Such a call would not have enabled us to discuss the result in detail, but it would at least have confirmed what the exact result was.

[10] Denim: the fabric from which jeans are made.

A telegram from Moscow

One evening in Minsk, a young male colleague took me to the restaurant "Zhuravinka" which had a dance floor. The place was crowded; the waiter ordered us to join two young women already seated at a table for four. None of the four of us seemed to mind; that's how I met Lida.

On Sunday, Lida and I took the train and visited nearby Vilnius, the beautiful capital of neighboring Lithuania (then a Soviet republic), where I had acquaintances: students I had met a few months earlier in Krakow, Poland, at an international student camp where I had accompanied a group of students from Budapest. (Not having a permit for Vilnius, I broke the law of the land with this side-trip. Lida bought the tickets, and she did most of the talking on the train.)

As the last day of my visit in Minsk approached, my friends arranged for a farewell party in a restaurant. I asked if I could take Lida along. They agreed, somewhat reluctantly. They asked if I was sure she wasn't a KGB agent. I was pretty sure; even if all her shining sincerity and blushes had been fake (which I could hardly imagine), why was she not allowed to enter my hotel? She told me why I could not visit her family or even see the apartment building where she lived: she felt embarrassed to show me their piece of "Soviet reality."

The atmosphere at the farewell party was generally good, although my colleagues could not entirely put their suspicion about Lida to rest. And at a certain point they called me to a sideroom, out of Lida's hearing range. The news they wanted to tell me, and especially my reaction to it, was not for the ears of the KGB.

The news was that they had just received a cable from Moscow. The International Office of MGU informed them that my visit to Moscow had been canceled; I should spend the remaining two weeks of my Soviet exchange in Minsk. My friends assured me that they would be more than happy to continue to be my hosts.

After a bewildered moment, I made up my mind. Minsk was friendly, but I was ready to explore my segment of the vast Moscow mathematical scene. I had myriad business to conduct there. In my mind, Moscow had been my primary destination all along. I thanked my hosts for their kindness but told them that I would ignore the telegram and would return to Moscow as planned. They sighed and wished me luck.

Lida's feelings were bruised by this secret conference, an obvious expression of mistrust. I could not tell her what it was about.

The ninety-minute flight that took three days

Having possessed a permit to enter Moscow, I had no difficulty boarding Aeroflot flight number 1992 Friday afternoon. Ninety minutes later I landed at the familiar Domodedovo Airport in Moscow.

I had no hotel reservation, however, nor could I make one. The reason was simple; in the hotel, they would ask my passport. Since I was on an official exchange, my host institution was supposed to provide a stamped form requesting

the hotel room for me. As I had learned the preceding day, such a stamp was not forthcoming from my purported host institution.

I was in no danger of having to sleep under the bridge, though. My friend Imre Bárány, whom I had visited for a few hours on my first arrival in Moscow, had at that time invited me to stay at his cozy apartment in Moscow, instead of a dull hotel room. I had gladly accepted, not knowing that in the end there would be no other option.

A few days later the Báránys would leave for a vacation, so I had both comfort and privacy.

All this, however, did not resolve a major headache. Although I had the right document to *enter* Moscow, I had no permit to actually *stay* there, even for a single night. That permit was only offered by police through the hotels, upon appropriate documentation of my status. I was not *supposed* to stay with friends; this was an *official visit*, after all.

While unlikely to be caught while in town, my undocumented status would inevitably get me, as well as my hosts, in serious trouble at the time of departure when my papers would be checked carefully. Not only would my transgression likely prevent me from traveling overseas a few weeks later, setting back my career by years, but worst of all, it could get my friends in Minsk in trouble for not reporting my defiance of the order to stay there.

I was hoping that with the help of Professor Skornyakov, I could persuade the International Office of MGU to change their minds; recognizing that I was there anyway, what else could they do, send me outside to freeze?

But it was Friday evening, there was nothing I could do about this during the weekend.

Saturday afternoon I met several colleagues led by Misha Klin, my old friend who was working on permutation groups, association schemes, and related subjects in algebraic combinatorics. The basic framework of my 1981 paper on primitive permutation groups [Ba81] owes some of its roots to Misha's work on maximal subgroups of the symmetric group [Kl].

Misha had largely been responsible for my Minsk connection and thereby for the feasibility of my entire trip. At this time (1978) Misha was at a relatively happier stage of his life; after a decade of isolation in Nikolayev, a military–industrial town near the Black Sea, he now lived in Kaluga, a town 200 kilometers south of Moscow. The "elektrichka" train would take him to Moscow in 3 or 4 hours, and he would make the trip rather frequently to maintain his close collaboration with Igor Faradzhev's group working in constructive and algebraic combinatorics.

While in Nikolayev, a town closed to foreigners, he obtained his Ph. D. under the long distance advisorship of group theorist L. A. Kaluzhnin in Kiev. Subsequently he worked at a university for ship-building engineers in Nikolayev; several of his important papers are buried among engineering papers in the inaccessible publications of that institute ([Kl]). The move to Kaluga, where he taught at the Pedagogical Institute, was just the beginning of his long odyssey. After six years of commute between Moscow and Kaluga, he was at last able to

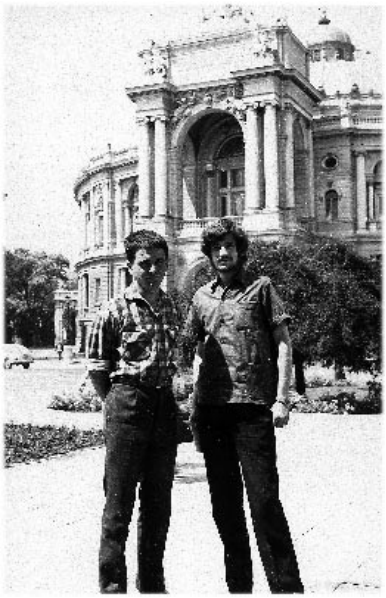


Fig. 1. With algebraic combinatorist Misha Klin in front of the Opera House in Odessa, U. S. S. R., 1971.

procure a "propiska" (residence permit) for Moscow. It would take another two years in a "communal apartment" before Misha and his wife Inna could move into a tiny Moscow apartment in a concrete-block complex and yet another year before Misha got a decent job at the Moscow Institute for Organic Chemistry. And in 1988, at the age of 42, he was able to make his first trip abroad (to a meeting in Calcutta).

Throughout the years of hardship, Misha remained the long-distance mentor of generations of algebraic combinatorists. Even while he was in Nikolayev, he already had students and close associates in Kiev, Minsk, and Moscow. His energy and determination seemed inexhaustible.

It would take another several years before he could emigrate to Israel, only to replace hardship with insecurity; at 46, he had to face stiff competition in a small country with a great surplus of superb mathematicians. Misha was sustained for years by various grants. But, having overcome his initial shock over the realization that the struggle would never seem to end, he appears at last to have arrived at the happiest stage of his life; his daughter Hana was born in 1997 in Suva, capital of the Fiji Islands, while Misha taught there at the University of the South Pacific. And Misha now has a "secure" temporary position at Ben-Gurion University. "As the old Soviet saying goes, 'Nothing is as permanent as a temporary arrangement'[11]," he reported cheerfully in a 1998 e-mail message.

[11] Hungarian newpapers always referred to the Soviet occupation forces as the "Soviet army units temporarily stationed in our country." – In Hungarian, an apartment

On that late October Saturday in Moscow two decades ago, we had some discussion of algebraic combinatorics and computational group theory (Igor Faradzhev was a leader in that area), then the talk turned to politics, and finally we spent a spirited evening during which my friends would sing an interminable series of songs from the operettas of Hungarian composer Imre Kálmán, whom they hailed as one of the greatest musical geniuses of all time. I stood there in embarrassment, not even recognizing most of the melodies. I was dumbstruck by the recognition that a genre for which I had little appreciation had brought my tiny country fame and honor across the vast land of Russia.

My urgent job Monday morning was to legalize my status in Moscow. I met professor Lev Anatolyevich Skornyakov at the Department of Algebra on the 13th floor of the massive "Stalin baroque" building of MGU. Together, then, we went to the *Inotdyel*, the International Office. The spacious office was dominated by the towering figure of the head of the office, a lady about six inches taller than professor Skornyakov. Hardly had I stuck my head in the door when she greeted me, yelling across the room, "What are you doing here? Did they not tell you to stay in Minsk?" How did she know who I was? I had never seen her.

Poor Lev Anatolyevich never had a chance to utter a word. "Goodbye, both of you," we were told. Now this was rude even by the standards of Soviet officialdom. Outside in the hall professor Skornyakov, intimidated and resigned, advised me to seek help from the Hungarian Embassy. I was grateful for his word of wisdom.

I followed up on his advice and managed to get in touch with Mrs. Kohánka, a consular officer. She understood and was prepared to help. The next day I had my paper with the all-important rubber stamp, and was directed to report at the desk of Hotel "Universitetskaya."

So I did. A young woman, slender and fair, greeted me with a heartwarming smile at the reception. She considered my documents carefully. Very carefully. The rubber-stamp

"Residence permit from (date) until (date)"

lay next to her on the desk. But she would not lift it. Instead, she looked up, deep into my eyes, and asked in a soft voice: "It has taken you *three days* to get here from Minsk?"

I was mortified. The residence permit stamped in my visa at the hotel in Minsk expired on Friday. Again, my stupidity. On arrival in Minsk, I could have asked them to enter the full length of my stay in the Soviet Union. But now, that entry gave me away, a burning testimony to my misdeed.

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you own (as opposed to rent) is called an "eternal apartment." During the decades of Communism, most city-dwellers resided in apartments rented from the state. In an incident in the 70s that caused many tongue-in-cheek comments in private, a government newspaper reported that the keys to a block of "eternal apartments" were handed to the happy new owners, "officers of the Soviet army units temporarily stationed in Hungary." – Well, the occupation did in fact not last forever. After a mere 47 years, our temporary masters were out.

A kind lady next to my tormentor observed this exchange and took my papers from her junior colleague. Within a minute I was on my way to room No. 1002, key in hand. I was deciphering the new rubber-stamp on my visa:

"Residence permit extended until (date)."

So I will not need to account for the three days. How clever. How thoughtful. Would such an act of mercy be possible in the age of computers, interlinked databases, bar-coded documents, and magnetic strips? I have seen numerous manifestations of a peculiar "civil liberty" in the Soviet Union which did not exist in the West: the "right" to lie to authority. (Privacy–rights warriors seem to be working hard on upholding the Western version of this waning liberty.)

I spent the next couple of days attending lectures and seminars, as well as meeting mathematicians from all over Moscow, including even the "Institute of Steel and Alloys." Among the interesting results I learned was Margulis's lovely explicit construction of small 4-valent graphs without short cycles, addressing a problem of Erdős. Back in Budapest, my friends László Lovász, Vera Sós, András Hajnal, and I were in the process of creating a new journal called *Combinatorica*. I solicited Margulis's paper for our journal. He sent me a greatly expanded version three years later, which I translated into English [Ma82]. This construction was one of Margulis's starting points for his celebrated construction [Ma88] of what have become known as "Ramanujan graphs," a term introduced by Lubotzky, Phillips, and Sarnak who simultaneously discovered the same remarkable class of objects. (Incidentally, the LPS paper was also printed in *Combinatorica* [LPS].)

I had known Margulis from his several earlier visits to Budapest. In Moscow, Margulis worked at the Institute for Problems of Information Transmission (IPPI), one of the bastions of free inquiry in Moscow, not affected by the madness of Pontryagin and company.

Misha Lomonosov, working in combinatorial optimization, was another friend from IPPI. A few years later Misha would join the ranks of "refusniks," those applying for emigration to Israel but whom the Brezhnev government would not allow to leave (which was the case for almost all). The typical consequence of application for emigration was that the applicants would lose their jobs and livelihoods and could only find menial labor. Not quite so at IPPI; when I saw Misha again in Moscow in 1981, he was barred from entering the IPPI building, but he still received his monthly paychecks.

Misha was a different person in 1981. The die having been cast, he was now a free individual. Presumably always a family man, he was enjoying his life at home, in a concrete-block medium-high-rise apartment in one of Moscow's crowded residential districts, yet outside of Soviet society. He and his family were in the process of converting from a previously secular life to orthodox Judaism in preparation for his aliyah in the indefinite future. Misha's mathematical work flourished. He was planning to submit a paper to *Combinatorica;* I would have to smuggle the paper out of the Soviet Union. A false name would be printed as author on the typewritten manuscript in case my suitcase should be examined on exit. (This was my advice, I had done similar service for other friends.)

I felt I had to discuss the manuscript in detail with Misha since communication would be very difficult after I left. So I decided to return to Moscow a day earlier than planned from a conference I attended in Leningrad (legally!). I arrived at Misha's Friday morning; we discussed his paper until sundown, at which time he put down his pen and retreated to prepare for the Sabbath. Shortly afterwards a boy from the neighbors' arrived, and we had a jolly Sabbath dinner, the first I had ever attended. Misha's paper appeared in *Combinatorica* in 1983 [Lo].

The Czar's pencil

Leningrad Railway Station, 11:30 p. m., November 3, 1978.

There were still several trains scheduled to leave before midnight so I hoped to be able to get on one of them. The last train would leave at 11:59 p. m. It is said that the reason for the midnight cutoff was in the law for reimbursement of travelers on official business. The *per diem* rate at the time was 2.60 Rubles (roughly, $ 2.60) if the train left before midnight, and considerably less (!) otherwise.

The timetable has remained stable for decades, even up to the present day. The trains bear odd numbers on their way from Saint Petersburg to Moscow and even numbers on the way back. Train number 1/2, "Krasnaya Strela" (Red Arrow) offers the most comfort, it is the train foreigners would generally use. *Krasnaya Strela* leaves Moscow (now as then) at 11:55 p. m. and arrives in Saint Petersburg at 8:25 in the morning, stopping only once for ten minutes in the town of Bologoye, roughly halfway between the two metropolises.

The Saint Petersburg – Moscow railroad was built between 1846 and 1851 on orders of ironfisted Czar Nicholas I.[12] Called the "Nicholas Road," the route follows a single 400-mile straight line, with just a small jog at Bologoye. According to legend, the Czar himself drew the line on the map, and his otherwise firm hand jiggled a bit halfways. Would an engineer dare to disobey the mighty emperor?

Technological progress came at an enormous human price. The poet Nikolai Alekseevich Nekrasov (1821–1877) described it, ostensibly for the edification of children, in his poem "The Railroad" (1864):

> 'There is a Tsar in the world – without mercy:
> Hunger his name is, my dear.
> ...
> 'He, too, it was, who drove multitudes hither,
> Crushing the task that he gave:
> In the grim fight to bring life to these deserts
> Many have found here a grave.
>
> 'Straight is the road, and the track is but narrow,

[12] Nicholas I: Czar of the Russian Empire from 1825 until 1855. His reign started with the bloody suppression of the Decembrist revolt.

Poles, rails, and bridges abound on the way,
Everywhere, too, Russian bones lie beside them,
Vanka, how many, my dear, canst thou say?'[13]

On arrival at the Moscow Railway Station in Leningrad, *Krasnaya Strela* used to be greeted by grandiose music thundering from loudspeakers, the kind of music that is supposed to elate your spirits and make you feel that you are part of a great human enterprise, be it the breaking of the sound barrier in aviation or entering the City of the Revolution. It may be no coincidence that A. Glier, the composer of this "Hymn to the Great City," is also known for his movie scores.

An unrecorded contribution to queuing theory

If you stand in a queue, you may progress once in a while for one of two reasons: the person at the head of the queue was served; or someone ahead of you was fed up with waiting and left the queue. You may also regress as people join the queue ahead of you for various reasons (they spot a friend already waiting there, or press their way in front of an easily intimidated person, etc.).

Assuming a patient crowd (few leaving as long as there is a glimmer of hope for progress) and a long enough queue, there will be a point in the queue where progress is no longer expected because the rate at which people join the queue ahead of that point is no less than the rate at which people in the front are served. This point is called the *Dobrányi threshold*, a fundamental invariant of any given queue, whose discovery is attributed to unaccomplished electrical engineer and tavern philosopher Elemér Dobrányi [pron. dob-chun-yee]. I heard about this magnificent concept from my friend József Pelikán[14] on an earlier trip to Moscow; the USSR seemed to provide unlimited inspiration and an inexhaustible source of working models for queuing theorists[15].

[13] Translation by J. M. Soskice[Ne]. The actual title in [Ne] is "The Railway." Other translations give the title as "The Railroad."

[14] József Pelikán, two years my senior, was my first teacher of group theory, an area that became one of my main research interests, especially in its combinatorial and algorithmic aspects. Pelikán, then a 12th grade student at Fazekas high school (the same school I attended), a formidable problem solver and a compulsive instructor, invited me to his home for a weekly "group theory class." I treasure the memory of these afternoons which later decisively influenced my choice of research direction.

[15] On my earlier trip to Leningrad I witnessed on several occasions how, in a matter of seconds, a line spanning several blocks would form when a pushcart carrying a big keg of cold *kvass* (Russian beer made from barley, malt, and rye) would drop anchor at a street corner. The sudden emergence of a supply of any other kind of goods, from shoes to groceries, would trigger the formation of similar instant lines. In fact, people would get in the line before even knowing what the line was for. – A highly visible change brought about by the new era of Capitalism is the disappearance of such lines from the post-Soviet scene.

For the purposes of this accounting, the place of the person right in front of the *"okoshko"* counts as position zero. It is to be noted that the Dobrányi threshold increases as service at the head of the queue speeds up; on the other hand, the threshold goes down at slower service rates.

The queue I joined (at its tail, of course) was quite long, and there was lively activity in the middle, so I was curious to assess whether my position exceeded the Dobrányi threshold. I soon decided that it did, but it took me a while to establish the precise value of the Dobrányi threshold, for this determination required keen observation of the events far ahead of me in faint light and through a fog of smoke. Lacking other optical devices, I had to strain my eyes somewhat, but my then-impeccable vision eventually helped to unlock this tantalizing scientific mystery. The exact answer turned out to be zero. No tickets were handed out. None, zilch, period. The *okoshko* in the obscure distance was shut.

I cannot say, though, that no entertainment was provided. Although, as usual in Moscow, the lines were generally quite disciplined, many patrons were well stocked with spirits and as time passed, more and more of them dug into their supplies to get a lift of their spirits. Considerable loud interaction occurred between the lines, mostly friendly, but some rough, too, with occasional scuffles. I hoped to stay out of trouble's way by avoiding eye contact, while trying to observe the performance.

I am found out

The Soviet nation was preparing for the celebration of the 61st anniversary of Lenin's "Great October Revolution" which took place on November 7, 1917 (October 25, 1917, according to the Orthodox calendar). November 7, 1978 was a Tuesday, and Sunday through Wednesday became a four-day national holiday. (November 8 was always added.) The long holiday would be the time for many Muscovites to visit relatives in other towns; the traffic between Moscow and Leningrad was at a peak. I was not particularly surprised that getting on a train should be difficult, and I had anticipated that returning from Leningrad would be no easier.

The trouble with the return trip was that, for a reason not clear to me, round-trip seat reservation was not possible. However, getting stuck in Leningrad was a risk I could not afford. I had to return to Moscow the next night. Lida took a few days off from work in Minsk around the November 7 holidays; she would visit a friend in Moscow. I had to meet her at 8:38 Sunday morning at the Belorussian Railway Station in Moscow. There was no way to change this appointment, no telephone contact, and no alternative dates to choose from.

I had procrastinated the decision about the Leningrad trip to the point that it became even more difficult than it should have been. Finally sometime Friday afternoon I decided that I could not afford to pass up the opportunity to learn first-hand about the "mystery theorem," even though the risks included jeopardizing my visit to the U. S.

Once resolved, I set about securing the return trip. Around 5 p. m. I called my friend Ruvim (Reuven) Gurevich in Leningrad and explained the situation to him. I asked him to reserve a seat for me on a return train for Saturday night.

Then I went about other business. I was interested in two seminars at Moscow State University late in the evening: Professor Rybnikov's on graph theory on the 16th floor at 6 p. m., and Professor Skornyakov's on abelian groups on the 13th floor at 7 p. m. After the seminars I called Ruvim again; he confirmed that he indeed managed to get a return reservation for me. So I rushed back to my hotel, packed up for the trip, and took off for the Leningrad Railway Station.

The very efficient subway system got me rather quickly to the destination. By 9:40 I was standing in the line, entertaining the hope that I might be able to get to the *okoshko* before the last train of the night would depart.

Alas, it was not to be. After I had spent nearly two hours in the line without any progress, the *okoshko* finally did open. "Grazhdanye[16], there are no more seats available on tonight's trains," I seemed to be hearing a strong voice through noise and distance.

My interpretation of the announcement was immediately confirmed by what I saw, the crowd began to disperse. Some cursed, but nobody protested the fact that the authorities failed to make the announcement an hour or two earlier. Within minutes, the hall was virtually deserted.

I was one of a handful of slow-wits who failed to recognize the single prudent course of action. Rather than moving vigorously to catch the last subway, I just hung on, alternately resting my weight on one foot and then on the other, looking to the right, looking to the left, as if waiting for some miracle to happen.

I noticed a man in his 30s, about fifty feet away, doing much the same: looking to the right, looking to the left. Our eyes locked. I was no longer concerned about eye contact.

The man walked up to me.

"You want to go to Leningrad?" he asked (in Russian), revealing remarkable insight at the Leningrad railway station. (Well, my destination could also have been Murmansk, within the Arctic circle, but no other place from this station.)

"Da," I agreed obligingly.

"You a foreigner?" How did he guess? Was my meek "da" enough, or even that wasn't necessary.

"Da," I acknowledged humbly.

Chervonyets sverkhu!

Then came the surprise.

"Chervonyets sverkhu!", he pronounced with a wide gesture.

I was clueless. I knew that something like "chervonnyĭ" had something to do with the color "red," and "sverkhu" meant "up." But how does "red up" fit in this discussion, I could not guess. The man seemed rather encouraging,

[16] Fellow citizens.

so I asked him to explain perhaps in some detail what exactly he had in mind. "Chervonyets" turned out to be the nickname of the (red) ten-ruble bill. Now the ticket, he explained, was 13 rubles. And he would get me a ticket if I paid him a chervonyets on top of the 13.

This sounded like a terrific bargain. The ruble was then officially on par with the dollar (and much less on the black market); imagine an overnight trip of 650 kilometers (400 miles) at 23 dollars. My new acquaintance also volunteered to let me in on his secret; his aunt was working at the ticket booth and they would split the profits.

I agreed to the deal and was wondering how we should go about executing it.

"Just give me the 13 rubles now, I take it to my aunt and I'll return with the ticket. Then you give me the extra ten."

"Sounds great, but what happens if you take the money and I don't see you again," I asked, emboldened by my partner's friendly business style.

In lieu of a reply, he unfastened his wristwatch and handed it over to me. The watch worked, and it seemed to be in rather good condition. I estimated that it might be worth twenty rubles. So I handed over the price of the ticket, and saw the man disappear behind a door at the far end of the hall.

He returned a short while later, ticket in his hand. I returned his watch and gave him the *chervonyets.* He took me by the arm, guided me to the platform and to the designated rail car, and wished me a good trip. It was shortly before midnight.

I boarded my "platskartnyĭ vagon" ("reserved seat rail car"). The car had 54 sleepers (the lower ones were used to serve as 72 seats during the day). The car had no walls to break it into compartments, all passengers were in the same hall. There were stacks of sleepers perpendicular to the train on one side, and an aisle with single sleepers on the other side at the window, parallel to the train.

I found my place in the crowded car and assumed my least inviting expression so as to avoid being engaged into conversation. My place was in the aisle. I was gazing out the window, as if anything of interest could be seen there. It seemed like everybody was talking to everybody. People were eating, drinking, strangers became acquaintances. I could not make my bed until there seemed to be a consensus about the timeliness of that operation. It seemed like an eternity until finally the lights went off and the danger of being approached by a friendly fellow traveler waned.

Slowly the chatter subsided and all I could hear was the rhythmic clacking of the rails. I tried to imagine the straight line we were traveling. It took me a long while to fall asleep.

When I awoke, not well rested, seven hours later, people were getting ready for arrival. Arriving at nearly the same time as the *Krasnaya Strela,* our train, too, appeared to be greeted by Glier's *Hymn.* I did feel some elation, having survived the trip without having to utter a word. Here I was, illegally in the City of the Revolution, and more importantly, in the City of Grigoryev. The chilly breeze on the platform quickly restored me to my senses.

Ruvim, Dima

I had been in Leningrad before. As an undergraduate, I spent a semester at Leningrad State University. I learned the theory of simple algebras in the classes of Professor D. K. Faddeyev, and the rigors of winter in my dorm room where snow collected between the double window-panes.

Waiting in the corridor for Professor Yakovlev's course on Galois theory to begin, I met Ruvim, two years my junior, a short and slightly stooped boy with very thick spectacles over his lively eyes. Ruvim was an ardent problem solver and on our first encounter, he immediately proposed some questions and asked me to reciprocate. He easily engaged me in conversation and was ready to discuss any subject in mathematics. Logic, topology, algebra were our favorite topics of mutual interest. We became friends. A few years later he visited me in Hungary.

Another four years passed, and there I was in Leningrad. Ruvim was there, too, after a detour to a provincial town. After graduation he had been assigned a job as a programmer in the town of Syktyvkar in the Ural mountains. He reported from there that there was no milk and the only vegetables in the store were half rotten cabbage and sometimes potatoes. As for meat, an occasional dried fish was on the menu. He did not like the job and did not like the town, he feared for his fragile health. So he moved back to Leningrad, although he knew, that, having left the track of assigned jobs, he would have difficulty finding employment in Leningrad. He told me that he wanted to emigrate to Israel; he had even written a letter to Brezhnev. That, of course, was of little help.

I called Ruvim from the railway station and took the streetcar to his place. Ruvim lived in a very small apartment with his parents and his sister. I received a warm welcome, took a hot bath and had a fine breakfast (I recall that the menu included pickled mushrooms), and was treated to a number of quotations from Russian playwright Griboyedov (after whom one of the great canals of this "Venice of the North" is named) and other great literary characters. The combination of hygiene, cuisine, and poetry made me feel civilized again after the dirt and the sweat of the trip.

While I took a much-needed nap, Ruvim called the Grigoryevs. He learned that Dima was at work, at LOMI. This was somewhat surprising since it was Saturday, no seminars were scheduled.

It turned out that Dima had some urgent editorial work to do: proofreading a volume he was editing for the publisher "Nauka." The manuscripts had been typed on large special paper to make a camera-ready copy; formulas were inserted by hand, by calligraphers employed for this purpose (invariably women).

Early afternoon we went to LOMI which was located on the bank of the small and peaceful Fontanka canal. Dima was busy checking the beautiful formulas when Ruvim and I showed up in his office.

Dima was very friendly and forthright. I told him that I was interested in his work on the Graph Isomorphism Problem under a restriction on the spectrum of the graph. I told him about my partial results (I had solved the case of simple eigenvalues) and about the rumors I heard in Minsk that compelled me to take this trip. It turned out that my colleagues in Minsk had quoted Dima's

result rather accurately; he had solved the problem under the more general assumption of bounded eigenvalue multiplicity, but with the side-condition of vertex-symmetry which I found so unnatural as to make me doubt the result. I expressed my conviction that less symmetry could make the problem only easier, so if his result was indeed correct, it should be possible to remove that extra condition, thereby obtaining a complete and appealing result.

After a couple of hours of discussion I was not only satisfied that Dima's result was indeed correct, but I managed to translate his ideas into "my world" in a way that Dima himself found enlightening. What seemed rather complicated at the beginning now appeared crystal clear. I felt that I had learned something; the *chervonyets* was worth its weight in gold.

It was clear to both of us that I had become the custodian of the problem. Dima's result seemed to be "halfway" to the real thing. I was grateful for the insight gained, and excited about the prospect of possibly finding the missing other "half," by eliminating the symmetry condition. Whether that "half" would be just a routine increment, or itself a new beginning, was yet to be seen. Of course, as with any unfinished work in mathematics, there lurked the logical possiblity that the residual problem was beyond my powers to tackle, but I was not (yet) willing to consider this danger as real.

The first attempt

I returned to Budapest on November 10. Two days later I found a way to overcome the obstacle.

I spent the next few weeks feverishly preparing for my planned year-long journey overseas. My plane landed at JFK on November 30. The afternoon before the transatlantic flight, I gave a seminar lecture in Budapest about the isomorphism test for graphs with bounded eigenvalue multiplicity. The details of the "second half" seemed a bit messy; indeed I was unable to fully reproduce them at the blackboard.

Most disturbing perhaps, I did not feel elated over the solution. It did not seem to make me any wiser.

A couple of months later, in the U.S., I returned to the problem. I realized that I needed to formulate a simplified situation in which to formalize the solution. To this end I considered the problem where the graph under consideration had colored vertices with bounded multiplicity of each color; isomorphisms would preserve colors by definition. The linear algebra was out, but the problem retained the combinatorial difficulty that separated Dima's result from the "real thing."

After a failed attempt to explain the idea to Gary Miller at M. I. T. in March 1979, I had to concede that my "solution" did not work even in this simplified model. The embarrassment was considerable since I had announced the result at several places.

For several weeks I was desperate. I felt that I had to solve a problem under the gun.

Las Vegas in Montreal

Finally, the the life-saving spark came in June 1979 during a conference in Montreal.

I clearly remember the moment of discovery. After a conversation with László Lovász, a friend from my high school and a combinatorist who has produced many of the most original thoughts in the field, I recalled what András Hajnal, the brilliant set theorist, combinatorist, and chess player used to say when confronted with an interesting chess position. "The question is, what would Alekhine[17] move in this situation," Hajnal would proclaim in deep thought. Paraphrasing Hajnal, I asked myself, how Lovász would approach my problem.

This thought was, of course, just as ridiculous as trying to guess Alekhine, yet it seemed to help.

Thanks to seminars conducted by Péter Gács[18] in Budapest in the preceding years, I was already familiar with the basics of the P/NP theory.

Rather than trying to find a polynomial time algorithm, it dawned on me that the right question to ask was to place the problem in $coNP$. Within hours, I saw the basic structure that would accomplish this: coset representatives for a certain chain of subgroups. From here, a *randomized* polynomial time algorithm was only a small step; the group on top of this chain was a direct product of groups of bounded size, therefore uniformly distributed random elements of the group were easy to obtain. A subsequent process of "sifting" down the chain[19] would fill the coset tables with large probability. Since the order of the group on the top was known, this algorithm had a deterministic verification and thus it would never give erroneous output (although with small probability it would report failure).

I found it important to distinguish this type of randomized algorithms from those which can produce (with small probability) an undetected error ("Monte Carlo algorithms") and a few months later (in Vancouver) I coined the term "Las Vegas algorithm" for the error-free variety. In spite of misgivings by American-born computer scientists ("why should Las Vegas be more respectable than Monte Carlo?" [Jo]), slowly the term caught on and is now generally accepted[20].

More importantly, the subgroup chain method fulfilled my expectation that the solution to the simplified case (bounded color-multiplicities) would extend to the problem of bounded eigenvalue multiplicity. The "unnatural condition" from Grigoryev's solution was, finally, removed.

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[17] Alexander Alekhine (1892–1946): legendary Russian chess player, world champion for 17 years. Noted for the variety and elegance of his attacks.

[18] Gács was influenced by Leonid Levin with whom he worked in Moscow in those years. Later both of them emigrated to the U. S. and found a permanent home at Boston University.

[19] I called this the "tower of groups method." The term "sifting" was subsequently introduced by Furst, Hopcroft, and Luks [FHL].

[20] Some Canadian and European computer scientists did not seem to share Johnson's reservations and picked up the term almost immediately [Me,Br].

The most significant aspect of the "tower of groups" method was that it represented the first application of group theory, albeit on a very elementary level, to the design of a polynomial time algorithm for a problem which did not involve groups in its formulation.

Canada and the 72 questions

I love Canada. The towns in the East which are reminiscent of the Old World, the breathtaking scenery of the West from the Rockies to Vancouver Island, the mathematical culture with which I had so many points of affinity. Canada was the first country I ever visited on the other side of the Iron Curtain[21], by invitation of algebraic graph theorist Gert Sabidussi of the Université de Montréal, in summer 1972 while I was still an undergraduate.

My summer 1979 in Canada was special in many ways, and exceptionally productive. It should not be surprising that to be admitted to the realm which offers all these pleasures, first I had to demonstrate my valor to the Government's satisfaction. Jousting before the royals being out of fashion, the Canadian government kindly devised another type of obstacle course which only the most resolved could successfully complete.

While in Nashville in Spring 1979, I had to subject myself to a rather thorough physical exam, complete with a 16×24 inch chest X-ray which I had to mail to Ottawa. The form my examiner had to complete contained 72 questions, probing all my organs and bodily functions, a task of obvious relevance to an eight-month work permit. I learned the English word "embarrassing" from my examiner, a shy but meticulous young female nurse, when it came to checking for *inguinal hernia.*

I was planning to leave Nashville on May 5, briefly visit Atlanta, and then spend some time in New York and Boston before arriving in Montreal at the beginning of June.

Just as I was preparing to leave for Atlanta and was anxiously awaiting the message from Ottawa that I can pick up my visa and work permit in Atlanta, I received a message instead that my urine test had to be repeated. The doctor who had administered the test was on vacation but luckily happened to be in the hospital that day, so I retook the test, the doctor assured me that there was nothing to be found in it (the previous sample had apparently not been a "clean catch finding," a concept I will never forget given the role it played on my road to Computer Science). So I took the new test results along to Atlanta where in addition to visiting Emory University and Georgia Tech, I also had to appear at the Canadian Consulate.

I showed the new finding to the consular officer. He informed me that he could not judge the doctor's finding, the report had to be sent to Ottawa where it would be read. I told him about my schedule, that I had to move on, I could

[21] This was just another irony of the Iron Curtain; Vienna, also on the other side, is about 4 hours by train from my native Budapest. But as a high school kid I had to travel to East Germany to practice my German.

not wait for the papers to return. So the officer wanted to give me a temporary visa valid for 6 weeks. However, I was planning to stay in Canada for 8 weeks, then go to the U.S., and return to Canada at the end of September. "I cannot give you more than one entry because I am bound by law," he declared solemnly. I begged him that I absolutely needed two entries because I had to visit Stanford and Berkeley (I was hoping the names of these places might ring a bell) and then return to Canada with no chance of seeing consulates in the meantime. To which he replied, without blinking: "I cannot give you more than two entries because I am bound by law." I was grateful for this elasticity of the law but I had one more question. I respectfully inquired why the government of Canada was interested in my hemorrhoids, of all things. The answer was as prompt as it was undeniably to the point: "You will have to do research. You will need to sit. If you cannot sit, you cannot fulfill your work obligations."

The temporary permit I finally received was good enough to cover my summer and the reentry to Canada in September. But it fell short of securing further reentries (which I needed for brief visits to the U.S. such as attending FOCS'79 in Puerto Rico). I ended up spending several days at the Canadian Immigration offices in Montreal, Vancouver, and Toronto, but eventually I had to leave Canada, go to the Canadian Consulate in New York, and receive my "not-so-temporary" visa there. By the time I had the right visa, a mere six weeks were left of my tour of Canada.

Montreal to Vancouver

Peter Frankl, a combinatorist with multiple talents[22] and an old friend of mine from Eötvös University, Budapest, was one of the participants in the Montreal conference. Peter and I already had several joint papers in algebraic combinatorics and later we would write a book on "Linear Algebra Methods in Combinatorics" [BFr]. But in Montreal, Peter's principal contribution to my welfare was not strictly mathematical; he introduced me to Maria Klawe, then a junior member of the University of Toronto C.S. faculty.

Meeting Maria was easily the highlight of the Montreal conference; she was one of the most brilliant, and definitely the most energetic, woman I ever met[23].

[22] The author of a large number of path-breaking papers in extremal combinatorics, Frankl is conversant in ten languages (Swahili is his most recent addition), juggles 7 balls, and is a celebrity in his chosen country, Japan, where he regularly appears on TV shows, edits mathematical columns for several magazines, and helps train the Japanese high school team for the International Mathematical Olympiad. He is the author of 11 books in Japanese.

[23] I met Maria before her stellar career took off: founder and manager of the Discrete Mathematics group at IBM Almaden, Head of the Computer Science Department and subsequently Vice President of the University of British Columbia, Vice-Chair of the Board of the Computing Research Association, Chair of the Board of Trustees of the American Mathematical Society, creator of an award-winning educational CD-ROM game, ..., the list is mind-boggling, and just watch what is yet to come.

In no time she arranged that I should appear at the Theory of Computing seminar in Toronto the day after I would leave Montreal. I spent half the preceding night packing and squeezing my stuff (mostly papers) into the car I had bought from a colleague for $200 in Nashville. I hit the road at 2 a.m.

After my head nodded for the third time at the steering wheel, I decided that I had to pull over and take a nap if I am ever to arrive in Toronto. I parked the car facing a brick wall and fell asleep, leaning on the steering wheel. I had a nightmare; I dreamt that I was approaching a wall at full speed. I grabbed the steering wheel, woke up, bumped my head, and saw with terror that right there, two feet in front of me was – the wall! I screamed and braced myself for the imminent crash.

Of course, nothing happened. Now wide awake, I had no difficulty negotiating the rest of the trip. After another nap in the comfort of my hostess's home, I went to the seminar early in the afternoon. I gave a talk on the algorithm I had found two days earlier. Al Borodin and Steve Cook were present, and I think they liked what they heard. Five years later, the opinion from Toronto was critical in my bid for a visiting position at the newly established Computer Science department at the University of Chicago.

The Toronto seminar was the first time I gave a talk in one of the research centers of the Theory of Computing anywhere.

From Toronto I had to move on to Vancouver, for a workshop in "Algebraic Graph Theory" held in July 1979 at Simon Fraser University. After the workshop I had a week of complete solitude in a home with a scenic view of the bay. During those halcyon days I completed the "Las Vegas" paper, along with two papers in group theory. One of the latter two was on a problem of combinatorist Michel Deza on "sharply edge-transitive graphs" (two more coauthors joined in later on[BCDS]); the other was motivated by the complexity of graph isomorphism, but its real significance lay in group theory (it solved a century-old problem on primitive permutation groups), an interesting reversal of the direction of influence [Ba81].

On the road again

My next stop was Stanford (Péter Gács was there at the time). I received much-needed technical assistance from Professor Knuth's secretary in typing, according to prescribed format, my first FOCS manuscript which I had to prepare for an imminent deadline. The paper (joint work with L. Kučera [BK]) considered the average case complexity of graph isomorphism. It was Ron Graham who had pointed out to me half a year earlier the significance of the FOCS and STOC conferences. I had not been aware of these meetings before, and only later did I find out the feverish competitiveness of submissions.

I first met Ron Graham in Hungary in 1969 at a combinatorics conference. Ron, a close friend and collaborator of Paul Erdős in number theory, combinatorics, and discrete geometry, was a frequent visitor to Hungary, attending many of the numerous international conferences organized by Paul Erdős's Hun-

garian friends. The lucid and entertaining style of Ron's lectures was something I found worth emulating. I was even more affected by a message carried by Ron's lectures: the connection between Erdős's world and the new concepts of polynomial time and NP-completeness. A lecture by Ron on "Multiprocessor scheduling anomalies" [Gr], given in Hungary in the mid seventies, was among the first to make me think about problems of computational complexity.

Paul Erdős's enormous influence on the Theory of Computing is somewhat paradoxical because Erdős himself never showed any interest in the subject. While Erdős often jokingly discouraged his closest associates from "vasting their time" on computational problems, many of his disciples had become major players in complexity theory and algorithms, often using techniques they had developed in the course of work on Erdős's problems[Ba97].

I consider myself a mathematical grandchild of Paul Erdős; most of those who shaped my mathematical interests were Erdős's close collaborators, including my foremost mentor during the college years, number theorist and combinatorist Vera Sós, one of Erdős's closest friends. Over the years, I benefited tremendously from the warm welcome by the large circle of Erdős's friends, among whom Ron was the first non-Hungarian, and one whose continued guidance helped me a great deal in finding my way in the U. S.

My first joint paper with Uncle Paul was on the average case complexity of the graph isomorphism problem [BES]. The paper with Kučera strengthened these results, and, thanks to Ron's advice, accorded me a stage, which would, in the 20 minutes allotted, complete my initiation into the Theory of Computing community.

This ritual would take place in November 1979 in Puerto Rico. Meanwhile back in California that summer I did my best to make smaller-scale introductions. Double misfortune befell me the night before my much-awaited debut at Berkeley; I got a sore throat and locked myself out of my car. The locksmith cost $40, not a small sum while I was making less than $300 a month; the sore throat was left to take its own course. I hope Dick Karp did not catch it; if he did, I hereby offer my belated apologies.

A week later came a combinatorics conference in Arcata, CA, in the middle of the scenic forests of Northern California. I was all awe and admiration as I witnessed Phyllis Chinn, the organizer of the meeting, in her kitchen, with a big spoon over a large boiling pot in one hand, a telephone on her shoulder, and two sweet children occupying the full length of her other arm.

The meeting was attended among others by group theorist Bill Kantor, Ron Graham, and Michel Deza. It was reassuring to see that each of the three manuscripts I had written in Vancouver had found an illustrious reader. Ron acknowledged my terminological invention, "Las Vegas," with a smile of encouragement, and even suggested that I put it in the title of the paper. (The title was, and regrettably remained, "Monte Carlo algorithms....")

After California, I had to drive across the continent once again, the back seat of my increasingly ailing old Javelin filled to the brim with boxes of paper. My Gypsy lifestyle continued: October 1979 in Toronto (complete with a

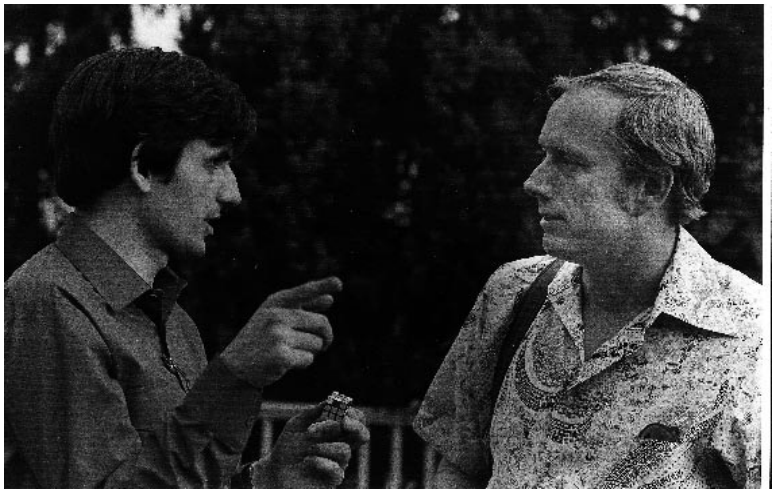

Fig. 2. At a conference with Ron Graham, around 1980. *Photo: Adrian Bondy.*

heartbreak, nothing accomplished that month), November in Waterloo, and December in Montreal. Meanwhile the "Las Vegas" paper turned into an Université de Montréal tech report [Ba79].

It was at FOCS 1979 in Puerto Rico that I had the occasion to explain the Las Vegas algorithm to a wide audience, although the paper that allowed me to speak was the joint work with Kučera on a different aspect of the Graph Isomorphism problem. Bending the rules somewhat, I spent only half the time on the paper accepted by the program committee; I used the other half to communicate the more recent results. From then on there was no question that I was a member of the Theory of Computing community. John Hopcroft was among the audience; shortly after the meeting he described the "tower of groups" method and the Las Vegas algorithm in a survey paper on "Recent directions in algorithmic research" [Ho].

On New Years day I visited Péter Gács who had meanwhile moved to Rochester, N.Y. From Rochester I drove to my next destination, Columbus, Ohio. Finally, I had a real job with a decent salary; invited by professor Dijen Ray-Chaudhuri, I was teaching at Ohio State University for the Winter quarter 1980. It was there that I first taught a course in complexity theory (in addition to the calculus course I was paid for).

After a rewarding term in Ohio, already an extension of my planned trip to the New World, I returned to Budapest. I could have stayed longer at Ohio State, perhaps indefinitely, but I was determined to return to my native Hungary. Throughout the 16 months I spent in North America, there had never been a shade of doubt in my mind that I would return home, to help the wonderfully nurturing community which had lifted me up and paved my way. It was time for me to repay my debt, helping younger generations.

While in Ohio, I received a letter from Bucknell University in central Pennsylvania that would change my life forever.

The sender was Gene Luks, an algebraist specializing in nilpotent Lie algebras. Gene had decided that for a change, he would pursue his side-interest in computer science, so he spent the fall of 1979 at Cornell studying the theory of algorithms with John Hopcroft. Gene was a quick study indeed. Hopcroft called Gene's attention to the graph isomorphism problem, and on Hopcroft's return from Puerto Rico, he described my Las Vegas algorithm to Gene. Subsequently Gene discovered that my randomized procedure for finding coset representatives can be replaced by a deterministic algorithm; this was the content of his letter of introduction.

That discovery had far reaching consequences and laid the foundations of the polynomial time theory of permutation groups [FHL]. Shortly afterwards, Gene communicated to me his seminal discovery: a polynomial time isomorphism test for trivalent graphs, involving an ingenious application of the *divide-and-conquer* technique.

Before returning to Hungary, I visited Gene at Bucknell. I brought him a copy of a handwritten letter by Peter Cameron containing marvelous new results on permutation groups [Ca81] based on the (by then nearly complete) *classification of finite simple groups*. Within two months, Gene was able to make profound use of the techniques found in Cameron's notes, extending his polynomial time isomorphism test from trivalent graphs to graphs of bounded valence.

I returned to Bucknell for brief visits during the subsequent three years. This was the beginning of our long friendship and prolific collaboration with Gene in the algorithmic theory of finite groups.

Epilogoue

Permutation groups and graph isomorphism

While I procrastinated writing up our result with Dima, David Mount observed that Luks's divide-and-conquer can also be used to obtain a polynomial-time algorithm for isomorphism of graphs with bounded eigenvalue multiplicity. The two proofs finally appeared together in [BGM].

The "sifting" method turned out not to be as new as I had thought; in the context of the stabilizer chain of permutation groups, it had been widely used in computational group theory following the pioneering work of C. C. Sims more than a decade earlier [Si70,Si71]. Three aspects of the "tower of groups" method [Ba79], however, remained novel: first, the use of a different chain of subgroups; second, the application to subcases of the graph isomorphism problem; and third, the complexity analysis.

The complexity aspect was extended by [FHL] to Sims's context (managing permutation groups given by a list of generators), ushering in the era of complexity theory in algorithms for finite groups. This area has combined the paradigms of algorithm design and analysis with a great variety of methods ranging from elementary combinatorial ideas to the deepest results of group theory

(cf. [Ba91a]). Much of the fundamental work in this area was done by Gene Luks; I feel fortunate to have had the privilege of a long period of collaboration with him. Some of the highlights are included in the bibliography. Ákos Seress, one of the chief architects of the theory, has also taken made invaluable contributions in turning the most efficient theoretical algorithms into code (using suitable heuristic shortcuts, of course); his programs are now part of the group theory system GAP [Sch+].

Graph Isomorphism is particularly intriguing for its unsettled complexity status. In spite of the considerable success of the group-theoretic approach, Graph Isomorphism is still not known to be solvable in polynomial time. The group-theoretic methods alone do not even yield a better than exponential (c^n) upper bound, where n is the number of vertices. Another input from Leningrad was needed to bring the exponent down to a fractional power of n (ultimately to $\sqrt{n \log n}$). On my 1981 visit to Leningrad I met a remarkable mathematician by the name of Victor Zemlyachenko (characteristically, Misha Klin made the introduction). Victor invented a strikingly elegant combinatorial trick which when combined with Luks's divide-and-conquer, allowed the reduction of the exponent (cf. [ZKT,BL]). The article [Ba95] contains an almost up-to-date survey on Graph Isomorphism.

Matrix groups and Interactive Proofs

Moving in a different direction, in a 1984 joint paper with Endre Szemerédi we considered the membership problem for matrix groups over finite fields, given by a list of generators [BSz].

The algorithmic obstacles to this problem are formidable, so the first goal was to settle the nondeterministic complexity. While a sweet combinatorial lemma puts the membership problem in NP, putting it in $coNP$ seems much harder; we seem to require a conjecture regarding finite simple groups (the "short presentation conjecture"). This conjecture has been verified for all but three of the infinite classes of finite simple groups ("rank-1 twisted groups") [BGKLP], so we can say we have an *almost theorem* that non-membership in matrix groups belongs to NP.

I then wished to complement this *almost theorem* with a genuine *theorem* that the non-membership problem belongs to *almost–NP*. The concept of *almost NP* would involve a marriage of randomization and nondeterminism; the attempt at defining the right combination of these two concepts naturally led to the "Arthur–Merlin hierarchy" of complexity classes [Ba85], the public coin variety of Interactive Proofs [GMR]. The connection of interactive proofs to many problems in algorithmic group theory is explained in [Ba92a].

The concept of interactive proofs, especially its generalization to multiple provers, has quickly led to a huge body of work with striking implications to the seemingly unrelated area of approximate optimization (cf. the survey [Ba92b]).

Curiously, interactive proofs made an important contribution to clarifying the complexity status of Graph Isomorphims (GI); it turns out that GI belongs to

the class *coAM*, and therefore GI *cannot be NP-complete,* unless the polynomial time hierarchy collapses [GMW].

I should mention one last feedback loop involving interactive proofs: the results in [Ba92a] rest on a lemma on "local expansion" in groups. Subsequently this lemma, conceived in a nondeterministic context, found many algorithmic applications, to polynomial time and even nearly linear time algorithms [Ba91b,BCFS,BeB].

Ruvim Gurevich (1952–1989)

After living as a *refusnik* for many years, Ruvim Gurevich eventually managed to emigrate to the U. S. in 1985. Already an accomplished logician, he become a graduate student at the University of Illinois at Urbana. He obtained his Ph. D. in 1988 and subsequently joined the mathematics faculty of the University of Wisconsin at Madison.

Earlier that year Ruvim was diagnosed with cancer. The Soviet government allowed his mother to join him in December 1988. Ruvim passed away[24] on October 10, 1989, three days after his 37th birthday.

Maybe someday there will be a reunion of the old friends. If not at LOMI, or POMI, as it is now called, then over the Internet. To interact, we don't need passports, visas, and rubber stamps, we don't have to take the *platskartnyĭ vagon.* (We may need passwords, though.)

We shall miss Ruvim at that virtual reunion.

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