ON THE ADEQUACY OF A GRAMMATICAL MODEL OF THE BRAIN

BY

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1. INTRODUCTION

A grammatical model of the brain behaviour was introduced in [4]. The core of that model is *a universal type-zero grammar*, i.e. a type-zero grammar in Chomsky hierarchy which can simulate every grammatical competence of the brain.

The model in [4] is consistent with Löfgren's *learning hypothesis* and with the abstract learning scheme as presented in [11]. We recall that following Löfgren, "an object A can *learn* from a surrounding S to the extent that it can extract order (regularities) from S so as to produce a description of S relative to A (A shall be able to make inferences from the description). The more regularities that have been found, the more *genuine* is the learning, and the shorter the description (which utilizes the regularities) compared with a representative part of S itself. *The learning ability* (capacity) of A reflects both the kinds of surroundings that A can learn and the genuinessess of the learnings". In our model the regularities extracted are expressed by the axiom (as an entry for the universal grammar); the learning ability corresponds to the capacity of constructing axioms for the universal grammar.

Let us notice that the grammatical model can be equally used as a possible answer to *the mind-brain problem* as formulated by Dubrovski [6]: "the relation between a phenomenon with subjective reality (a sensation, a perception etc.) and the neuro-dynamic system of the brain (the material support). This relation is thought of as a relation between an information and its material support".

The above remarks stress the explanatory power of the grammatical model. In the sequel we shall analyse new properties of the model and we shall check its adequacy, its fitting with the modelled reality (the informational brain).

First, we shall analyse the logics of the following informational paths in the model: (1) environment — relay — grammars generator, (2) parser — universal grammar, (3) universal grammar — environment. We prove that the logic of the first path is Boolean, the logic of the second path is the epistemological logic of O. Onicescu [14] and the logic of the last path is the ontological logic (non-commutative) of O. Onicescu [14].

Then, we shall examine the possibility to rewrite some parts of the model at a higher level in Chomsky hierarchy, taking into account that the level of type-zero grammars is too general for many brain algorithmical competences. More specifically, we try to re-build the model at the level of type-1 grammars. As we shall see, this is possible only introducing some limitations of the model capabilities generality.

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2. NOTIONS AND NOTATION FROM FORMAL LANGUAGE THEORY

We shall specify only some basic notions necessary in presenting the grammatical model of the brain. For details, the reader is referred to [18].

For a vocabulary V we denote by V^* the free monoid generated by V under the operation of concatenation and the null element λ. A Chomsky grammar is denoted by $G = (N, T, S, P)$, where N is the non-terminal vocabulary, T is the terminal vocabulary, $S \in N$ is the grammar axiom and P is the set of rewriting rules. In what follows we allow S to be a string over $N \cup T$; clearly, this does not change the generative capacity of the four types of grammars in Chomsky hierarchy. The four families of languages in the Chomsky hierarchy are denoted by $\mathcal{L}_i$, $i = 0, 1, 2, 3$ (respectively, the recursively enumerable, context sensitive, context-free and regular languages).

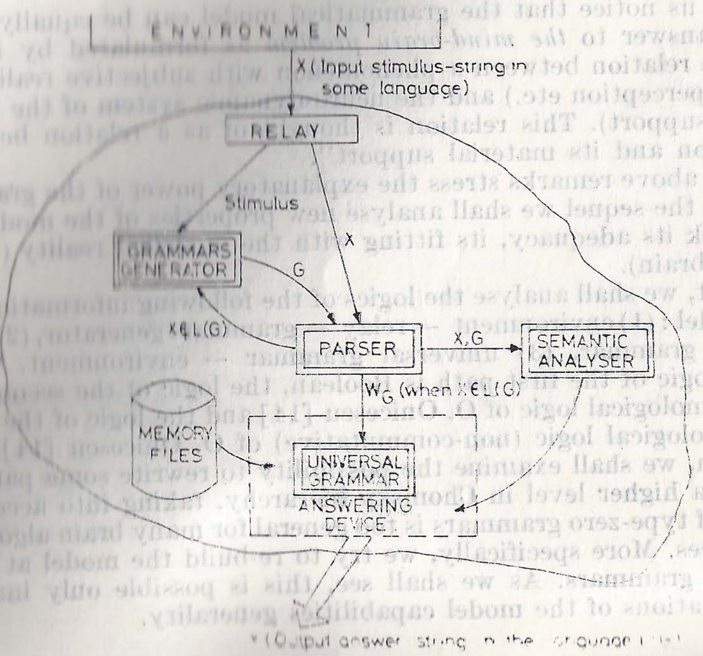
For a grammar $G = (N, T, S, P)$ we write

$$Var(G) = \text{card}\, N$$

and for a language L we define

$$Var(L) = \inf \{Var(G) \mid L = L(G)\}$$

A triple $U = (N, T, P)$, with given N, T, P, is said to be a *universal type-0 grammar* if and only if for every language $L \in \mathcal{L}_0$ there is a string $w_L \in (N \cup T)^*$ such that the grammar $U(w_L) = (N, T, w_L, P)$ generates the language L. (Although N, T, P are fixed, by varying the string-axiom we can obtain all type-0 languages over T.)



A universal type-0 grammar was constructed in [5] in the following way. Let $L \in \mathcal{L}_0$ be a language generated by the grammar $G = (N, T, S, P)$, $N = \{S, A_1, \ldots, A_n\}$. We replace each occurrence of A_i in the rules in P by AB^iA. In this way we obtain a grammar $G' = (\{S, A, B\}, T, S, P')$ also generating L (hence $Var(G') = 3$). Let us suppose that $P' = \{x_1 \to y_1, \ldots, x_k \to y_k\}$. The grammar G' can be precisely identified by a string

$$z_{G'} = x_1 \Rightarrow y_1/x_2 \Rightarrow y_2/ \ldots /x_k \Rightarrow y_k$$

The type-0 derivation procedure can be codified using a type-0 set of rewriting rules in such a way that starting from a string of the form

$$w_{G'} = u_1 S u_2 z_{G'} u_3$$

we can obtain all strings in $L(G')$. Let P_0 be this set of type-0 rules and N_0 be the symbols in P_0 which do not belong to T. Then (N_0, T, P_0) is a universal type-0 grammar and $w_L = w_G$ is a string for which (N_0, T, w_L, P_0) generates the language L.

3. THE GRAMMATICAL MODEL OF THE BRAIN

A universal grammar can be used in building a very "economical" model of the human brain as a question answering device. Indeed, let us remember the picture in figure 1 [4].

The model works in the following manner. An external stimulus (a question, for example) comes in the brain under the form of a string x in some language. Then the exciting relay activates the grammars generator which produces strings of the form

$$x_1 \to y_1/x_2 \to y_2/ \ldots / x_k \to y_k$$

corresponding to the rewriting rules of a grammar. In fact, we can assume that the grammars generator generates exactly strings of the form w_G as that needed by the universal grammar described in the above section. The string w_G and the input string come in the syntactic analyser which answers the question whether x belongs to the language identified by w_G or not. If the answer is affirmative, that is the right competence corresponding to x was found, then x and w_G come in the universal grammar which is the core of the answering device. This universal grammar behaves now as the grammar of the competence needed to "answer" the stimulus x. The answering device, using some information provided by a semantic analyser and some memory files, generates the answer and sends it to the environment.

The whole informational functioning of the brain can be accomplished thus using only these few components of the model, without actually retaining all the grammars corresponding to the many competences the brain has.

4. THE LOGIC OF THE MODEL

An interesting problem is to find the logic laws of the information transmission in the model. A careful examination of the problem shows its complexity and, consequently, the difficulty of a direct approach. In the sequel we shall analyse some important paths in the model and we shall establish their corresponding logics. We shall deal with the following informational paths: (1) the environment relay — the grammars generator, (2) the parser — the universal grammar, (3) the universal grammar — the environment.

In the first informational path, to every input stimulus—string (in some language) corresponds a generated grammar. *The logic of this path is the Boolean logic* (an analogous law corresponding to K. Lucas well-known principle works; see [1] and [3]).

Let us examine the second informational path. The strings acting in this path are of the form

$$u_1 \to v_1/u_2 \to v_2/ \ldots /u_n \to v_n,$$

where every u_i contains at least a nonterminal symbol. Every string of the above form codifies a particular grammar. Let us denote by L_A the set of all strings corresponding to the rewriting rules of particular grammars codified as above. In view of the fact that the working space is the set of all type-zero languages the membership problem is unsolvable; consequently, *in the logic of this path one cannot define a negation operation.*

In order to derive the logical laws of this path we shall note that every two strings in L_A which differ by a permutation of the components $u_i \to v_i$ produce the same informational effect on the universal grammar. In this way one can introduce an equivalence relation on L_A; let $\bar{L}_A$ be the set of all equivalence classes.

Let

$$w = u_1 \to v_1/u_2 \to v_2/ \ldots /u_n \to v_n,$$

be an element of L_A. We shall define a sub-string class of $\bar{w}$ to be the class of a string

$$w' = u_{i_1} \to v_{i_1} / u_{i_2} \to v_{i_2} / \ldots / u_{i_m} \to v_{i_m},$$

for $1 \leq i_j \leq n$. In a similar manner one can define a supra-string class.

In $\bar{L}_A$ we introduce two binary operations as follows: $\bar{u} \oplus \bar{v}$ = the class of the longest common sub-string of $\bar{u}$ and $\bar{v}$, $\bar{u} \odot \bar{v}$ = the class of the shortest common supra-string of u and v. These operations act on the set of axioms of grammars. They capture the "logical operations" of a process of learning the grammar of a given language, more precisely, the hesitations between various languages of sub/supra-strings. By means of the operations $\oplus$ and $\odot$ we define the relation $\Rightarrow$ by enumeration:

$$\bar{w} \Rightarrow \bar{w} \oplus \bar{u},\ \bar{u} \Rightarrow \bar{u} \oplus \bar{u},\ \bar{u} \odot \bar{w} \Rightarrow \bar{u},\ \bar{u} \odot \bar{w} \Rightarrow \bar{w},$$

for all $\bar{u}$ and $\bar{w}$ in $\bar{L}_A$. It is clear that the definition of the relation $\Rightarrow$ does not depend on the choice of the names for the classes of $\bar{L}_A$.

A routine proof shows that the following properties hold for every $\bar{u}$, $\bar{v}$ and $\bar{w}$ in $\bar{L}_A$:

1. $\bar{u} \oplus \bar{w} = \bar{w} \oplus \bar{u},\ \bar{u} \odot \bar{w} = \bar{w} \odot \bar{u},$
2. $(\bar{u} \oplus \bar{v}) \oplus \bar{w} = \bar{u} \oplus (\bar{v} \oplus \bar{w}),\ (\bar{u} \odot \bar{v}) \odot \bar{w} = \bar{u} \odot (\bar{v} \odot \bar{w}),$
3. $\bar{u} \oplus \bar{u} = \bar{u},\ \bar{u} \odot \bar{u} = \bar{u},$
4. $(\bar{u} \oplus \bar{v}) \odot \bar{v} = \bar{v},\ (\bar{u} \odot \bar{v}) \oplus \bar{v} = \bar{v}.$

We obtain: *The logic of the path the parser — the universal grammar is the epistemologicl logic of O. Onicescu* [14]. Let us notice that, in the grammatical model, the epistemological logic models the value of knowledge of a given language in a learning process. This logic does not describe an *a posteriori* phenomenon; it is a component of the process of learning / knowledge / creation.

Let us pass to the third path. At this stage one has chosen an axiom-string w_G and one activates, by means of the universal grammar, the particular grammar G which generates the language $L(G)$. In this path one works with strings. The logical operations are defined as follows: $x + y$ = the longest common sub-string of x and y, xy = the usual concatenation of the strings x and y. The implication relation $\to$ is defined as follows: $x \to y$ if there exists a string z such that $y = z + x$. The above logical operations model the information transmission from the generated language $L(G)$ to the environment. The operations defined above and the implication relation have the following six properties (for all x, y, z):

1. $x + x = x,$
2. $x + y = y + x,$
3. $(x + y) + z = x + (y + z),$
4. $(xy)z = x(yz),$
5. $(x + y)y \to y,$
6. If $x \to y$, then for all z, $x + z \to y + z$ and $xz \to yz$.

Let us write that $xx \neq x$ and $xy \neq yx$. Consequently, *the logic of the path the universal grammar — the environment is a non-commutative ontological logic of O. Onicescu* [14].

The main difference between the logics of string-axiom transmission and string-output transmission relies on the presence / absence of the commutativity and idempotence properties of the operations $\odot$ and concatenation. The first operation concerns the extension of the string-axiom; it acts on the grammar associated, and consequently it determines the modification of the language. The second operation concerns the linear organization of the outputs.

The absence of negation in our models can be avoided in many interesting concrete cases, most of them having a great frequence. An important space in which we can recapture the negation operation is the class of type-one languages. This fact motivates the investigation of the possibilities to realize the model at this level of generality.

5. BRINGING THE MODEL TO A HIGHER LEVEL IN CHOMSKY HIERARCHY

The model in [4] contains four grammatical components: the grammars generator, the parser, the semantic analyser and the universal grammar. The semantic analyser is a weakly investigated topic and in what follows we shall ignore it.

As it was pointed out in [4], the grammars generator can be realized as a regular grammar (the only condition about type-0 grammars is that a nonterminal to appear in each left-hand side of the rules), but the parser and the universal grammar are essentially of type-0 in Chomsky hierarchy.

However, there are many arguments against this maximal generality of the model. Here are some of them:

— there is no known grammatical feature in natural languages wich surpasses the context sensitive level,

— all the usual artificial languages are context sensitive: the programming languages, the languages of the propositional calculus (including the language of theorems [15]), the individual algorithmic competences [19], the economic algorithmic processes [16], the languages describing the various lectures of dramatic and folkloric works [12], etc.

— the type-0 grammars request an unbounded work-space; however, the brain seems to work more economically. For handling some given information it uses a bounded number of neurons. (Indeed, only a small fraction of the 10—15 billions of neurons of the brain is activated at a given moment.)

All these remarks raise the significant question whether the grammatical model of the brain can be realized at the level of type-1 grammars or not (this level seems to be sufficient for the current questions the brain deals with).

We shall examine the question with respect to the three grammatical components enumerated: the grammars generator, the universal grammar and the parser.

Let N, T be two given vocabularies and let $L_0 = \{x_1 \to y_1/x_2 \to y_2/\ldots/x_k \to y_k/ \,|\, k \geqslant 1,\ x_i, y_i \in (N \cup T)^*,\ x_i$ contains a symbol in N and $|x_i| \leqslant |y_i|,\ 1 \leqslant i \leqslant k\}$ ($|x|$ denotes the length of the string x.). This language contains all the strings z_G associated to all type-1 grammars.

The language L_0 is not regular. Indeed, let $A \in N$, $a \in T$ be two given symbols and let us consider the regular language

$$R^n = \{A^n \to a^m / \,|\, n, m \geqslant 1\}.$$

Clearly, we have

$$L_0 \cap R = \{A^n \to a^m / \,|\, 1 \leqslant n \leqslant m\}.$$

The congruence relation $\delta_{L_0 \cap R}$ (see [18]) has an infinite index because no string A^r is equivalent to a string A^p with $r \neq p$ (if $r < p$, then A^r is accepted by $\langle \lambda, \to a^r/\rangle$, but A^p is not accepted by this context). Consequently, L_0 is not a regular language.

However, L_0 is a context-free language. Indeed, let us consider the following grammar (its rules are denoted by $x \dashv y$, in order to avoid the confusion between the symbol $\to$ in this grammar and that in L_0):

$$G_0 = (N_0, N \cup T \cup \{\to, /\}, S_0, P_0)$$

with

$$N_0 = \{S_0, A, B\}$$

and P_0 contains the following rules:

1. $S_0 \dashv A/S_0$

   $S_0 \dashv A/$

(One generates a string of the form $(A/)^n$, $n \geqslant 1$.)

2. $A \dashv A\beta$, $\beta \in N \cup T$,

   $A \dashv \alpha A\beta$, $\alpha, \beta \in N \cup T$,

   $A \dashv \alpha B\beta$, $\alpha \in N$, $\beta \in N \cup T$,

   $B \dashv B\beta$, $\beta \in N \cup T$,

   $B \dashv \alpha B\beta$, $\alpha, \beta \in N \cup T$,

   $B \dashv \to$

(From each A we can develop a derivation of the form

$$A \overset{*}{\Rightarrow} xAy \Rightarrow x\alpha B\beta y \overset{*}{\Rightarrow} x\alpha Bw\beta y \Rightarrow x\alpha z \to w\beta y$$

where $x, y, z, w \in (N \cup T)^*$, $\alpha \in N$, $\beta \in N \cup T$, $|x| \leqslant |y|$, $|z| \leqslant |w|$. Consequently, $|x\alpha z| \leqslant |w\beta y|$.)

It follows that $L(G_0) = L_0$, hence $L_0 \in \mathcal{L}_2$.

We consider now the universal grammar.

As pointed out in [5], the codification of A_i by AB^iA cannot be used for type-1 grammars because such a substitution unboundedly increases the length of the sentential forms (we have to deal with type-1 grammars with arbitrarily many nonterminal symbols). Therefore, the construction in [5] cannot be used in order to obtain a universal type-1 grammar (although the algorithm of derivation in a grammar can be codified in such a way).

However, the construction in [5] can be slightly modified in order to obtain a universal type-1 grammar with respect to the set of type-1 grammars G with $Var(G) \leqslant k$ for given k. (We work with nonterminals $A_1, \ldots, A_k$, thus the codification $A_i \to AB^iA$ is not necessary in this case.)

Consequently, the model of the brain can be based on a type-1 universal grammar providing that all the grammatical competences correspond to type-1 grammars having at most k auxiliary symbols. This restriction postulates the existence of a characteristic constant k for each brain, an upper bound of the (syntactic) complexity of the brain competences. This raises the natural question: are there competences of arbitrarily large *Var?* The problem has two sides: the theoretic one (are there languages $L \in \mathscr{L}_1$ with arbitrarily large *Var(L)?*) and the practical one (are there human competences which need grammars with arbitrarily many auxiliary symbols?) It is likely that both these questions have an affirmative answer. On one hand, for any k there are regular languages for which all the context-free grammars have more than k auxiliary symbols [7]. On the other hand, the brain competences seem to have no apriori limit of grammatical complexity. Let us think, for instance, to the "chess grammar" owned by a chess champion.

At this point, let us remark that the fact that the universal grammar deals with grammars with at most k nonterminals does not mean that all the grammars the brain manipulates are of equal complexity. The "basic" competence (of a chess player, for example) can reach the threshold k while other competences (the mathematical ability of a chess player) may be of a lower complexity.

What can one see about the syntactic analyser? The usual parsing algorithms [18] (the algorithms in the proof that each context sensitive language is recursive) need an exponential space compared with the length of the input string; such an algorithm cannot be realized by a type-1 grammar. On the other hand, the linear bounded automata recognize type-1 languages in bounded space. A problem arises here: can the brain incorporate a nondeterministic automaton or only a deterministic one? The real brain seems to involve nondeterministic procedures, but the nondeterminism is to be avoided in actually dealing with computers, for instance. The problem is related to the well-known *LBA* problem [10]: are the deterministic linear bounded automata (*LBA*, for short) as powerful as the nondeterministic *LBA?* The problem is open. In connection to our model it is of interest to evaluate the power of deterministic *LBA* (let us denote by $\mathscr{L}_{1-det}$ the corresponding class of languages).

One may assert that $\mathscr{L}_{1-det}$ is a very large family of languages. Indeed, $\mathscr{L}_2$ is contained in $\mathscr{L}_{1-det}$ [10], the simple matrix languages [8], the finite index matrix languages [17], as well as all Szilard languages (associated to arbitrary type-0 grammars) [9] are in $\mathscr{L}_{1-det}$. Moreover, each language in L_0 is the homomorphic image of a language in the family $\mathscr{L}_{1-det}$.

Therefore, restricting the syntactic analyser to type-1 level (even by considering it as being realized by a deterministic *LBA)*, seems to be an unrestrictive hypothesis.

In conclusion, providing that we assume the brain to have an upper limit of grammar complexity, we can realize the model at the level of a type-1 grammar.

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