A Multi-Criteria Metric Algorithm for Recommender Systems

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Abstract. Information overload and an abundance of choices create situations where selecting one option becomes extremely difficult or even worse, a guessing game. Collaborative ranking systems are widely used to alleviate this problem by creating intelligent rankings of items based on an aggregation of user opinions. Current ranking systems can still be improved in a number of areas, including accuracy, transparency and flexibility. This paper presents a multi-criteria ranking algorithm that can be used on a non-rigid set of criteria. The system implementing the algorithm fares well with respect to the above qualities.

Keywords: Metric algorithm, intelligent ranking, non-rigid criteria

1. Introduction

This paper presents a multi-criteria ranking algorithm that can be used on a non-rigid set of criteria (that may be defined by the users of a system). The approach uses a novel concept of an ideal candidate, which is an aggregation of users’ digital belief systems. While most systems produce a ranking based on the origin as a pivot point, we present an algorithm that first calculates a hypothetical ideal candidate, which is then used as the pivot point.

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The paper is structured as follows. First, we briefly present the motivation and related results. Then, an overview of the algorithm is informally described followed by a detailed description of the algorithm itself. We then describe a proof of concept implementation of the algorithm and a user study evaluating the perceived accuracy of the algorithm and usability of the system. We finish with conclusions and a brief discussion of future work.

2. Motivation and related facts

The biggest motivating factor for recommendation systems in general is that of information overload. Our society produces more information than it produces anything else \cite{12, 27, 28}. Information overload leads to situations where the inputs to one’s decision making process exceed the “capacity to assimilate and act on the information as well as the ability to evaluate every alternative” \cite{25}. Information overload has also been linked with negative psychological impacts created by the illusion that more choices lead to better results \cite{19}. Recommendation systems generally use single criteria ratings that define how good an entity is. For example \cite{14} uses a single 10 star rating for each movie for their recommendations. More recently, multi-criteria recommendation systems have become popular, as evidenced by Yahoo! Movies’ recent movie recommender system. Various surveys and papers \cite{1, 2} have indicted the need for multi-criteria rating systems and have shown the increase in accuracy they can achieve. Transparency, “the opposite of secrecy” \cite{17}, is important because it goes hand-in-hand with trust and accountability. Transparency increases trust, hence recommendations acceptance \cite{10}. Transparency increases accountability too, as seen in numerous situations presented in Wikileaks (http://www.wikileaks.org). Fundamentally, a multi-criteria system allows for more transparency because one sees how each rating is broken down to create the overall rating. Flexibility is also paramount: it allows users to participate with their own preferences and knowledge. There has also been extensive work done on content, collaborative and hybrid based recommender systems and social information filtering, see for example \cite{4, 22, 6, 2, 24}. Demographic, utility and knowledge based systems have been proposed by \cite{6}. Recently, matrix factorisation methods have been used in \cite{16}.

Multi-criteria approaches to recommendation and ranking systems have been considered in \cite{1, 11}. The authors crawled Yahoo! Movies and extracted a number of movie ratings decomposed into 4 criteria; they found that using multi-criteria ratings allows for more accurate ratings than single ratings. One approach they used was to simply divide the multi-criteria problem into $M$ single criteria recommendation problems, thus treating each criteria as a single rating. Another approach taken by \cite{21} treats tags as multiple dimensions and first infers users’ preferences for tags and then resultantly for items (movies).

Multi-criteria approaches to recommendation and ranking systems are subject to limitations which were first proved for voting systems. The most famous result, Arrow’s Impossibility Theorem \cite{3} (also known as Arrow’s Paradox), states that no voting system can turn individual preferences into a global (community) ranking if there are three or more options to choose from and a few “innocent-looking” conditions (such as, non-dictatorship, Pareto efficiency, and independence of irrelevant alternatives) are satisfied. Another limitation may appear because of the lack of independence of preferences; this is related to phenomena studied in complexity theory where different seemingly unrelated computational tasks can be in fact related in a subtle way \cite{15}.

\footnote{The amount of digitally stored information in 2009 was estimated to be 500 exabytes \cite{29}.}
3. Algorithm overview

The algorithm applies to a system composed of 5 parts: the users, the entities, the value dimensions, the belief system and the ideal candidate. Entities are anything that the system is recommending or ranking. For example in a movie recommender system, movies are the entities. Value dimensions are a set of factors that influence the ratings of an entity. For example, taste and price are value dimensions that influence the ratings of menu items in a restaurant. All entities are defined over a set of value dimensions. Users collaborate within the system by rating an entity over the set of value dimensions. For example, a user may rate price high and taste poor, or price low and taste excellent.

The belief system is personal to each user. Each user is allowed to tell the system what ideal value they want a value dimension to have, and how important that value dimension is to them. For example, most people’s belief system would have a value of ‘low’ for the dimension ‘price’ but depending on the level of income, the importance of price may vary. Finally, the ideal candidate is the vector of ideal values for each value dimension. The system determines the ideal value dimensions by aggregating all users’ belief systems into an average. That is, if there were 2 people in the system, and user one’s belief system had the ideal value for price set to high, and user two’s belief system had the ideal value for price set to low, then the ideal candidate will have its ideal value for price set to ‘in between low and high’. The ideal candidate can be thought of as the belief system of a hypothetical user that takes everyone’s opinions into account.

Value dimensions can be either hard or soft. Hard value dimensions are factual, such as the price of an item, or the location of a building. Soft value dimensions are subjective, i.e. an opinion. The major difference between a soft and a hard value dimension is that a hard dimension cannot be rated, while a soft one can. While a hard value dimension cannot be rated, its belief weight can still be set. Price is an example of a hard value dimension because the price of an entity is a factual piece of information (bar bargaining practices). On the other hand, quality is a subjective (hence soft) value dimension as there are no standard measurements to quality and are subject to individual perspectives.

The ideal candidate discussed above is the global ideal candidate (i.e. all users’ belief systems aggregated into one). The system also uses a local ideal candidate, which is simply equivalent to a single user’s belief system. The distinction between a local and global ideal candidate results in two different types of rankings and two different types of recommendations—a global and local ranking and a global and local recommendation. A global ranking of entities is one which calculates distances between an entity and the global ideal candidate, and a local ranking calculates distances between an entity and a local ideal candidate. Likewise, a global recommendation uses the global ideal candidate, which represents a community at large, and a local recommendation uses the local ideal candidate (personal to each individual). The ideal candidate refers to the global ideal candidate unless explicitly stated otherwise.

The algorithm developed for the multi-criteria recommendation process is based on a distance metric that calculates the distance between an entity and the ideal candidate. The distances are then weighted to take into account importance levels. Two types of recommendations can be performed. One recommends similar items by finding similar entities to a pivot entity. The second recommends items that match each
user’s belief system. The second method is the same as the ranking algorithm, except instead of using the ideal candidate, it uses the specific user’s belief system.

4. The algorithm

The algorithm developed for the multi-criteria recommender system uses a weighted sum approach which is defined in multi-objective optimization literature [13]. The goal of a recommendation system is to construct the Users × Items 2-dimensional matrix by predicting the missing values in the matrix. The approach we take involves the reconstruction of a 3-dimensional matrix, with the third dimension being the set of criteria defined over the items, i.e. the value dimensions. Additionally, we use a weighted approach allowing more important value dimensions to count more in the final calculations.

For the rest of this section we use the following notation. The set of users $U$ has $n = |U|$ elements. The set of entities denoted by $E$ has $m = |E|$ elements. The set of value dimensions is denoted by $V$; let $l = |V|$ be the number of dimensions for each entity. Finally, let $W$ be the set of weights such that $|W| = |V|$.

Our goal is to predict values in the $n \times m \times l$ matrix. There are three concepts used by the proposed algorithm: 1) value dimensions (i.e. criteria), 2) the belief system, and 3) the ideal candidate. Value dimensions determine the ratings of each entity in $E$. Each entity is defined by $l$ value dimensions, which are collaboratively rated and then normalized to the range $[0, 1]$ before being used as the input to the algorithm. Therefore, each entity is a vector $e = (v_1, v_2, \cdots, v_l)$ for all $v \in V$. A belief system allows each user to define their beliefs using two components: 1) the values for each criteria and 2) the weights attached to each criteria. The weights are normalized to $[0, 1]$ with 0 representing no importance and 1 indicating utmost importance. Formally, each user $u$ has a belief system $B_u$, which is the ordered pair $B_u = (v_u, w_u)$ where $v_u = (v_1, v_2, \cdots, v_l)$, $w_u = (w_1, w_2, \cdots, w_l)$ are vectors and $v_i, w_i$ represent the user’s preferred value for value dimension $i$ and weight $i$, respectively. The ideal candidate is used as the pivot point for all distance calculations. Instead of calculating the distance of entities from the origin or from other entities, the algorithm makes use of a hypothetical ideal entity that is an aggregation of each users’ belief system. Formally, the ideal candidate is an ordered pair $I = (v_I, w_I)$, where $v_I = (v_1, v_2, \cdots, v_l), w_I = (w_1, w_2, \cdots, w_l)$ are calculated as follows:

$$I = \frac{1}{N} \sum_{i=1}^{N} B_i = \left( \frac{1}{N} \sum_{i=1}^{N} v_i, \frac{1}{N} \sum_{i=1}^{N} w_i \right).$$

Equation (1) is referred to as the global ideal candidate $I_G$, which takes into account every user’s belief system. The local ideal candidate $I_L$ is specific to each user and is simply equal to $B_u$. The ideal candidate is an entity as well, hence any algorithm that can calculate the distance or similarity between two entities can operate similarly with the ideal candidate.

Let $\mathbb{M}$ be a nonempty set of nonnegative real numbers with the greatest element $a = 1$. Then $d : \mathbb{M} \times \mathbb{M} \rightarrow \mathbb{R}$ is a metric on $\mathbb{M}$ and the ordered pair $(\mathbb{M}, d)$ is a metric space [7, 8]:

$$d(x, y) = \begin{cases} \frac{1}{2}(1 + |x - y| - |1 - x - y|), & \text{if } x \neq y, \\ 0, & \text{if } x = y, \end{cases}$$

(2)
The multi-criteria metric operates on an \( l \)-dimensional metric space \( \mathbb{M}^l \). From [7, 8], it is shown that since \( d \) is a metric on \( \mathbb{M} \), then:

\[
d_l(x, y) = \frac{1}{l} \sum_{i=1}^{l} d(x_i, y_i), x, y \in \mathbb{M}^l
\]

is a metric on \( \mathbb{M}^l \).

A weighted multi-criteria distance metric \( d_w : \mathbb{M}^l \times \mathbb{M}^l \times \mathbb{M}^l \to \mathbb{R} \) is defined as:

\[
d_l(x, y, w) = \frac{1}{l} \sum_{i=1}^{l} (1 - w_i) d(x_i, y_i), x, y, w \in \mathbb{M}^l
\]

The weights have to be normalised to \([0,1]\). The higher the weight \( w_i \) is, the more important it is, and the less it increases the distance results. Therefore, as a weight approaches 0, its importance is reduced and the distance is increased. From equation (4) we can define a multi-criteria rating function in terms of \( e \in E, u \in U \) and \( I \). We define two rating functions, the global rating function \( r_G : E \times I \to \mathbb{R} \):

\[
r_G(e, I_G) = d_w(e, v_I, w_I),
\]

and the local rating function \( r_L : E \times I \to \mathbb{R} \):

\[
r_L(e, I_L) = d_w(e, v_u, w_u).
\]

The difference between \( r_G \) and \( r_L \) is that the global rating function calculates the distance between an entity and the global ideal candidate \( I_G \) while the local one calculates the distance between the local ideal candidate \( I_L \).

There are two ways in which recommendations can be made using the algorithms. The first is a ranking of items obtained by using function (6). The second method uses a similarity metric to recommend entities that are similar to other entities. The similarity method can also be divided into two functions, one which uses the global ideal candidate \( I_G \) to obtain global recommendations, and one which uses a local ideal candidate \( I_L \) to obtain personalized recommendations.

From [9], any normalized distance metric \( d \) can be converted into a similarity metric \( s \) defined as follows: \( s = 1 - d : 0 \leq d \leq 1 \). Equations (2), (3) and (4) are normalized distance metrics that return a value in \([0,1]\). If \( d \) is a normalized distance between two entities \( e_1 \) and \( e_2 \), then we can define a global similarity metric \( s_G \) as:

\[
s_G(e_1, e_2, I_G) = 1 - |d_w(e_1, v_I, w_I) - d_w(e_2, v_I, w_I)|,
\]

and the local similarity metric as:

\[
s_L(e_1, e_2, I_L) = 1 - |d_w(e_1, v_u, w_u) - d_w(e_2, v_u, w_u)|.
\]

Since \( s_G \) and \( s_L \) are both normalized similarity metrics, because \( d \) is normalized, hence they both satisfy the following “coherence” properties [9] for all \( x, y, z, I \): 

\[
s_{G/L}(x, y, I) \geq 0, s_{G/L}(x, x, I) \geq 1, s_{G/L}(x, y, I) = s_{G/L}(y, x, I), s_{G/L}(x, y, I) + s_{G/L}(x, z, I) \leq s_{G/L}(x, z, I) + s_{G/L}(y, z, I),
\]

\[
s_{G/L}(x, x, I) = s_{G/L}(y, y, I) = s_{G/L}(x, y, I) \iff x = y.
\]

The following functions showcase part of the pseudo code used for the developed prototype system. The function \textit{distance} represents equation (4), the function \textit{rating} represents equations (6), (5), and the function \textit{similarity} represents equations (7), (8).
Function distance( rating, ideal, weight )

min = 1
max = 5
x = (min - rating) / (min - max)
y = (min - ideal) / (min - max)
if x == y, return 0
return (1 - weight) * (0.5 * (1 + |x - y| - |1 - x - y|))
End

Function rating( entity )
num_dims_used = 0
total_distance = 0
For each dimension dim in entity.category
    rating = average_rating(dim)
    total_distance += distance(rating, dim.ideal, dim.weight)
    num_dims_used += 1
End For
// Return a rating in between 0 and 1 (1 means perfect)
return 1 - (num_dims_used / total_distance)
End

Function similarity( entity1, entity2 )
return 1 - |rating(entity1) - rating(entity2)|
End

5. Implementation

The algorithms have been implemented in a proof of concept system developed for a local restaurant in Auckland, New Zealand. Users are required to sign up to obtain personalized recommendations. Signed in users can add or edit a value dimension, set their beliefs, rate a menu item or view details about a menu item.

Adding a value dimension and editing a value dimension use the same input screen. End users can only add soft value dimensions. Only administrators can add hard value dimensions, such as Price, via another input screen. Once a hard value dimension is added it becomes part of an entity’s profile. For example, having added Price, each priced entity has attached to it a new property, called price, the value of which can be changed on the entity’s edit page.

Once value dimensions are provided, users can set their beliefs. Two input columns are presented to the user, one for the ideal value for a belief and the other for the weight of the value dimension. There are two scale types: one is an ordinal scale with a low and high range and the other a nominal scale. These ideal values are used to calculate the global ideal value and importance of a dimension.

The user can also obtain valuable statistics at this point. For example clicking on the value dimension ‘healthiness’, you will be informed that 44 people in the system care about this dimension and that most people think that the healthiness of mains, desserts and starters should be, ideally, almost as healthy as one can get and that while healthiness matters, it is not vital nor is it very important in determining the
rating of food. We can also learn how much healthiness matters for various groups of customers (divided, for example, by countries, gender, age, etc.).

The prototype allows users to rate menu items over all soft value dimensions entered into the system; hard value dimensions are facts and can only be changed by the administrators of the system. Users are free to ignore certain value dimensions. The more a value dimension is ignored, the more obvious it becomes that this value dimension is not worth having in the system. A straightforward extension would be for the system to provide a confidence level to each value dimension’s importance level.

The system performs recommendations based on entity-to-entity similarities. It can either compare entities directly, or use an ideal candidate as the pivot point for comparison to make the recommendations more personal. The system that was implemented uses the user’s belief system as the pivot point for the recommendation process (i.e. local recommendations). That is, the system tells the user which menu items are similar to the Lamb Shanks dish by finding the qualities of the Lamb Shank that are closest to the user’s ideal values and then finding other entities that match those qualities. In one specific case, the value for salt content for Lamb Shank is low, which for the top recommended item, Satay Chicken, is high. However, the user had specified that salt content is not an important value, so even though Satay Chicken is not similar to Lamb Shanks when it comes to salt content, that aspect of dissimilarity does not matter to the specific user. On the other hand, Lamb Shank and Satay Chicken are both rated high along the tastiness value dimension, and the user has set the importance of taste as vital, so in this respect, Lamb Shank and Satay Chicken are similar.

The system will list all the details about the entity in question. This includes how many have rated it, what the global rating is, what the predicted rating for a user is and any hard values. It will then show the rating screen in the case that a user wants to rate it, and this will be followed by a number of item-to-item recommendations based on the similarity calculations between this entity and every other entity, with the user’s belief system used as the pivot point.

6. Evaluation

A user survey was carried out on a random sample of 20 patrons of the restaurant that the prototype was developed for. Each user was asked to use the system and rate items that they had tried. They were then given a list of 5 recommendations and asked to rank the algorithms in order from most to least accurate. We obtained 85 unfiltered orderings of the algorithms. After filtering them out for incomplete orderings we had 78 orderings. In addition to ranking the algorithms that were employed, the participants of the survey were also asked a number of questions regarding the usability of the system. The following five algorithms were rated:

1. Algorithm A: Rating based on global ideal candidate and average ratings based on (5).
2. Algorithm B: The predicted rating based on user’s belief system and average ratings based on (6).
3. Algorithm C: Weighted sum from [2].
4. Algorithm D: Adjusted weighted sum from [2].
5. Algorithm E: Rating based on global ideal candidate and user’s personal ratings.
The results of the algorithm rankings are presented in Table 1. The first 5 columns of the table represent how many times that algorithm was ranked at that position. Algorithm B—proposed in this paper—was ranked first 29 times, second 14 times, etc. It can be seen from the results that it does better than all the others. Algorithm A is also used by the system, not for personal results, but for global rankings of items. The only difference between algorithms A and B is that B uses the user’s personal belief system to calculate the ratings and A uses the global ideal candidate. As it turns out, the personalised algorithm, based on a digitised belief system is the most accurate of them all. 

A number of other statistics were also calculated. The \( \%^{1st} \) column represents the percentage that the algorithm came first. Algorithm B dominates this by being the most accurate 33 percent of the time out of all 5 algorithms and algorithm E comes in second with 27 percent. The 2 algorithms together, both of which add in a personal aspect to the rating process, are the most accurate 60 percent of the time.

The next 4 columns are an additive ranking that was calculated for each algorithm by taking into account the number of times the algorithms came in each position rather than in just first spot. Each position was given a weighting and then a total rating was calculated. The algorithm with the highest rating wins, and again it shows that B came in on top with A and E a close second. One may note here that even though algorithm E came in first place almost 2 times more frequently than algorithm A, their linear rating is exactly the same. This happens because algorithm A came in third place almost 3 times more frequently than algorithm E, which linearly made a significant difference.

The linear rating for algorithm L was calculated as $\text{rating}_L = \sum_{i=1}^{M} (M - i) \cdot L_i$ and the exponential rating was calculated as $\text{rating}_{L'} = \sum_{i=1}^{M} 2^{M-i} \cdot L_i$, where $L_i$ represents the number of times algorithm L came in the $i^{th}$ place. The linear rating gives a consistent weight to each of the positions, which assumes that the most accurate algorithm is worth as much more than the second most accurate algorithm is worth over the third most. The exponential model is, however, more accurate to the laws of nature and gives more value to the difference in weight between the first and second place than the difference in weight between the second and third place.

### Table 1. The rankings of each of the algorithms and their additive scores.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$1^{st}$</th>
<th>$2^{nd}$</th>
<th>$3^{rd}$</th>
<th>$4^{th}$</th>
<th>$5^{th}$</th>
<th>Linear rating</th>
<th>Exp rating</th>
<th>% exp</th>
<th>% lin</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>13</td>
<td>21</td>
<td>32</td>
<td>8</td>
<td>4</td>
<td>16.7</td>
<td>265</td>
<td>1048</td>
<td>21.4</td>
</tr>
<tr>
<td>B</td>
<td>26</td>
<td>14</td>
<td>12</td>
<td>24</td>
<td>3</td>
<td>33.3</td>
<td>273</td>
<td>1250</td>
<td>25.6</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>21</td>
<td>18</td>
<td>21</td>
<td>7</td>
<td>15.4</td>
<td>247</td>
<td>962</td>
<td>19.7</td>
</tr>
<tr>
<td>D</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>58</td>
<td>9</td>
<td>133</td>
<td>460</td>
<td>9.4</td>
</tr>
<tr>
<td>E</td>
<td>21</td>
<td>19</td>
<td>13</td>
<td>19</td>
<td>7</td>
<td>27</td>
<td>265</td>
<td>1170</td>
<td>24</td>
</tr>
</tbody>
</table>

7. Conclusions

We have presented a multi-criteria ranking algorithm that can be used on a non-rigid set of criteria. The system implementing the algorithm has been implemented as a proof of concept for the envisioned final system. The current implementation provides a ranking based on any number of value dimensions, with weights and ideal values definable by the users. There are many directions the work on this framework can take. There are a number of areas which need to be researched in regards to the current implementation of the system.
First, the ontology system, which provides the ability for users to arrange the entities under categories and link categories if need be, can be made much more robust with the addition of being able to categorise categories. It is still not clear if a universal ontology would be best for the framework, or if other methods should be used to categorise entities into classes of value dimensions. One such method would be collaborative tagging as described in [26]. That is, instead of arranging the structure in a directory-like manner, the users can tag a category instead, and the entities within that group will implicitly have the same tags, creating a folksonomy. The value dimensions could perhaps be attached to the tags instead of the category as well. This may also better enable the ability to share value dimensions across categories, and enable impressive recommendations.

Secondly, governance mechanisms have not been implemented in the proof of concept in a way that is scalable. Governance mechanisms are essential in a collaborative system to determine how the system governs itself at the micro and macro level. Concepts such as policy citations on ‘talk-pages’ enhance governance at the micro-level on websites such as Wikipedia [5]. The macro-level governance mechanisms (namely the aggregation of the belief systems and rankings) are in place, but there is a lot of work that needs to be done at the micro-level.

Thirdly, work is needed on trust and reputation support. Incorporating trust into users’ profiles would be one direction to go in. Perhaps the framework should take a user’s reputation into account when applying the value dimension weightings in the algorithms—perhaps just for the global rankings. These extra calculations could, however, improve performance.

Enabling users to have to do less within the system, i.e. by automating certain processes, is also desirable. One example may be the weighting system for value dimensions. The trust system [30] developed for Wikipedia may perhaps be used to create implicit weightings over the value dimensions by determining collaborative importance through analysing a value dimension’s revision history.

Knowledge extraction mechanisms are needed for the framework to be useful. This is the component that will provide the most value to users in the long run. The ranking algorithm is a central part of the entire knowledge extraction system. This part can answer questions such as: what is a positive contribution to the world? What is a negative contribution to this world, and why? What is the most important value-dimension for universities, for countries, or for various businesses?

Future work may enable iterative aggregation; in other words, how to include nominal value dimensions.

The evaluation section showed that users found the accuracy of the proposed algorithm higher than the one of other algorithms that were implemented. It would be interesting to compare the present system with other classes of algorithms, for instance, with algorithms based on statistical learning theory.

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References


