Chapter 26

The Universe is lawless or "Pantôn chrêmatôn metron anthrôpon einai"*

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The belief that the physical Universe is a *knowable* system governed by rules which determine its future *uniquely* and *completely* has dominated the Western civilisation in the last two and a half millennia. The goal of this paper is to provide new arguments in favour of the hypothesis that the Universe is lawless, a hypothesis proposed and discussed in our papers.[7,9,11,14,15,18]

1. Introduction

The endeavour to discover and determine the laws presumed to govern the physical Universe is as old as Western civilisation itself, as are the difficulties herewith associated. Witness the anecdote transmitted by Plato (in his dialogue *Theaetetus*) concerning Thales of Miletus, the first mathematician to accurately predict a solar eclipse (for the 28th May 585 BC):

> While Thales was studying the stars and looking upwards, he fell into a pit, and a neat, witty Thracian servant girl jeered at him, because he was so eager to know the things in the sky that he could not see what was there before him at his very feet.

Nowadays we continue "to look upwards", albeit with the help of the latest technology and its fabulous instruments. This process is unavoidably marked by the human "measure" which biases the laws we presume to hold in the entire Universe.

In what follows we provide new arguments in favour of the hypothesis that the Universe is lawless, a hypothesis proposed and discussed in our pa-

*"Man is the measure of all things", Protagoras, 5th century BC.

pers.[7,9,11,14,15,18] We start by describing the notions of (physical) Universe and law of the Universe (sometimes called natural law or the law of nature), then we discuss the lawfulness hypothesis and lawlessness hypothesis. We continue by arguing in favour of the lawlessness hypothesis in various types of Universes. Finally we discuss the provability of the lawlessness hypothesis.

2. The Universe

The dictionary definition of the term, "all that exists", is a tautology. For the scientific endeavour ("looking upwards") the term covers two quite separate domains of the reality accessible to humans: I) the Solar System and II) the electromagnetic radiation signals from beyond the Solar System captured by the antennas of our instruments.

The Sun and the plethora of planets, moons, comets, asteroids, and other directly detectable objects have been intensely scrutinised by humans—from the very dawn of their history—by means of their innate radiation-detection antenna: the retina. Remarkably, beyond the naked eye, no further light-amplifying instrument was available to Thales, or to Ptolemy, until Galileo's invention of the telescope radically changed this way of "looking upwards". The telescope and Copernicus presented humans with a different Universe, a Universe gigantic but still reasonably comprehensible by minuscule humans, although definitely removing them from the central position. In the following centuries, great physicists discovered the first "laws" of nature, in domains so different as optics, electricity, movement, gravity, chemistry, etc., giving rise to the idea that there might exist rules of "universal" validity. But, as with Galileo's telescope, advances in technology again changed the entire outlook. Now a "large" bracket of the electromagnetic spectrum, not just the narrow visible-light window, is available for scrutiny.

Contemporaneous with these technological advances, theoretical physicists developed the two fundamental explanatory models of physical "reality", the standard model of quantum mechanics and general relativity of gravitation. These advances again fundamentally changed what was meant by the term "Universe".

In the first place, the human "measure" vanished: the reality encompassed by this term is enormous, both in time as in space. Just one tiny example: the distance from the Solar System to the nearest star, Alpha Centaurii, is approximately $40,000,000,000,000,000$ m. Further, if the Solar

System inspected with the modern instruments revealed itself as being of an unsuspected complexity, the electromagnetic signals now detected in all frequencies of the spectrum showed—not just an extraordinary complexity—but also what could only be interpreted in the light of presently admitted theories as incredibly gigantic phenomena, for which even new names had to be coined: super-massive black holes, pulsars, quasars, neutron stars, and a variegated catalogue of supernovae, to name only a few.

Nevertheless, the search for a universal explanatory theory—"the laws"—went on. It had to incorporate the two fundamental theories: quantum mechanics and gravitation. But the first is a probabilistic theory, the second a deterministic theory, and their marriage has so far resisted all efforts. That is to say, these efforts now take outlandish forms: in them the Universe has more dimensions than the traditional four, eleven, for instance. Worse: there is not just one Universe—"ours"—but many of them, although completely detached and unreachable for us.

If these conundrums were not enough, further observations have created even more problems. Several decades ago it was discovered that the movement of the stars in a galaxy, including our Milky Way, do not comply with the speed values assigned to them by Newton's or Einstein's gravitation laws. Neither do groups of galaxies. The remedy: "black matter", an undetectable ("black") gravitating component of the Universe. Then recently it was found that the Universe expands faster than what was allowed by the latest theories. The remedy: "black energy", a concept originally put forward by Einstein albeit in a different context. What are these mysterious matter-energy forms? Until today nobody knows and of course nothing of that kind has so far been detected. However, based on more and more exact measurements of the cosmic microwave background, initially predicted to exist as a fossil remnant of the Big Bang[†] itself, the following composition of the Universe is presently put forward by cosmologists: dark matter 23.3%, dark energy 72.1%, ordinary matter, of which stars, planets and people are made: 4.6%.

In a quite abbreviated form, this is what the term Universe stands for nowadays. Clearly, not a well-defined concept but a patchwork of observations not yet understood, theories and prejudices.

[†]To arrive at a cosmology of the Big Bang type, many additional postulates are required, see, for example,.[5]

3. The laws

What are the laws of the Universe? Are they just metaphors (cf. Zilsel[36]) or "like veins of gold, [...] that scientists are extracting the ore, " (cf. Johnson[24])?

According to Feynman[22] and Davies,[21] the physical laws or the laws of the Universe—shortly, the laws, are expressed in simple mathematical terms; further on, the laws are universal (they apply everywhere in the Universe), infinite, absolute, stable, omnipotent (everything in the Universe must comply with them).

When the adjective "lawful" is predicated from the term Universe, what is thereby meant? Here all the usual human prejudices impinge. To determine the laws that rule the changes and development of said object is equated with acquiring *knowledge* about this entity. In other words, one jumps from the *how* to the *why*. But these laws, assuming we will ever find them, are not *causal* laws at all. It is true that Aristotle has defined knowledge of something as *knowledge* of the cause (or causes) *why* that thing is as it is. Probably because the assumption of causality is an innate—fitness enhancing—trait of humans, causality is in most cases immediately associated with knowledge. But causality can only be observed by humans in the form of short causal chains, short as measured from a particular here and now (*hic et nunc*) and basically only in the past-time direction.

We give a trivial example. A man passes under a balcony from which a flowerpot falls killing him. The cause of his death? The flowerpot, of course. But also the red traffic light: had it been green he would have passed earlier under the balcony ...etc. It is clear that from every *hic et nunc* sprout exponentially many interconnected "causal chains", and the whole idea becomes meaningless at a short past distance from any *nunc*. In the opposite direction, towards the future, causality changes into prediction, always a probabilistic affair in the best case. Finally, let us repeat again that the dimension of the Universe makes any reasonable reference to the human measure, as required by Protagoras, if not directly absurd, at least untenable.

Consequently, it seems that predicating "lawful", in any common-use sense of that term, with the noun Universe, cannot be reasonably made. In fact, it is precisely *not* in the common-use sense that the term is applied in most cases. Loaded with centuries of religious belief, the search is not for *laws* but for a *design*, or at least a *design principle*, of that Universe. This is again a very old idea. Divinities were always credited with a superior knowledge, including the possession of the ultimate account and justification of the world.

In Plato's *Timaeus* we have one of the most famous examples. The Universe in the *Timaeus* is fashioned by a divine craftsman, the demiurge, of whom it is repeatedly stated that he was "good" and that he designed the Cosmos with the view to make it as "good" as he possibly could (the platonic demiurge is not omnipotent). Note however, that nowhere in the *Timaeus* (or elsewhere) does Plato neatly define the "good". But the idea is clearly expressed: the difference between a chaotic, lawless Universe and a Universe that can be claimed to be "lawful", i.e. to be a Cosmos, is the existence of some overlying, unifying concept presumably of "divine" origin (such as "the Good", *t'agathon*, for Plato).

Plato's ideas directly influenced Brahe, Kepler, Galileo who put forward what became the official program of science: Find the lawful part of the Universe, and, if lucky, try to formulate the (mathematical) laws describing its "kinesis" (change). In this spirit some audacious present-day physicists and cosmologists are looking for a "theory of everything" or whatever name they may choose.

4. The lawfulness hypothesis

From millennia-old aspirations "to know more" comes the idea that the Universe is lawful. This hypothesis seems to be supported by our daily observations: the rhythm of day and night, the pattern of planetary motion, the regular ticking of clocks. The stage is set at the beginning and everything follows "mechanistically" without the intervention of God, without the occurrence of "miracles". The future is determined from the past by *universal, infinite and eternal laws*:

> [The] *entire history of the Universe is fixed*, according to some precise mathematical scheme, *for all time*, cf. Penrose ,[28] p. 558–559.

Most importantly, the laws are *knowable* by means of observations/measurements and reason/logic. It is up to us to discover them.

The great law, the law of cause and effect—*a thing cannot occur without a cause which produces it* in Laplace's words—transcends all known laws and is ever at work with chains of causations and effects governing all of manifested matter and life.

5. The lawlessness hypothesis

It is a simple matter of reflection to point out some limits of the lawfulness hypothesis: the vagaries of weather, the devastation of earthquakes or the fall of meteorites are "perceived" as fortuitous.

The great law of cause and effect is illusive and could not be proven, just observed. In fact, it is possible to disprove its universality as we shall see soon.

The lawlessness hypothesis—according to which there are no laws of the Universe—does not exclude the existence of *local rules* functioning on large, but finite scales. Local regularities are not only compatible with randomness, but in fact a consequence of randomness. Following[15] we will illustrate our arguments for a Universe crudely represented by an infinite binary sequence.

For example, every Martin-Löf random sequence[‡] contains every possible string (of any length) and every such string must appear infinitely many times.[6] The fact that the first billion digits of a Martin-Löf random sequence are perfectly lawful, for instance by being exactly the first digits of the binary expansion of π, does not modify in any way the global property of randomness of the (infinite) sequence.

These facts are consistent with our common experience. Space scientists can pinpoint and predict planetary locations and velocities "well enough" to plan missions months in advance, astronomers can predict solar or lunar eclipses centuries before their occurrences, etc. All these results—as impressive as they may be—are only true *locally* and within a certain *degree of precision.* They are not "laws of the Universe".

The hypothesis that the Universe is lawless is not a new idea. Twenty-four centuries ago, Plato in the *Timaeus* invented a cosmology (see more in[14]) which states that in the beginning the demiurge finds a completely chaotic substrate, "Chora", which has only one property: it is the material substrate of the Universe in a primordial state, a state which we would call today random. Faithful to the law of cause and effect, Plato proposes an acting principle of disorder, a cause of randomness, which he calls "Anagke" (necessity). The demiurge is trying to "persuade" Anagke to accept a mathematical order. If successful, one arrives at a finite set of purely mathematical elementary building blocks—Plato's perfect polyhedra—which, when combined by simple mathematical rules, constitute the ordered Universe,

[‡]A sequence is Martin-Löf random if there is a constant c such that all its finite prefixes are c-incompressible with respect to a self-delimiting universal Turing machine.

the "cosmos" (order). But only the part where the demiurge succeeded in persuading Anagke is ordered. In fact, the demiurge is not all-powerful, hence in Plato's Universe, order is only partial. And an irreducible disorder, chaos, randomness remains, so irreducible that nothing can be *said* about it. Plato does not indicate anywhere what part of the Universe is lawful, and what part is entirely random.

We note that the demiurge is not the God of Genesis, as later interpreters hoped to prove. In fact, the demiurge does not "create" anything at all, it is only the sufficient cause of order, where such order exists. Instead of saying, "there is a law which underpins the order detected in this context", Plato says, "the demiurge caused ...", and then he adds the mathematical expression describing in rigorous terms, this partial order.

Twenty four centuries later, Poincaré also suspected the chaotic, random nature of the Universe when he wrote:[§]

> If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that universe at a succeeding moment. But even if it were the case that the natural law no longer had any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, that [it] is governed by the laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.

In our time Barrow[2] has proven that Einstein's equations exhibit a formal chaotic behaviour, which means that the evolution of the Universe becomes unpredictable after a time short in cosmological scales. Hawking's views (see[23] p. 26) are even stronger:

> The intrinsic entropy means that gravity introduces an extra level of unpredictability over and above the uncertainty usually associated with quantum theory.[¶] So Einstein was wrong when he said, "God does not play dice." Consideration of black holes suggests, not only God does play dice, but that he sometimes confuses us by throwing them where they can't be seen.

[§]Quoted from Peterson,[29] p. 216.

[¶]A massive star, which has exhausted its supplies of nuclear energy, collapses gravitationally and disappears leaving behind only an intense gravitational field to mark its presence. The star remains in a state of continuous free fall, collapsing endlessly inward into the gravitational pit without reaching the bottom.

A detailed account of unknowables in physics is given by Svozil.[34]

Are there better ways to describe the Universe than the mathematical one? In retrospect, mathematical formalisms seem to be inevitable, In any case, there is nothing to indicate better candidates. The growing preference to move from analytical descriptions of physical laws to algorithmic ones (see for example Wolfram[35] or the discussions in[9,11]) is not a paradigm shift as programs are fundamentally mathematical entities.

6. Arguments in favour of the lawlessness hypothesis

We concentrate on continuous models for the Universe. First we will argue that even if the Universe is lawful then we won't be able to know this; secondly, we shall discuss reasons why the Universe cannot be lawful.

As the tools to understand the laws are mathematical and much of the elementary intuition about numbers derives from our linguistic abilities to assign names to objects[‖] it is not surprising that our arguments will focus on numbers. This point of view is consistent with Landauer's[26]

> The laws of physics are essentially algorithms for calculation. These algorithms are significant only to the extent that they are executable in our real physical world. Our usual laws depend on the mathematician's real number system.

To what extent is the system of real numbers contaminated by "chaoticity" and "randomness"? A real number in base b is *disjunctive* (cf. Jürgensen and Thierrin[25]) in case its b-expansion sequence contains all possible strings over that alphabet $\{0, 1, \ldots, b-1\}$. A *lexicon* is a real number which is disjunctive in any base. A lexicon contains all writings, which have been or will be ever written, in any possible language. A lexicon expresses a strong qualitative idea of randomness.

According to the law of large numbers, in every binary expansion of almost every real number in the unit interval every string appears with its "natural" probability. For example 1 appears with probability 1/2, 0 appears with probability 1/2, 00 appears with probability 1/4, and so on. This happens for almost all, but not exactly all of them: the law of large numbers is false in the sense of Baire category with respect to the natural topology of the unit interval,[27] but it is still true for a small modification of this topology.[13] Lexicons form residuals[19] for the natural topology,

[‖]According to Barrow (,[3] p. 4), "linguistic abilities are far more impressive than our mathematical abilities, both in their complexity and their universality among humans of all races."

hence *most reals do not obey any probability laws*. This shows that the system of real numbers, our very basic language of expressing laws, is fully contaminated by randomness.

Martin-Löf randomness, a stronger quantitative form of randomness, while "less pervasive" than disjunctivity, is still omnipresent among real numbers: with probability one every real number is Martin-Löf random.[6] Even more, Martin-Löf random reals are in a sense the "bricks" of the whole set of reals: by Gács theorem improved by Hertling (see,[6] p. 155–165) every real is effectively reducible to a Martin-Löf random one.

The law of cause and effect breaks down with the advent of algorithmic information theory: mathematics, even elementary number theory, is full of facts true for no formal reason as Chaitin has proved:[6,20,30]

God not only plays dice in physics but also in pure mathematics.

Randomness not only exists, it is everywhere.[10]

The lawlessness identified in the system of reals appears in quantum mechanics. This is no news, except that now one can go beyond the mere postulation of quantum randomness: one can prove some mathematical facts about the quality of quantum randomness. Consider a quantum random number generator generating bits produced by successive preparation and measurement of a state in which each outcome has probability one-half. By envisaging this device running ad infinitum, we can consider the infinite sequence $\mathbf{x}$ it produces. If we assume: a) a standard picture of quantum mechanics, i.e. a Copenhagen-like interpretation in which measurement irreversibly alters the quantum state, b) the "many-worlds" interpretation and other "exotic" possibilities including contextual hidden counterfactual observables are excluded, and c) the experimenter has freedom in the choice of measurement basis (the "free-will assumption"), then $\mathbf{x}$ is incomputable,[17] that is no Turing machine can reproduce exactly the bits of the sequence $\mathbf{x}$. For example, $\mathbf{x}$ can start with a billion of 0's, but cannot consists of only 0's. In fact, one can prove a stronger property: the sequence $\mathbf{x}$ is bi-immune, i.e. only finitely many bits of $\mathbf{x}$ are computable. Every bi-immune sequence is incomputable, but the converse is not true. Experimental confirmation of this theoretical result was obtained in.[12]

7. Digital universes are also lawless

As the "free-will assumption" used in the previous section excludes a digital Universe[**], it is natural to ask whether such a Universe is lawful or not?

Digital physics distinguishes three possible scenarios: a) the Universe[††] is (may be) continuous, but our model is digital, say a universal (prefix) Turing machine working with a discrete infinite time, b) the Universe is a universal (prefix) Turing machine working with a discrete infinite time, c) the Universe is a universal (prefix) Turing machine working for a finite, albeit huge, time only.

A law of a Universe in cases a) and b) can be expressed by an infinite sequence while for c) the law has to be expressed by a finite string. All results regarding qualitative and quantitative randomness described in the previous section apply for the scenarios a) and b). The status of a "law" in the scenario c) is not so clear. The lawlessness of such a Universe comes from the fact that strings coding programs expressing laws of such a Universe cannot be distinguished from algorithmic random strings.[6]

The influential NKS programme initiated by Wolfram's book[35]—the systematic, empirical investigation of computational systems for their own sake—is relevant for understanding the Universe in all three possible scenarios described above, irrespective of the particular philosophical views of researchers in NKS. Proposed digital versions of various parts of continuous physics have consistently revealed various forms of randomness; see for example the work in digital statistical mechanics in.[1,16,31–33]

8. Can the lawlessness hypothesis be proved?

In spite of many unknowables in physics,[34] the relevance of incompleteness of mathematics for physics is still unclear.[4] It is unlikely that a formal proof for the lawlessness hypothesis can be found. Of course, the hypothesis can be experimentally illustrated and tested (see[12] and the discussion in Zenil[37]).

In agreement with Hawking (,[23] p. 3–4):

> I take the positivist viewpoint that a physical theory is just a mathematical model and that it is meaningless to ask whether

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[**]In a truly deterministic theory—sometimes called super-determinism—the experimenter might have the illusion of exercising her independent free choice, but in reality she just obeys the rules of the theory.

[††]Note that the term Universe, as described in Section 2, is a model itself.

> it corresponds to reality. All that one can ask is that its predictions should be in agreement with observation.

one can say that our partial and provisional understanding of the Universe comes through measurements, so ultimately through numbers. With extremely rare exceptions, the real numbers representing the outcome of measurements are lexicons, so they are devoid of any order or law. Can such a system express any "laws" of the physical Universe?

Finally, does the lawlessness hypothesis mean the end of science? Should one definitely abandon the hope of finding sense and meaning in the Universe? The answers to both questions are negative. With the lawfulness hypothesis we leave in a dream of global, universal order and law, when, according to the lawlessness hypothesis, there is only Chora (chaos) with local laws only. These are just hypotheses and their merits should be pragmatically judged only. If one feels elated to discover the laws of the Universe, then the traditional assumption fits better; the alternative hypothesis is preferable for the more realistic and humble minds. Science is and will be alive, and progress in answering fundamental questions and developing applications will continue.

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