Diophantine Representation for $\Omega$

Cristian S. Calude

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The $\Omega$ number, see [2] is one of the most important concepts in Algorithmic Information Theory [1]. Chaitin has presented a first exponential Diophantine representation for a natural $\Omega$ number in [2]. His equation was automatically generated from a complex register machine program and is very large, e.g. it has approximately 17,000 unknowns. While it is has been shown that this can be reduced to just three [3], doing so would be a very challenging task.

In [4], Ord and Kieu have shown how to determine the $k$th bit of $\Omega$ by solving $k$ instances of the halting problem. From this they reduce the problem of determining the $k$th bit of $\Omega$ to determining whether a certain Diophantine equation with two parameters, $k$ and $N$, has solutions for an odd or an even number of values of $N$. They further construct an exponential Diophantine equation with a parameter $k$ which has an odd number of solutions iff the $k$th bit of $\Omega$ is 1, and a polynomial of positive integer variables and a parameter $k$ that takes on an odd number of positive values iff the $k$th bit of $\Omega$ is 1.

The projects seeks to improve the Ord and Kieu constructions and to write a program to automatically generate “smaller” Diophantine representations for $\Omega$.

References


