# Exercises in De-Quantisation

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## 1 Simulation of quantum algorithms based on superpositions

Consider a Boolean function $f : \{0,1\} \to \{0,1\}$ and suppose that we have a black box to compute it. Deutsch's problem asks to test whether $f$ is *constant* (that is, $f(0) = f(1)$) or *balanced* ($f(0) \neq f(1)$) allowing *only one query* on the black box computing $f$.

In a famous paper published in 1985, Deutsch [5] obtained a "quantum" *partial affirmative answer*. In 1998, a complete, probability-one solution was found by Cleve, Ekert, Macchiavello, and Mosca [4]. In [3] it was shown that the quantum solution can be *de-quantised* to a deterministic simpler solution which is as efficient as the quantum one.

Th core technique used by the quantum solutions is to *coherently* embed the classical black box into a quantum black box (the quantum black box produces the same outputs as the classical black box when the inputs are the pure Qbits $|0\rangle, |1\rangle$), then perform a special computation with the quantum black box on a *superposition* of carefully chosen quantum states (this computation has no classical meaning for the original black box), and finally *measure* the output produced. The analysis proposed in [3] showed that the same quantum technique, embedding plus computation on a "superposition", leads to a classical solution which is as efficient as the quantum one. More, the quantum solution is *probabilistic*, while the classical solution is *deterministic*. Other examples are in [2, 8, 9].

**Question 1.** How does the classical solution compare with the quantum one in terms of physical resources? A simple analogical scheme can implement the classical solution with two registers each using a real number as in the quantum case when we need just two Qbits. However, a more realistic analysis should involve the complexity of the black box, the complexity of the implementation of the embedding, as well as the complexity of the query performed.

**Question 2.** The simulation of superposition doesn't scale with the idea below. Show how to obtain a similar solution for a fixed $n$, but not uniformly (in each case a different function is used). Of course, the uniformly the solution discussed in this note is not scalable, because $n$ Qbits can represent $2^n$ states at the same time, which outgrows any linear function of $n$ (see [6]).

**Question 3.** Find other quantum algorithms based on the "superposition" technique only and try to construct classical algorithms as efficient as the quantum ones.

## 2 Simulation of quantum algorithms based on entanglement

- Short description of quantum entanglement.
- Example of the simplest quantum algorithm based on entanglement. Quantum Fourier Transform?
- An example of dequantisation [2].

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