Gregory Chaitin[1]

CONCEPTUAL COMPLEXITY AND ALGORITHMIC INFORMATION[2]

1. Introduction

In this essay we propose that the fundamental philosophical concept of *conceptual complexity* is captured mathematically by the notion of *algorithmic information content*, and we discuss the complexity of physical and mathematical theories, the complexity of biological mutations, and the most complex system in biology, the human brain. These are steps in the direction of a mathematical philosophy, by which we mean a mathematical approach to philosophical questions, not a philosophy of mathematics. For as Leibniz said:

> Without mathematics we cannot penetrate deeply into philosophy.
> Without philosophy we cannot penetrate deeply into mathematics.
> Without both we cannot penetrate deeply into anything.
>
> Sans les mathématiques on ne pénètre point au fond de la philosophie.
> Sans la philosophie on ne pénètre point au fond des mathématiques.
> Sans les deux on ne pénètre au fond de rien.

2. Origins of the Concept of Complexity in the Philosophy of Science and in Biology

2.1 Complexity in the work of Leibniz

We begin the intellectual history of the concept of complexity with Leibniz's brilliant *Discours de métaphysique* (1686). In particular, we are interested in Sections V and VI of the *Discours*, in which Leibniz explains why science is possible and what it means for facts to be governed by law rather than be random. Let's paraphrase Sections V and VI of the *Discours*.

God has created the best of all possible worlds, in that the richness and diversity – including us! – that we see in the world around us is the consequence of a small, simple set of concepts and laws. God *maximizes* the richness of the world and at the same time *minimizes* the complexity of the laws that determine this world. In other words, the world is comprehensible, science is possible, there are elegant theories. God's perfection resides in the fact that He uses the smallest possible number of bricks, the simplest possible tools, in order to create the world.

By the way, this excludes miracles, which would be like amendments to the constitution, exceptions that have to be added to the constitution and unnecessarily complicate it, and analogously would unnecessarily complicate the fundamental laws of physics.

Furthermore, what is a law, how can we distinguish the lawless from the lawful? Well, said Leibniz, if we have a finite set of points on a graph representing the behavior of a physical system, there is **always** an equation passing precisely through those points. Hence the mere existence of a mathematical law is not enough. Real laws have to be simple, because anything can be explained if one allows sufficiently complicated, *ad hoc* laws.[3]

In our modern reading of Leibniz, Sections V and VI both assert that the essence of explanation is compression. An explanation has to be much simpler, more compact, more concise, than what it explains.

2.2 Complexity and evolution

Leibniz is a tremendously deep thinker. We now turn to biology, a more obvious source of the notion of complexity. Please see Peter Bowler's book *Darwin Deleted* for an analysis of the many threads in the development of ideas on evolution and progress in Nature.

One of these threads was Lamarck, who proposed the inheritance of acquired characteristics in order to explain the evolution, the increase in the complexity of organisms, that was becoming increasingly apparent in the fossil record. Another precursor of Darwin was his own grandfather, Erasmus Darwin. Even before Charles Darwin, many realized that evolution was taking place. The problem was to find a mechanism, to explain how evolution works. That it was taking place was not at issue. Many even assumed that Nature's goal was to progress, a notion that is rather submerged – but not entirely eliminated – in Darwin's theory.

2.3 Complexity and spontaneous generation

A further impetus to the crystallization of the concept of complexity in biology was provided by the dispute over spontaneous generation. Spontaneous generation is immediately seen to be impossible once the enormous complexity of an individual cell was properly grasped.

2.4 The immensely complicated human brain

Later in this essay, we shall attempt to analyze human intelligence and the brain. That's also connected with complexity, because the human brain is the most complicated thing there is in biology. Indeed, our brain is presumably the goal of biological evolution, at least for those who believe that evolution has a goal. Not according to Darwin! For others, however, evolution is matter's way of creating mind.

Fundamental ideas that have inspired us may be discredited and slip out of sight, but they have a way of reemerging years later in more modern dress. Bowler sees this process at work in Darwinism and it certainly applies to our next topic, the search for the perfect language.

3. The Search for the Perfect Language

In the previous section, we discussed four important applications of the concept of complexity but neglected to specify what complexity is. The time has come to define complexity rigorously with mathematical precision.

To show how that's done, let's review the saga of *The Search for the Perfect Language*. Umberto Eco's book with this title reminds us that in the Middle Ages it was thought that knowing the language used by God to create the universe – perhaps Hebrew – would give us a way to analyze ideas into their basic conceptual components and provide a path to all truths. An early effort in this direction was the *Ars magna* of Ramon Llull. Inspired by Llull, Leibniz converted this project into the search for a *characteristica universalis*.

This project is mocked by Jonathan Swift in *Gulliver's Travels*, in his account of Gulliver's visit to Laputa. A later version of this dream is Hilbert's program for a single consistent, complete formal system for all of mathematics which would solve the *entscheidungsproblem* by providing an algorithm for deciding if any mathematical statement were true or false, merely by running through all possible proofs in the formal system until finding a proof of correctness or a refutation.

Thanks to Gödel and Turing we know that this cannot be done: There is no universal language for formalizing all possible mathematical proofs. However, Turing also showed that there are universal languages for formalizing all possible mathematical algorithms, and algorithmic information theory tells us which are the most concise, the most expressive such languages.

Clearly these would be the languages of choice for creating the world! In this essay we define the *conceptual complexity* of an object X to be the size in bits of the most compact program for calculating X, presupposing that we have picked as our complexity standard a particular fixed, maximally compact, concise universal programming language U. This is technically known as the *algorithmic information content* of the object X, denoted $H_U(X)$, or simply $H(X)$ since U is assumed fixed. In medieval terms, $H(X)$ is the minimum number of yes/no decisions that God would have to make to create X.

As Leibniz said, "As God calculates, so the world is made," "*Cum Deus calculat, fit mundus*."[4]

4. Mathematical Definition of Conceptual Complexity

Whence cometh this definition? Who invented the necessary mathematics? Here is a brief summary. If we sum

Alan Turing (1936) + Claude Shannon (1948)
computation **information**

this gives us

algorithmic information
[R. Solomonoff, A. N. Kolmogorov, G. J. Chaitin (1960's)]
[G. J. Chaitin, L. A. Levin (1970's)]

And the concept of algorithmic information in turn gives us a handle on the deep notion of conceptual complexity, which is fundamental in epistemology. (In contrast, I think of the more practical topic of *time complexity* as a branch of software engineering.) To illustrate the power of this new intellectual toolkit, we shall now discuss applications in metaphysics, in metamathematics, and in a new research program that I have dubbed *metabiology.*

5. Conceptual Complexity in Software Models of Physics, Mathematics and Biology

Now you know how to define conceptual complexity mathematically and a little about its history. In this section, we start by examining the complexity of physical theories, take a detour to consider the complexity of mathematical theories, and then finally arrive at a model of biological evolution. In later sections we shall present some thoughts on the brain and consciousness.

5.1 Complexity of a physical theory

> *A physical theory T is a computer program that calculates and exactly reproduces a finite set of experimental data by using the equations of physics and the initial conditions of the physical system in question. Then T halts. The data must be reproduced precisely; no errors are allowed.*
> *Thus we can only deal with* ***deterministic*** *physical theories in this model, not with* ***statistical*** *theories, which necessarily cannot exactly reproduce the experimental data.*
> *The conceptual complexity $H(T)$ of the theory T is the size in bits of the program T.*

Why is this definition of complexity of any value? Because it enables us to formalize and study mathematically the idea that the best theory is the simplest theory.

Following Ray Solomonoff, there are numerous practical applications of this criterion in machine intelligence, in Bayesian statistics, and in data mining, but in this essay I am interested in philosophy and not in practical applications.

5.2 Complexity of a formal axiomatic theory

> *A formal axiomatic theory T is an unending computer program that calculates one by one the infinite set of all theorems that follow from the axioms by applying the rules of symbolic logic [David Hilbert, Emil Post]. T does this by systematically running through all possible proofs.*

The conceptual complexity $H(T)$ of the theory T is the size in bits of the program T.

The surprising fact that you can almost never prove that you have the best – the simplest possible – physical theory, gives us a new kind of Gödel incompleteness, an information-theoretic incompleteness theorem in fact.

To be able to prove that an N-bit physical theory is the most concise computer program that calculates the output that it does (which I like to refer to as an "elegant" physical theory or an "elegant" program), you need to use a formal axiomatic theory T whose conceptual complexity $H(T)$ is at least N bits.

Proof Sketch: Given a formal axiomatic theory T with conceptual complexity $H(T)$, consider the paradoxical program P that calculates the output of the first large provably elegant program Q produced by T – the first such Q you find when systematically running through all possible proofs that programs are elegant in theory T. More precisely, P searches for a provably elegant Q with the property that the size in bits of Q is greater than the size in bits of P. P calculates the same output as Q, which is supposed to be the smallest program that produces the output that it does, but P is smaller than Q!

This easily shows that in a given formal axiomatic theory T a provably elegant computer program Q cannot be larger than the size in bits of P. And what is the size of P? One can show that it's $H(T) + c$ bits, where c is a constant independent of T and is negligible if $H(T)$ is large.

5.3 *Complexity of an* algorithmic mutation

An algorithmic mutation M is a computer program that calculates the new organism from the old organism.
The conceptual complexity $H(M)$ of the mutation M is the size in bits of the program M.

Note that in the toy model of evolution given below the organisms are also software, but we are not too concerned with their conceptual complexity. We concentrate on the conceptual complexity of the mutations.

Defining a mutation in this very general and somewhat abstract way is the key idea that we shall use to model Darwinian evolution mathematically. It's crucial for the success of our model. It's what enables us to prove that Darwinian evolution works in this model and yields open-ended, endless evolution.

6. Modeling Darwinian Evolution

Since this is a little-known, new area of application of the notions of conceptual complexity and algorithmic information, we shall sketch here some of the basic features of our highly-simplified toy model of evolution.

6.1 How do we pick a random mutation?

As we stated before, in our model an algorithmic mutation M is a computable function that takes as input the original organism A and produces as output the mutated organism A':

$$A' = M(A)$$

Here M, A and A' all consist of binary software, of bit strings:

M = algorithmic mutation
A = original organism
A' = mutated organism

But what is the probability of the mutation M? What probability distribution shall we use over this very rich space of possible mutations?

In our model, simple mutations are much more probable than complex mutations. In fact, if the conceptual complexity of the mutation M is $H(M)$, then its probability is $2^{-H(M)}$. In other words, if the binary software for the mutation M has K bits, then its probability is $1/2^K$, for each bit of the program for M is chosen by using an independent toss of a fair coin.

If M is a K-bit program, then probability of $M = 2^{-K}$.

If the *conceptual complexity* of M is K bits, then the probability of $M = 2^{-K}$.

This is a very natural way of assigning probabilities to programs and it has been used in theoretical computer science since the 1970's.

(Although there is an important technical proviso: The program for M must be "self-delimiting" – otherwise the total probability for all possible mutations is infinite instead of being less than one as a probability should be. We shall encounter self-delimiting programs again later when discussing the halting probability Ω.)

Please note that for any A and A' there is a non-zero probability that the mutation M will transform A into A'. But most mutations are extremely unlikely. The fact that we allow arbitrary *algorithmic* mutations M has some surprising consequences. For example, the global change M that consists of inverting each bit of a program A yielding the complementary bit string A' is a very simple and therefore a highly probable mutation.

Furthermore, M's probability does not depend on the size of A and A', since the same program M will work for inverting arbitrarily large bit strings A. Unfortunately M will probably completely wreck the program A. This is not a useful mutation, even though it is a highly probable one.

We've explained the mutational model we're using. We are now ready to talk about the evolution of mutating software.

6.2 Life as evolving software

Now let's explain the rest of our evolutionary model. It's just a *hill-climbing random walk in software space*, for our organisms are programs too, and a mutation is accepted if and only if it increases the fitness of the organism.

In fact, our organisms are programs that calculate a single positive integer, and the bigger the integer, the fitter the program. In other words, the well-known computer science Busy Beaver problem consisting of naming the largest possible integer is the challenge that will force our organisms to evolve.

There is just a single software organism A at a time, and we try mutating it at random using algorithmic mutations as previously described, until we obtain a fitter organism A', one that calculates a bigger number. Then everything continues as before but with this new fitter organism A' replacing the previous software organism A.

Darwinian evolution has been described as

"Design without a Designer!"

Here what we have instead is

"Programming without a Programmer!"

In other words, our organisms consist only of hereditary material, they do not have bodies or a metabolism. And in our model, instead of randomly mutating DNA, which is the actual software of life, we are mutating computer programs that are working on an extremely difficult computational problem, the Busy Beaver problem.

Simple as it is, this model exhibits some of the features of Darwinian evolution, in fact provably so. If it weren't so simple, we would not be able to prove anything. One important concept that emerges from this analysis is *mutation distance*.

6.3 Mutation distance

We are modeling evolution as a random walk in software space. How far does one have to walk to get from the organism A to the organism A'? Is there a distance measure associated with this model? Yes indeed, the mutation distance. What is mutation distance? Well, there are several different but ultimately equivalent ways of defining this important concept:

Mutation distance = $H(A' \mid A)$
= the *relative information content* of A' given A
= the number of bits of algorithmic information needed to transform A into A'
= the *conceptual complexity* of the mapping from A to A'
= $-\log_2$ of the probability that a random mutation M carries A into A'

For more information about this model of evolution and its remarkable properties, please see the book *Proving Darwin* or my article “Life as evolving software” in Hector Zenil’s Turing centennial volume *A Computable Universe*. Now let’s change topic.

7. Designing a Brain

We have discussed applications of the concept of conceptual complexity in metaphysics, in metamathematics, and in a toy model for Darwinian evolution. Now let’s turn to the most complicated object there is, the human brain.

What we present here may or may not be how our brain works, but I believe it is a possible design for a brain.

7.1 Clues for solving the puzzle

Our analysis of the brain will follow John von Neumann’s posthumous classic *The Computer and the Brain* (1958) and will be based primarily on considering processing power and memory capacity, the two key parameters in a computer.

And the RNA/DNA molecular biology level has much greater memory capacity and processing power than the neuronal level, so why not use it?

We also take into account some intriguing experiments with planaria that were formerly dismissed but which have recently received a certain amount of confirmation.

There is a deep remark in von Neumann, *The Computer and the Brain*, that in computers the technology used for memory and for logic is always different, so maybe the same is true in the brain.

Von Neumann also makes this anti-connectionist remark in the question and answer discussion at the end of his paper "The general and logical theory of automata" (presented 1948, published 1951), the paper in which he proposes thinking of DNA as software.

Following von Neumann, we shall explore anti-connectionist models for a brain, ones in which logic may be encoded in the connections of neurons but perhaps a different encoding is used for memory, especially long-term memory, which has a huge capacity, as illustrated by von Neumann himself, who reportedly had a photographic memory.

In a nutshell, we propose that

logic = neurons = conscious mind,

whilst

memory = RNA/DNA level = unconscious mind.

This proposal is also inspired by some old but intriguing planaria learning through cannibalism experiments which were greeted with skepticism by the scientific community but have just received some confirmation. Please see the following papers:

"Planaria: Memory transfer through cannibalism reexamined" *Science* **146**, 9 October 1964: 274–275.

"An automated training paradigm reveals long-term memory in planarians and its persistence through head regeneration" *J. Exp. Biol.* **15**, October 2013: 3799–3810.

7.2 Is the brain a two-level system?

Here then is our two-level proposal:

- Conscious, Rational, Serial, Sensual Front-End Mind: **Neurons (Fast)**
- Unconscious, Intuitive, Parallel, Combinatorial Back-End Mind: **Molecular Biology (Slow)**
  (much greater compute and memory capacity)

This proposal was also inspired by three other works, which we now list.

On the hugeness of human memory capacity and the fact that people seem to remember everything even if they can't recall it, see *Borges and Memory* by Rodrigo Quian Quiroga.

See also *Dissertatio de arte combinatoria*, an early work by Leibniz (1666). Following Llull, the *Dissertatio* considers intelligence and the power of invention as the ability to quickly search through all possible combinations of a set of ideas.

And see Jacques Hadamard, *The Psychology of Invention in the Mathematical Field* (1945). This book emphasizes the fundamental role of the unconscious in mathematical creation. Having a problem in your research? Sleep on it! Consult with your pillow!

On the basis of these readings, we tentatively conclude that the processing power and memory capacity of the unconscious mind is much greater than that of the conscious mind.

7.3 Discussion of the model

After all, the immune system does information processing at the molecular level. If the brain worked only at the neuronal level, for example by storing one bit per neuron, it would have roughly the capacity of a pen drive, far too low to account for human intelligence. But at the RNA/DNA molecular biology level, the total information capacity is quite immense.

In the life of a research mathematician it is frequently the case that one works fruitlessly on a problem for hours then wakes up the next morning with many new ideas. The intuitive mind has much, much greater information processing capacity than the rational mind. Indeed, it seems capable of exponential search.

We can connect the two levels postulated here by having a unique molecular "name" correspond to each neuron, for example to the proverbial "grand-mother cell." In other words, we postulate that the unconscious "mirrors" the associations represented in the connections between neurons. Connections at the upper conscious level correspond at the lower unconscious level to enzymes that transform the molecular name of one neuron into the molecular name of another. In this way, a chemical soup can perform massive parallel searches through chains of associations, something that cannot be done at the conscious level.

When enough of the chemical name for a particular neuron forms and accumulates in the unconscious, that neuron is stimulated and fires, bringing the idea into the conscious mind.

And long-chain molecules can represent memories or sequences of words or ideas, *i.e.*, thoughts.

To end this discussion, I should repeat that whether or not the human brain actually works this way, it seems to me that this is a design for a brain that just possibly might be made to work. At any rate, I think it is important to escape from the current cul-de-sac and propose new lines of research.

8. The Mystery of Consciousness

We talked about consciousness above. Now we discuss some ideas about consciousness put forth by other authors, ideas that are totally unrelated to – and perhaps even contradict – the brain design speculations that were just presented.

I once fell asleep at the wheel while driving, then woke up and wondered where I was and "who" had continued driving while my conscious mind was asleep. Fortunately *someone* had continued driving!

According to Christof Koch's book *Consciousness*, the level of my brain that continued driving was **also** conscious. Similarly, Koch believes the immune system must be conscious. In fact, he believes that everything is conscious; this is called *panpsychism*. What's important is the degree of consciousness.

According to Giulio Tononi's book *PHI*, consciousness can be measured in terms of the integrated information Φ. (Tononi's Φ is also discussed in Koch's book.) The greater a system's Φ, the more integrated and conscious it is. A binary switch has one bit of consciousness.

I suspect Φ has something to do with what in algorithmic information theory is called *mutual information*

$$H(X : Y) = H(X) + H(Y) - H(X, Y)$$

which is the extent to which X and Y are simpler when seen together than when seen separately.

In a 1979 paper "Toward a mathematical definition of 'life'" I discuss using the mutual information of the parts of a system to determine the extent to which the parts are integrated into a whole.

These ideas might also help with the question of "the creation of the self:" what it means to isolate a certain physical system from the rest of the world and declare it to be a unity. How do we decompose the world into things? How do I distinguish you from me? And how can we define this mathematically?

9. Digital Ontology

We have discussed epistemology (of physics and mathematics), evolution, the brain and consciousness. Our final topic is ontology.

9.1 The new Pythagoreans

The digital, algorithmic approach inspired by the computer suggests that everything is discrete, everything is 0's and 1's. The continuum is deemphasized, dethroned. Is the world a computer?

Instead of having a world built of number (Pythagoras) we now have a world built of information, **a new substance**. This view of the world is also known as *digital philosophy*, and other practitioners are Stephen Wolfram and Edward Fredkin. See Longo and Vaccaro, *Bit Bang: La nascita della filosofia digitale*.

9.2 Digital physics

There are also hints from physics.

The so-called "holographic principle" has emerged in the field of quantum gravity, as a consequence of results on the thermodynamics and entropy of black holes. According to this principle, the total number of bits of information in any physical system is finite, and moreover grows only as the surface area of its boundary, not with its volume.

These tentative results – quantum gravity is a relatively new field – suggest both that the physical world is in some sense discrete, and that perhaps at some level space is more two-dimensional than three-dimensional. See Lee Smolin, *Three Roads to Quantum Gravity*.

9.3 The halting probability Ω

In order to continue our discussion of ontology we have to define the number Ω and say a few words about why it is so interesting. Ω is the total probability of all the self-contained (without any input) programs *p* that eventually halt, assuming that the bits of *p* are picked using independent tosses of a fair coin:

$$\boldsymbol{\Omega} = \sum_{p \text{ halts}} 2^{-|p|} = \sum_{U(p) \text{ halts}} 2^{-(\text{size in bits of program } p)}$$

This infinite sum defines Ω, but does not enable us to calculate its numerical value, because in fact Ω is wildly, extravagantly uncomputable.

Technical Note: For this infinite sum to converge and be between zero and one it is essential that we stipulate that the programs *p* for our universal computer *U* be "self-delimiting" so that

U knows by itself where to stop reading the bits of p and no extension p' of a p that halts is ever included in the sum for Ω. Otherwise this sum will diverge to infinity. For a full explanation of this please consult any treatise on algorithmic information theory.

Imagine writing the numerical value of Ω in base-two notation. This would give us its binary expansion, for example,

$$\mathbf{\Omega = .01110\ldots}$$

Knowing the first N bits of this expansion would enable us to discover all the programs up to N bits in size that ever halt, see what output they produce before halting, and output something different. Therefore the algorithmic information content of the first N bits of the binary expansion of Ω is greater than $N - c$:

$$H(\textbf{first } N \textbf{ bits of } \Omega) > N - c$$

Leibniz in his *Monadolology*, Sections 33-35, says that mathematical proof consists in analyzing a complex statement into the consequence of simpler statements, continuing this analysis until one reaches statements that are so simple that their truth is self-evident and no longer requires any proof – otherwise there is an infinite regress.

However, it takes an N-bit theory – a formal axiomatic theory whose conceptual complexity is N – for us to be able to determine N bits of Ω. To be able to prove what are the precise values of N bits of the binary expansion of Ω, we must use a formal axiomatic theory which itself has at least N bits of complexity.

Therefore the precise values of the successive bits of Ω are *irreducible mathematical facts*, mathematical facts that are true for no reason, more precisely, true for no reason simpler than themselves. They thus violate Leibniz's *principle of sufficient reason* and seem to be *contingent* rather than *necessary* truths.

Does the number Ω exist? Does anything with infinite complexity exist? If we can never know something, why should we believe it exists? Well, we **can** prove lovely theorems about Ω. For example, Ω is Borel normal, which means that in the binary expansion for Ω, in the limit we encounter 0's and 1's with exactly the same relative frequency. And hopefully Ω gives us some intuition about more down-to-earth situations and how things may behave in somewhat more realistic contexts. Remember, the world of pure mathematics is much simpler and much more understandable than the messy real world!

All by itself, Ω shows that pure mathematics is biological. Because one of the defining characteristics of the field of biology – as opposed to the field of theoretical physics which is based on equations – is that there are few simple organizing principles in biology. In other words, biology is very complicated. That may be so, but Ω is infinitely complicated, and therefore, in a sense, more biological than biology itself.

Furthermore, the Platonic world of pure mathematics has infinite conceptual complexity, whilst if Hilbert had been correct, the world of pure mathematics would only have finite complexity, namely the complexity of the universal consistent, complete formal axiomatic theory that Hilbert was searching for. Mathematics is open-ended, creative, not closed as Hilbert thought. And this makes mathematical truth more tentative and not as totally black or white as Hilbert thought it would be.

This complexity-based point of view also emphasizes the similarities between pure mathematics and theoretical physics rather than the differences, and suggests, as I have argued elsewhere, a *quasi-empirical* view of mathematics. Mathematics and physics may be different, but they are not **that** different – at least not from an information-theoretic point of view.

10. Summary

We have used the program size $H(T)$ to measure the *conceptual complexity* of a physical or a mathematical theory T. The fact that you can never prove that you have the simplest physical theory yields a new version of Gödel's theorem on the limitations of formal axiomatic mathematical theories.

In modeling Darwinian evolution, we have used *relative* program size $H(A' \mid A)$ to measure the conceptual complexity of an A to A' mutation. We can prove that our model evolves. We have a Pythagorean world in which life provably evolves.

What about the brain and consciousness? Following von Neumann, maybe a computer engineering approach is relevant. And following Tononi and Koch, maybe *mutual* program size complexity $H(X : Y)$ is related to *integrated information* as measured by Φ.

We now have a new fundamental substance, information, that comes together with a digital world-view.

And – most ontological of all – perhaps with the aid of these concepts we can begin again to view the world as consisting of both mind and matter. The notion of mind that perhaps begins to emerge from these musings is mathematically quantified, which is why we declared at the start that this essay pretends to take additional steps in the direction of a mathematical form of philosophy.

The eventual goal is a more precise, quantitative analysis of the concept of "mind." Can one measure the power of a mind like one measures the power of a computer?

Notes

[1] Invited talk at IACAP 14: Conference of the International Association for Computing and Philosophy, Thessaloniki, July 2-4, 2014.

[2] Federal University of Rio de Janeiro, Brazil, https://ufrj.academia.edu/GregoryChaitin. The author is grateful to the Brazilian government for supporting this research with a CAPES PVE grant.

[3] As Hermann Weyl put it, if arbitrarily complex laws are permitted, then the concept of law becomes vacuous because there is always a law. See Weyl, *The Open World* (1932).

[4] The actual statement by Leibniz seems to be "*Cum Deus calculat et cogitationem exercet, fit mundus*" (When God thinks things through and calculates, the world is made). But the abbreviated version is more to the point, or at least more to *our* point.

Further Reading

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