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Is the Universe Lawful?

CRISTIAN S. CALUDE † and **F. WALTER MEYERSTEIN** ‡

† Computer Science Department, The University of Auckland, Private Bag 92109, Auckland, New Zealand
‡ Departament de Filosofia, Universitat Autonoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

Abstract—The last 2500 years have been dominated by the belief, expressed in different forms, that the Universe is lawful, that is, the Universe is a *knowable* system governed by rules which determine the future *uniquely* and *completely*. An extreme way to express this belief is to claim overconfidently that the study of some branches of science will soon be completed, will soon attend an *end*. Our aim is to challenge this apocryphal hypothesis by arguing, with complexity-theoretic arguments, that the *Universe is lawless*, that is, the Universe is lacking any kind of general ordered structure implied by the term "law".

There are currently two opposing views of the scientific enterprise. First of all, the overconfident one proclaiming that science is converging towards a "final theory," a "theory of everything"† which will describe the basic laws governing the Universe.‡ The opposing view of the scientific enterprise is the skeptical one that questions the power of science to ever arrive at some kind of "Truth". "Is the world just too complex for the human mind to fully comprehend?" (Casti [3,30]). Casti and Karlqvist's conclusion [4] is negative, that is, the physical world is not too complex for the human mind. Our thesis is that the inability of the human mind to fully understand the world of mathematics necessarily implies the impossibility for the human mind to completely understand the Universe.

In what follows we will speak about the "Universe", and its presumed "lawfulness". We will start by trying to "describe" (not define) both terms. The fact that these descriptions are extremely vague has not escaped our attention; nevertheless, we believe them to be precise enough for the aim of this paper.

1. THE UNIVERSE

For this paper, the Universe is the solar system plus the sum of electro-magnetic radiation and some rare particles reaching that system, like neutrinos arriving from a supernova.§

† We have in mind Deutsch's [1] "theory of everything" which is supposed to explain the "fabric of reality". This theory, which will enable us to understand emergent phenomena as life, thought and computation, is far wider than the "theory of everything" aimed to uniformly describe the basic forces known in physics, i.e. gravity, electromagnetism and nuclear forces.

‡ An extreme reading of Laplace's scenario—*a thing cannot occur without a cause which produces it*. The stage is set at the beginning and everything follows "mechanistically" without the intervention of God, without the occurrence of "miracles". It is not just a matter of the future being determined by the past; the *entire history of the Universe is fixed*, according to some precise mathematical scheme, *for all time*, cf. Penrose [2], pp. 558–559.

§ Observable, in principle, by humans during their recorded history and in some reasonable future.

The solar system is more or less directly accessible to experimentation; claiming its lawfulness can, at least to a certain extent, be the subject of experimental verification.† This part of space-time which can be seen from the Earth is relatively insignificant in comparison to the part which lies in the future.‡ What happens beyond the solar system is less clear§; it is precisely this part of the Universe that interests most this paper.

Barrow ([7], p. 3) observes that *the fact that the Universe is so big and old has influenced our religious and metaphysical thinking in countless ways*. These metaphysical and mythological attitudes towards the unknown have, in turn, determined many of the general ideas that we now accept about the Universe, Barrow [8].

In keeping with these considerations, and for the sake of an illustration, we will very briefly summarize some fundamental assumptions which modern cosmology puts forward in order to fulfill its program: arrive at a set of reasonably compact mathematical expressions representing the geometry and the dynamical relations of the extra-Solar-System Universe.¶ These are extremely strong and, in a way, shocking assumptions which are certainly not self-evident, and very difficult if not utterly impossible to prove:‖

- *The Universe is isotropic with respect to the Solar System.* This does not apply to small** dimensions, but only if one averages over gigantic dimensions, of the order of hundreds of Mpc (1 Mpc = 3.26 million light years). Stated, equivalently, an observer cannot distinguish one of his space directions from the other by any local physical measurements.
- *The solar system does not occupy a privileged position in the Universe*, consequently, the Universe is isotropic for all observers, anywhere, anytime.††
- *The Universe is homogeneous; it is spherically symmetric overall.*

2. WHAT IS A NATURAL LAW?

For some (e.g. Zilsel [10]), natural laws are just metaphors (taken over from laws in the juridical sense); for others, they are "like veins of gold, and that scientists are extracting the ore," (cf. Johnson [11]).

One distinguishes between physical processes *as they are* and *as they are known*. One distinguishes, again, between what is *real*, what is *known* and what is *knowable*. Natural laws represent our best attempt to represent approximately the actual/real physical processes by means of observations/measurements and reason/logic.

Where do the natural laws come from? Why do they operate universally and unfailingly? Nobody seems to have reasonable answers to these questions. We can argue that one decides to adopt a natural law by reflecting on our observations, which are compared with the observable consequences of the law in order to get or not our support. This scenario points to the three factors involved in this process: the physical processes, the observations, and the physical laws. The observations produce *numbers* which result from various measurements (length, temperature,

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†The fact that the sun is one of the many stars seems to be as irrefutable as the roundness of Earth or the fact that all elements are made by atoms.

‡"...we humans have come into existence in the very early childhood of the cosmos", Tipler [5], p. x.

§Without further assumptions, the extra-solar system Universe appears to be a patchy space-time collection of uncorrelated sources of electro-magnetic radiation, cf. Ellis and others [6]. It seems that we receive information exclusively from thin slices or from isolated points of the Universe. For instance, we may detect objects as they emitted such information 100, 1,000, 100,000, or a billion years ago. Where are these objects now? We have no idea.

¶Based on which, at least partial predictions have become feasible.

‖To arrive at a cosmology of the Big Bang type, many additional postulates are required, see, for example, Brisson and Meyerstein [9].

**In the astronomical sense.

††A fantastically strong assumption, considering the enormous size of the Universe.

weight, etc.). For example, quantum theory suggests that an electron's trajectory is ambiguous, but the ambiguity suddenly ceases when the particle is measured, that is, when the wave function "collapses" and the particle's state assumes one of its possible values.[††] *The measurement problem implies that the physical realm is defined in some sense by our perception of it.* These perceptions have a finite precision limit, and, consequently, the natural laws, in their observable consequences, have a finite precision. In contrast, no natural law can be universal if it avoids the use of infinity; coping with infinity is, currently, [††] *beyond the capability of any measurement.*

So, what do we mean by "lawful"? Actually, this is just a modern way of speaking, as we are not only interested in what we can know about the Universe, but in what does it mean to "know" something at all. To avoid millenia-old philosophical discussions, we will say that to know the Universe, in the context of this paper, means to be able to establish some globally valid mathematical laws allowing a certain amount of short-term predictions about the dynamics of the Universe or some restricted domain of it, or some particular phenomenon appearing in it, predictions which can be tested at least to some adequate extent. Are there better formalisms than the mathematical one? In retrospect, the mathematical formalisms seem to be inevitable and irrevocable: in any case, for the moment there is nothing to replace them.[§]

3. THE LAWLESSNESS HYPOTHESIS

What are the arguments justifying the hypothesis that the Universe is lawful?[¶]

The most we can do is to explain that this hypothesis is supported by our daily observations: the rhythm of day and night, the pattern of planetary motion, the regular ticking of clocks. It is a simple matter of reflection to point out some limits to this type of argument: the vagaries of weather, the devastation of earthquakes or the fall of meteorites are "perceived" as fortuitous. How can the same physical process, for example the spin of a roulette wheel, obey two contradictory laws, the laws of chance and the laws of physics?

Perhaps a different hypothesis can better explain this type of behaviour. As our direct information refers to finite experiments, it is not out of question to discover *local rules*, functioning on large, but finite scales, even if the global behaviour of the process is truly random.[‖] But, to perceive this *global randomness* we have to go beyond the finite; we have to access *infinity*, which is not physically possible!

It is time now to state the alternative view, the hypothesis that the Universe is lawless.

The fact that the Universe is lawless is not a new idea. Twenty-four centuries ago, Plato in the *Timaeus* invented a cosmology, based on a different set of fundamental assumptions than the ones presented in Section 1. Because the mathematical science of his time was much less developed, and because he wanted to transmit his ideas to the general—although philosophically

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[††] A particle can evolve on many paths and the probabilities to find it on a specific path are defined by the wave function. To be more precise, the wave function, that is the Green function of the corresponding nonstationary wave equation, is defined by the multiplicative integral over all possible paths, which begin at the initial point x_0, where the particle was at the initial moment, and end at the point x: $\int_{x(0)=x_0, x(t)=x} \int_0^t \prod_s \mathrm{d}x(s) e^{i \mathrm{d}t[(\frac{\mathrm{d}x}{\mathrm{d}s})^2 + q(x(s))]} = G(x, t; x_0, 0)$. The wave function which corresponds to the nontrivial initial condition $\Psi(0) = \Psi_0$ is represented as a convolution of the initial value $\Psi(0) \equiv \Psi_0$ with the Green function: $\Psi(x, t) = \int G(x, t; u, 0)\Psi_0(u)\, \mathrm{d}u$. Here we have assumed that the factor in front of the Laplacian is equal to 1.

[††] That is, assuming the Church–Turing Thesis.

[§] An algorithmical model is considered just another kind of mathematical model, even though not a traditional one.

[¶] The problem of "rationality" is beyond the aim of this paper; see more in Brisson and Meyerstein [12].

[‖] In the context of algorithmic information theory, cf. Chaitin [13,14], Calude [15], proves that every random sequence is a *lexicon*, i.e., in every random sequence every word—of any length—appears infinitely many times. The fact that the first billion digits of a random sequence are perfectly lawful, for instance by being exactly the first digits of the decimal expansion of π, does not modify in any way the global property of randomness.

trained—public, he had to adopt a language which later interpreters confused, first with a myth, and later with theology.**

We cannot hope to summarize Plato's assumptions in the *Timaeus*, so, just as for present-day cosmology, we will only highlight some of the basic, but unprovable, statements put forward in the *Timaeus*:

- In the beginning—this term will be clarified below—the demiurge‡ finds a completely chaotic substrate, for which Plato uses the word "Chora". The Chora has only one property: it is the material substrate of the Universe in a primordial state, a state which we would call today *algorithmically random*. Since everything ought to have a cause, including total disorder, Plato proposes an acting principle of disorder, a cause of randomness, which he calls "Anagke".§
- The demiurge is trying his best to "persuade" Anagke to accept a mathematical order. Where he succeeds, one arrives at a finite set of purely mathematical elementary building blocks—Plato's perfect polyhedra—which, when combined by simple mathematical rules, constitute the ordered Universe.¶ But only the part where the demiurge succeeded in persuading Anagke is ordered. In fact, the demiurge is not all-powerful. Thus, in Plato's Universe, order is only partial. And an irreducible disorder, chaos, randomness remains, so irreducible that nothing can be *said* about it. Plato does not indicate anywhere what part of the Universe is lawful, and what part is entirely random.

Plato, and 2,000 years later, Kepler and Galileo, put forward what became the official program of science: Find the lawful part of the Universe, and, if lucky, try to formulate the mathematical laws describing its "dynamics". Plato would have said the "kinesis", i.e., change, in the widest meaning of this word.

One more comment. The demiurge is not the God of Genesis, as later interpreters hoped to prove; in fact, the demiurge does not "create" anything at all, he is only the sufficient cause of order, where such order exists. That is, instead of saying, "there is a natural law which underpins the order we can sometimes somewhere detect", Plato says, "the demiurge caused . . . ", and then he adds the mathematical expression describing in rigorous terms, this partial order. But Plato does also speak of a "beginning". What does that mean?

> *When the demiurge ordered the heaven, he made, of eternity that abides in unity, an everlasting likeness moving according to number, that to which we have given the name "time".*†

Thus time is the imperfect image of a perfect object, of eternity considered as unity. Both eternity and unity are what later interpreters have called Platonic ideas. Since time can be numbered, it is not an idea, but only an imperfect copy of an idea. It also follows that time must appear simultaneously with the ordering of the "heaven" (the extra-Solar-System Universe), since without the order imposed by numbers, there can be no Cosmos, only total randomness.

This explains the term "in the beginning" used above: it is not an act of "creation", but simply the obvious point that in the absence of a "time" that can be measured‡ one cannot speak of order, and thus no Cosmos can exist. Last, but not least, Plato's definition of time allows the use of mathematics in the formulation of the laws of nature: if time can be numbered, then change can be reduced to the only symbolic language that permits the rigorous manipulation of

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**We claim, however, that this kind of unproven and intrinsically unprovable assumptions should basically be classified in one and the same category, that is we believe it is irrelevant if currently one set of such statements is called a myth and another one science. What only makes a real difference between assumptions is whether or not they lead to predictable experimental results. Furthermore, these basic assumptions have a "temporary validity" as there is no guarantee that the next experiment will not produce an aberrant result.

‡The name Plato gives to the efficient cause of the Universe.

§In Greek, Anagke means "necessity". Here, as in many other places, instead of inventing a neologism Plato provides a word from the normal language with a new meaning, the source of much misunderstanding.

¶In Greek, "cosmos" actually means order, but order with beauty, such as still detectable in the present-day term "cosmetics".

†Translation from Brisson and Meyerstein [9].

‡Measured in days, years, seasons, etc.: if the Solar System is totally random, then there is no time, and no Cosmos.

measures. (Plato's scheme is to pass from the perfect, continuous time to the discrete, measurable time. Consequently, the mathematical picture given by the discrete is but an imperfect copy, only an *eidolon* of reality, a reality for ever out of reach for human brains, because to grasp reality one has to grasp the infinite!) Based on these considerations, Plato offers the observation of the regular celestial (circular, or combination of circular) movements as validation for his axiomatic system (i.e., for the "existence" of a "demiurge" ordering the "heaven" with a time that can be numbered).

We have thus tried to show that Plato's explanation as well as modern cosmology are constructions crucially based on a set of assumptions that are not proved and cannot be proved. Such statements, we maintain, are equivalent to a myth. In both cases, however, some degree of experimental verification is possible. Based on his set of assumptions, Plato predicted that the movement of the seven "planets" could be accounted for by a combination of circular movements. This was the generally accepted world-view for 2,000 years, until Kepler could make use of the careful measurements made by Tycho Brahe.‡ Modern cosmology makes a large number of predictions, of which actually only three tests have been more or less unambiguously validated: the existence of a cosmic microwave background radiation, the Hubble expansion of the galaxies, and some predictions concerning the proportion of the light elements resulting from primordial nucleosynthesis. It must however be stressed that each of these observational "proofs" indispensably requires the interpretation of the data in the light of additional very complex and elaborate theories incorporating a much wider and controversial set of assumptions; thus they are not proofs *stricto sensu*.

Twenty four centuries later, Poincaré has also suspected the chaotic, random nature of the Universe when he wrote:†

> *If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that universe at a succeeding moment. But even if it were the case that the natural law no longer had any secret for us, we could still only know the initial situation* approximately. *If that enabled us to predict the succeeding situation with the* same approximation, *that is all we require, that [it] is governed by the laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.*

We, obviously, do not deny that present-day cosmology gives us a much more "modern" world-view than Plato's; what we are trying to say is that both cosmological models are pure inventions of the human mind based on sets of statements that we suggest should be classified as "myths".

4. CAN THE LAWLESSNESS HYPOTHESIS BE PROVED?

Most probably, it cannot be proved. As we have argued earlier, our *partial, incomplete and provisional* understanding of the Universe comes through measurements, so ultimately through numbers.§ As much of the elementary intuition about numbers derives from our linguistic abilities to assign names to objects‡ it is not surprising that our arguments will focus on numbers; see also Calude and Salomaa [18].

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‡This combination, careful measurements, expressed in numbers, plus a mathematical theory, allows science in the modern sense. Incredibly, this was also clear to Plato, in the fifth century B.C.!

†Quoted from Peterson [16], p. 216.

§According to Landauer [17] "The laws of physics are essentially algorithms for calculation. These algorithms are significant only to the extent that they are executable in our real physical world. Our usual laws depend on the mathematician's real number system."

‡Barrow ([7], p. 4) claims that the our "linguistic abilities are far more impressive than our mathematical abilities, both in their complexity and their universality among humans of all races."

Our first argument is given by Gödel's Incompleteness Theorem ([19,20]):

> *Any sufficiently rich, sound, and recursively axiomatizable theory is incomplete.*

In Gödel's terms, incompleteness is a relative result, i.e., it essentially depends upon the fixed axiomatic system. Using an information-theoretic version proof of the undecidability of the halting problem (see Chaitin [25]) an *absolute* claim has been proved, namely the

> *lack of a decision procedure for the truth/falsity of statements expressible within the formal axiomatic system.*

In fact a more quantitative statement has been proven by Chaitin [13] (see, also, Calude [15]):

> *An N-bit set of axioms cannot yield a theorem that asserts that a specific object is of complexity substantially greater than N.*

It is only natural to ask the questions: How "ubiquitous" is the incompleteness phenomenon? Is it just a curiosity, an exotic and artificial phenomenon? Is it a very bizarre pathological case, or is it pervasive and quite common?

It seems that the general feeling was that independent, true but unprovable, statements are rather unusual, if not abnormal. This impression is recounted by Solovay ([26], p. 399): "The feeling was that Gödel's theorem was of interest only to logicians"; Smoryński ([26], p. 399) is quoted as stating: "It is fashionable to deride Gödel's theorem as artificial, as dependent on a linguistic trick." Contrary to these claims, Calude, Jürgensen and Zimand [27] have proved that in a strong qualitative[§] way,

> *the set of unprovable and true statements is large.*[¶]

These results show that unprovability is a common phenomenon. In some sense, of course, Gödel's incompleteness phenomenon indeed depends on a "linguistic trick," essentially based on the possibilities to assign names to problems: one simply cannot change this fact. Instead, the above mentioned analysis shows that this linguistic trick is widely applicable and inevitable.

We need only to make one step to get randomness from incompleteness. Chaitin [13,14] has constructed an exponential Diophantine equation $P(i, X_1, \ldots, X_m) = 0$ having the following property: the binary sequence whose jth term is 1 if for the parameter $i = j$, the equation has infinitely many solutions in $X_1, \ldots, X_m$, and 0 in case the equation has only finitely many solutions, is *random*. Any formal theory can answer the question "Has $P(i, X_1, \ldots, X_m) = 0$ infinitely many solutions?" for finitely many values of i only. No matter how many additional answers we learn, for instance, by experimental methods or just flipping a coin, this will not help us in any way as regards the remaining infinitely many values of i. As regards these values, mathematical reasoning is helpless, and a mathematician is not better off than a gambler flipping a coin. This holds in spite of the fact that many mathematical problems are *finitely solvable*[‖] and every finitely solvable problem can be solved by simply checking that it holds true for finitely many cases—to decide whether the problem is true we only need to check that it holds for the first N cases, where N is a constant depending upon the problem, cf. Calude, Jürgensen, Legg [28]. The catch is that we cannot even place a computable upper bound on N.

We will close this analysis with the following question: To what extent is the system of real numbers contaminated by "randomness"? Following Jürgensen and Thierrin [29] we say that a real number in base b is *disjunctive* in case the sequence of its base b-digits contains all possible words over that base. A *lexicon* is a number which is disjunctive in any base. A lexicon

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[§] Currently, there is no measure-theoretical counter-part for this result.

[¶] More precisely, for any reasonable topology, the set of unprovable and true statements is dense, and in many cases even co-rare.

[‖] Goldbach's Conjecture, *every even positive integer is a sum of two primes* and Riemann's Hypothesis, *the function* $\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$ *has all non-real zeros on the axis* $x = \frac{1}{2}$ are finitely solvable.

contains all writings, which have been or will be ever written, in any possible language.** A lexicon expresses the strongest qualitative idea of randomness; pure randomness is a much more demanding property, but the natural, typical realization of a lexicon is by tossing a pure coin, i.e., with probability one a lexicon real is purely random.††

Let x be a real number in [0, 1] written in base b, and let $x_{(n)}$ have precisely n digits, namely the first n digits of x (completed with zeros if necessary).

If the word a appears (without overlaps) exactly k times in $x_{(n)}$, put

$$p_{a,n}(x) = \frac{kL(a)}{n},$$

where $L(a)$ means the length of (number of digits in) a. Let

$$p_a^-(x) = \liminf_{n\to\infty} p_{a,n}(x), \qquad p_a^+(x) = \limsup_{n\to\infty} p_{a,n}(x).$$

According to the law of large numbers, in almost every real number from $[0, 1]$, every word appears with its "natural" probability, so for example the 1's appear with probability $1/2$ if base two is used. This happens for almost all, but not exactly all of them. In a qualitative sense, that is in the sense of Baire categories, how do most numbers behave? Calude and Zamfirescu [21] have (constructively) proved the following theorem:

For most numbers $x \in [0, 1]$, i.e., all, except those in a set of first category, using any base and choosing any word a, written in the same base,

$$p_a^-(x) = 0 \quad \textit{and} \quad p_a^+(x) = 1.$$

Actually, the above relations show how un-ordered, chaotic, most numbers are, when one adopts the point of view of topology rather than measure theory. As a consequence, one constructively shows that the typical number is a lexicon (the class of lexicons is larger than a set which is residual and of measure-one). Consequently:

Constructively most numbers do not obey any probability laws.

This result (from which we can immediately deduce the Oxtoby and Ulam's theorem [22] stating that the *law of large numbers is false in the sense of category*) says that:

The system of real numbers, our very basic language of expressing the natural laws, is fully contaminated by randomness.

In Chaitin's words,

God not only plays with dice in quantum mechanics,† but even with the whole numbers.

5. DOES THE LAWLESSNESS HYPOTHESIS IMPLY THE END OF SCIENCE?

The answer is negative. Consistent with our common experience, facing global randomness does not imply the impossibility of making predictions. Space scientists can pinpoint and predict planetary locations and velocities "well enough" to plan missions months in advance, as-

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**For a musical analogue see Karlheinz Essl 1992 interactive, real-time composition for computer-controlled piano titled "Lexikon–Sonate" at url `http://www.ping.at/users/essl/Lexikon- Sonate.html`.

††Cf. Martin-Löf's theorem, see Calude [15], p. 129; in fact, the situation is more dramatic, as by Gács' theorem, see Calude [15], pp. 155–165, every real is effectively reducible to a random one, see also Szovil [32].

†What about a clockwork of Newtonian determinism which sits in the world of certainty and objective reality and creates the appearance of uncertainty and subjectivity to our imperfect senses and inadequate measuring apparatus? This possibility was ruled out by the experiments carried out in 1980s by a group headed by Alain Aspect working along the lines suggested twenty years before by Bell. Their conclusion is strong: there is no underlying clockwork, the spooky action at a distance, that Einstein hated so much, is correct.

tronomers can predict solar or lunar eclipses centuries before their occurrences, etc. However, we have to be aware that all these results—as impressive as may be—are only true *locally* and within a certain *degree of precision.* Of course, in the process of solving equations, say of motion, small errors accumulate, making the predictions less reliable[§] as the time gets longer. We face the limits of our tools and methods! Why are they so imperfect? Algorithmic information theory gives researchers an appreciation of how little complexity in a system is needed to produce extremely complicated phenomena and how difficult is it to describe the Universe.[‡]

6. TENTATIVE CONCLUSIONS

Now we come to the questions: What have we achieved in this paper? Have we proved anything at all?

No human endeavour leading to knowledge is safe; no human knowledge is certain. Truth is a metaconcept. As W. V. O. Quine said: "The proposition *snow is white* is true if and only if snow is white". Any bit of knowledge achieved by scientific methods, that is by hypothesis–deduction–prediction–testing methods, is necessarily based on a set of *unproven and unprovable assertions.*[§] It follows that any conclusions are always provisional, as any change in the set of hypotheses may lead to different results.

This paper suggests the importance of an alternative scenario, discussed in Calude and Salomaa [18]. Assume the following basic "myth": the Universe, in particular, the extra-Solar-System portion of it, is, in the strong sense of the verb "to be", a very long bit-sequence, the numerical output produced by suitable devices measuring the electromagnetic radiation arriving from various sources. These measurements, past, present and future, must be considered, constitute a gigantic sequence of bits, i.e., these measurements taken together form the first N bits of the infinite expansion of some real number. As such a sequence is typically a lexicon,[¶] the conclusion follows: the *Universe is lawless.*[‖] Simply tossing a fair coin is all that is required to produce everything, to produce as outcome the entire Universe. Everything is there, on the lexicon. All Shakespeare, every galaxy, each human brain .... Where are we on this Universe-lexicon? We will never know. But maybe we "live"[**] on a very long finite sequence that is ordered, where "Anagke" has been thrown out, where we may find many laws of nature, where science has a literally enormous domain of investigation, where we may arrive one day to some theory of everything?[††] But, and this is the key point, but everything lies in that particular *finite* section of the lexicon. And, *if* the Universe is a lexicon, and since we obviously must be located on some section of it, we can always *hope* that we live on a partially ordered finite section, allowing us to progress in its scientific exploration for many centuries yet. But even then, no guarantee can be expected, no particular cosmos/"Anagke" mixture can be assumed. In fact, if the Universe is a lexicon, the only and exclusive fact we can rationally assume, is that anything can arrive, anytime, anywhere, we must admit that anytime, anywhere, the "order" may suddenly, without transition, switch back to pure randomness, to "Anagke".

[§]Barrow [23] has proven the Einstein's equations exhibit a formal chaotic behaviour, which means that the evolution of the Universe becomes unpredictable after a time short in cosmological scales.

[‡]A random sequence cannot be "computed", it is only possible to approximate it very crudely.

[§]Called "myths" in this paper.

[¶]In Calude and Salomaa [18] the model was Chaitin's omega number.

[‖]The idea that the Universe behaves like a lexicon can be traced to Nietzsche (cf. Tipler [5], p. 78.):*... the Universe must go through a calculable number of combinations in the great game of chance which constitutes its existence. In infinity, at some moment or other, every possible combination must once have been realized; not only this, but it must once have been realized an infinite number of times.*

[**]Meaning as far as we can judge from observation, history, etc.

[††]That is "everything" on this restricted and partial section of the lexicon.

Casti [3,30] has asked the question “Is the world just too complex for the human mind to fully comprehend?”. For Hawking reality is not a quality you can test with litmus paper, so we cannot demand that a theory “correspond” to reality because we don’t know what the “real world” is.[‡‡] In opposition, Penrose thinks that simply comparing predictions with experiments is not enough to explain reality. For Casti and Karlqvist [4] the physical world is not too complex for the human mind, even though they admit that the mathematical world *is* too complex.edefYou′velostsometext.Trytyping<return>toproceed.■Ifthatdoesn′twork,typeX<return>toquit.

We think that the arguments presented in this paper show that, while Penrose is right, due to the fact that the only language one can use to understand the “real world” is mathematics and because of the inability of the human mind to fully understand the world of mathematics, theories *cannot* do better than confirm experiments. Quantum mechanics and relativity have shown that it is impossible to know everything: quantum mechanics argues that we cannot know because we are part of the system and relativity suggests that the Universe is too big to be known as we cannot have enough energy. Mathematics teaches us that, with extremely rare exceptions, any real number representing the outcome of the measurements is a lexicon. A lexicon, although we can define it, although we understand what we mean by that mathematical expression, is way beyond the capabilities of the human mind because the bits in the sequence of a lexicon are devoid of any order, any law, even the law of large numbers. Thus we assert that the impossibility of the human mind to fully understand the world of mathematics must necessarily imply the impossibility of the human mind to completely understand the Universe as *Universal natural laws do not exist*. Or, to put it differently, it necessarily implies that the Universe, defined as the set of measures describing it, is devoid of any overall lawfulness, of any order; consequently, the Universe can be the outcome of the most elementary random process, *the toss of a fair coin*.

So our answer to Casti’s question is affirmative: *it is impossible for the human mind to completely understand the Universe*. Finally, the “credo” in universality, adopted by many scientists[†] should be revised.

But this in no way proves the end of science (see Horgan [31], nor the impossibility of finding the theory of everything, as long as we always remember: This applies exclusively to some finite part of the lexicon—which is infinitely long!—in the same way that some part of it reads exactly like everything ever written by Shakespeare.[‡]

Maybe our conclusion is echoed in Hawking’s recent views[§]:

> *The intrinsic entropy means that gravity introduces an extra level of unpredictability over and above the uncertainty usually associated with quantum theory.*[¶] *So Einstein was wrong when he said, “God does not play dice.” Consideration of black holes suggests, not only God does play dice, but that he sometimes confuses us by throwing them where they can’t be seen.*

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[‡‡] “I take the positivist viewpoint that a physical theory is just a mathematical model and that it is meaningless to ask whether it corresponds to reality. All that one can ask is that its predictions should be in agreement with observation”, cf. Hawking and Penrose [24], pp. 3–4.

oesn′twork,typeX<return>toquit.“The provisional message coming out of these disparate efforts is that, unlike mathematics, there is no knock-down, airtight argument to believe that there are questions about the real world that we cannot answer—in principle.”

[†] See, for example, Tipler [5], p. 73: “until an experiment shows otherwise, a scientist should always assume that the firmly established laws of physics are true in all circumstances.”

[‡] We thus equally affirm that Shakespeare’s work could be just the outcome of fair coin tosses (!), since he, Shakespeare, and the world he lived in, are just parts of the lexicon.

[§] Hawking and Penrose [24], p. 26.

[¶] A massive star, which has exhausted its supplies of nuclear energy, collapses gravitationally and disappears leaving behind only an intense gravitational field to mark its presence. The star remains in a state of continuous free fall, collapsing endlessly inward into the gravitational pit without reaching the bottom.

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