The Mathematical Theory of Information

by Jan Kåbre

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REVIEWED BY CRISTIAN S. CALUDE

Undoubtedly, the title of the book was well chosen: it is provocative, promising, and full of information. Syntactically, the title can be viewed as a variation on the titles of both the seminal paper [11] ("a" is replaced by "the") and the book [12] ("Communication" is replaced by "Information"). It provocatively questions Shannon’s theory; according to [11] (page 215), “no prophet remains unchallenged for ever”. And it promises “a new mathematical theory of information, built on a single powerful postulate: The Law of Diminishing Information.”

The book was praised—"a bold new approach to classical information theory"—by von Baeyer [1], who dedicated a special chapter of his book to it; details about Kåbre and the fascinating autonomous islands Aland on which he lives are presented too.

Do we need a new information theory? Unsurprisingly, there is no single theory of information, but several theories: semantic theories [2], algorithmic information theory [5,4], logic of information [7], information algebra [9], philosophy of information [8], information flow [3], quantum information theory [10], evolutionary information [13], etc. The information measure used in the book is defined by inf(B@A) = the information B gives about A (author’s notation). Here inf(B@A) is a real function satisfying the Law of Diminishing Information (or, simply, the Law, as it is referred in the book): Compared to direct reception, an intermediary can only decrease the amount of information. If A→B→C denotes a transmission chain, then the Law reads: inf(C@A) ≤ inf(B@A). The “theory of information” developed in the book is based on probability (as defined below) plus the Law (page 14):

[The Law] will be used as the fundamental axiom of the mathematical theory of information. The Law is the pruning knife of information theory: we will argue that the Law is the necessary and sufficient con-

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1It takes no fewer than 235 pages to realise that probabilities, defined in this way, apply only to finite sets (see section 8.4).
tation for a mathematical function to be accepted as an information measure, i.e. qualify as inf(B@A).

The Law is easy to understand and informally seems correct (for example, to anyone who has played the Telephone game in which one person chooses a sentence, whispers it into the ear of her left neighbour, who in turn whispers it into the ear of her left neighbour, and so on down the line). It ties in well with other principles such as the second law of thermodynamics, the data processing inequality [6], and the invariance of algorithmic complexity under computable transformations [4]. Moreover, it is not difficult to see that there are infinitely many functions satisfying the Law (trivially, each constant function satisfies the inequality). Finally, a weaker form of the Law has already been discussed in [6] (page 32) as a consequence of the data processing inequality.

According to the book, the Law is ubiquitous. It makes physics possible: "systems are forgetting their past as they reach equilibrium, or rather, the initial conditions can be eliminated from their description. Otherwise, physics would be complicated beyond comprehension." It also explains evolution. It applies to information technology, game theory, legislation, logic of research, algorithmic information, chaos theory, control engineering, medical tests. It can even be used as a legitimacy test: any acceptable information measure must satisfy the Law.

Is the Law true and should it be adopted? First, there are exceptions. The author himself discusses one: the Chinese paper. Assume dmt the channel B is independent of the article A ---> B---> C consists of A = an English paper, B = an interpreter translates the article into Chinese, C = the English translation of the article. Clearly, inf(C@A) > inf(B@A), hence the Law fails. Second, similar but less subjective violations of the Law can be easily constructed using algorithmic complexity. This signals a problem: which restrictions should be imposed?

The book, which covers more than 500 pages, discusses a wealth of topics grouped in 14 chapters, from specific information measures, statistical information and algorithmic information to control and communication, information physics and quantum information and applications. Some topics are better presented than others. The chapter on algorithmic information, which is close to my expertise, is far from satisfactory, as one can see by browsing the paragraphs 2, 3, and 4 on page 238. The main aim, a grand unification theory of information, is certainly not achieved. Despite this, the book, written by an original thinker, contains a number of interesting ideas which may inspire mathematically oriented readers to continue the project.

REFERENCES

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Negative Math: How Mathematical Rules Can Be Positively Bent
by Alberto A. Martinez

Extricating the book from its packaging, I was greeted by a picture of a large spoon on the dust wrapper. What could this mean? Some sort of reference to spoon-bending in the last word of the subtitle? Is the reader going to be spoon fed? Surely the book isn't written in spoonerisms. Immediately after the title page, came the following:

You can use a spoon to drive a screw into a wall. With practice, you can become skillful at it. You can also learn many juggling tricks with the spoon, and thus impress and bewilder people who don't juggle spoons. And you can make all of this more puzzling by calling the spoon a 'fork'. And you can write books about it and form societies with other people who also juggle spoons. And even then, sure, you can use a spoon to drive screws into a wall. But a screwdriver is better. And even if you've never seen a screwdriver, you can just as well invent one. It might resemble the spoon in some ways though not in others. So you can keep your spoon as well; for eating soup, for juggling, or even, occasionally, for driving screws into walls. At least until you have more skill with a better tool.