

Incompleteness

April 9, 2014

By $N(P, v)$ we mean that the (Turing) program P will *never* halt when begun with input v .

1. Assume $N(P, v)$ is undecidable. Is $N(P, v)$ true or false?
2. Assume $N(P, v)$ is undecidable. Add $N(P, v)$ to the axioms of the theory. Is the new (augmented) theory decidable?
3. Is the set $\{N(P, v) \mid P, v \text{ with } N(P, v) \text{ false}\}$ computable?
4. Is the set $\{N(P, v) \mid P, v \text{ with } N(P, v) \text{ true}\}$ computable? Computably enumerable?
5. Can you extract an infinite computable subset T of $\{N(P, v) \mid P, v \text{ with } N(P, v) \text{ true}\}$?
6. Which of the following two statements could be unprovable? a) The string x is not incompressible. b) The string x is compressible. Gives reasons for your answer.
7. Illustrate various logical fallacies from the attached chart in proofs.