

Incomputability in Physics: Does It Really Matter?

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What this talk is not about...

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- A full picture of incomputability in physics.
- A fatal flaw discrediting the concept of incomputability.
- A cure for incomputability.

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and their meaning, significance and limits have been critically discussed by Svozil (1993, 2009), Penrose (1989, 1994), Traub (1998), Barrow (1998, 2000), Cooper and Odifreddi (2003), Reif (2009).

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Hehner (2007) analysis concludes that *there are no incomputable (or uncomputable) functions*.

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For Traub (1998) the answer to the first question is negative.

Penrose (1989): *it is hard to see how ... non-computability ... could be of any "use" in a device which tries to "harness" possible non-computable elements in physical laws.*

Is it possible to “harness” the
incomputable elements in physics?

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- There is a total independence of previous history, such that no correlation exists between the i th outcome and previous or future outcomes. The system carries no memories of previous or expectations of future events. All outcomes are temporally “isolated” and free from control, influence and determination. They are both unbiased and self-contained.

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No two-valued measure exists for more than two mutually exclusive outcomes per experiment in a Hilbert space of dimension three and higher, as the Kochen-Specker theorem shows.

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How “random” are sequences of quantum bits?

Probabilistic algorithms and the quality of randomness

To probabilistically test whether an integer $n > 1$ is prime, we take a *sample* of k natural numbers *uniformly distributed* between 1 and $n - 1$, inclusive, and for each chosen i we check whether a simple predicate $W(i, n)$ holds. If $W(i, n)$ holds for at least one i then n is composite; otherwise, the test is inconclusive, but in this case if one declares n to be prime then the probability to be wrong is smaller than 2^{-k} . Predicates W have been proposed by Miller-Rabin and Solovay-Strassen.

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Chaitin and Schwartz (1978) proved that if the *sample* is not only uniformly distributed, but *algorithmically random*, then for almost all n the algorithm returns the **correct** answer.

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- **(Open)** Are probabilistic algorithms powered by quantum randomness *correct*, not just highly probable correct?
- **(Open)** A Turing machine with an oracle of quantum random bits has hypercomputation power. How powerful is such a machine?