# APPENDIX A

# *Publications*

This thesis resulted in the following refereed international journal publications:

- *The Visualization and Measurement of Left Ventricular Deformation Using Finite Element Models*, Burkhard Wünsche and Alistair A. Young, International Journal of Visual Languages and Computing - Special Issue on Biomedical Visualization for Bioinformatics, Vol.14 No.4, August 2003, pages 299-326.

- *A Survey and Analysis of Common Polygonization Methods & Optimization Techniques.* Burkhard Wünsche, Machine Graphics & Vision, Vol.6 No.4, 1997, pages 451-486.

This thesis resulted in the following refereed international conference publications:

- *The 3D Visualization of Brain Anatomy from Diffusion-Weighted Magnetic Resonance Imaging Data*, Burkhard Wünsche and Richard Lobb, Proceedings of GRAPHITE 2004, 15-18 June 2004, Singapore, organised by ANZGRAPH and SEAGRAPH, sponsored ACM SIGGRAPH, (accepted for publication).

- *The Visualization of Myocardial Strain for the Improved Analysis of Cardiac Mechanics*, Burkhard Wünsche and Richard Lobb and Alistair A. Young, Proceedings of GRAPHITE 2004, 15-18 June 2004, Singapore, organised by ANZGRAPH and SEAGRAPH, sponsored ACM SIGGRAPH, (accepted for publication).

- *Advanced Texturing Techniques for the Effective Visualization of Neuroanatomy from DTI Data*, Burkhard Wünsche, Proceedings of the Second Asia-Pacific Bioinformatics Conference, APBC '04, 18-22 January 2004, Dunedin, New Zealand, ACS, pages 303-308.

- *A Comparison and Evaluation of Interpolation Methods for Visualising Discrete 2D Survey Data*, Burkhard Wünsche and Ewan Tempero, Proceedings of the 2004 Australasian Symposium on Information Visualisation, InVis.au 2004, January 23-24 2004, Christchurch, New Zealand, pages 1-7.

- *A Survey, Classification and Analysis of Perceptual Concepts and their Application for the Effective Visualisation of Complex Information*, Burkhard Wünsche, Proceedings of the 2004 Australasian Symposium on Information Visualisation, InVis.au 2004, January 23-24 2004, Christchurch, New Zealand, pages 17-24.

- *Texture-based Visualization Methods for Diffusion-Weighted MRI Images*, Burkhard Wünsche, Proceedings of the 17th International Congress and Exhibition on Computer Assisted Radiology and Surgery (CARS 2003), 25-28 June 2003, London, UK, Excerpta Medica International Congress Series 1256, Elsevier, page 1326.

- *New Techniques for Visualising and Evaluating Left Ventricular Performance*, Burkhard Wünsche and Alistair A. Young, Proceedings of the 17th International Congress and Exhibition on Computer Assisted Radiology and Surgery (CARS 2003) - 2003 International Symposium on Cardiovascular Imaging, 25-28 June 2003, London, UK, Excerpta Medica International Congress Series 1256, Elsevier, pages 1173-1178.

- *A Toolkit for Visualizing Biomedical Data Sets*, Burkhard Wünsche, Proceedings of GRAPHITE 2003, 11-14 February 2003, Melbourne, Australia, organised by ANZGRAPH and SEAGRAPH, sponsored ACM SIGGRAPH, pages 167-174.

- *The Visualization and Measurement of Left Ventricular Deformation*, Burkhard Wünsche, Proceedings of the First Asia-Pacific Bioinformatics Conference, APBC '03, 4-7 February 2003, Adelaide, Australia, ACS, pages 119-128.

- *A Field Data Structure for Improved Interactive Exploration of Scientific Data Sets*, Burkhard Wünsche, Proceedings of IVCNZ '02, 26-28 November 2002, Auckland, New Zealand, pages 13-18.

- *A Scientific Visualization Schema Incorporating Perceptual Concepts*. Burkhard Wünsche and Richard Lobb, Proceedings of IVCNZ '01, 26-28 November 2001, Dunedin, New Zealand, pages 31-36.

- *The Visualization of Diffusion Tensor Fields in the Brain*. Burkhard Wünsche and Richard Lobb, Proceedings of the International Conference on Mathematics and Engineering Techniques in Medicine and Biological Science, METMBS '01, CSREA Press, Las Vegas, Nevada, USA, June 25-28 2001, pages 498-504.

- *A Toolkit for the Visualization of Stress and Strain Tensor Fields in Biological Tissue.* Burkhard Wünsche and Richard Lobb, Proceedings of VIP '99, Sydney, Australia, November 1999, pages 6-15.

- *A Survey and Evaluation of Mesh Reduction Techniques.* Burkhard Wünsche, Proceedings of IVCNZ '98, November 1998, Auckland, New Zealand, pages 393-398.

- *The Visualization of Two-Dimensional Second-Order Tensor Fields.* Burkhard Wünsche, Proceedings of IVCNZ '98, November 1998, Auckland, New Zealand, pages 174-179, (2nd place in the best poster competition).

# APPENDIX B

## *CD*

The accompanying CD has the following content:

- The complete LaTeX source code of this thesis including all images and figures.
- The source code of our visualization toolkit and finite element modelling package.
- Several simple example data sets such as the plate with a hole under an uniaxial load. The medical image data sets used in the case studies are not included for privacy reasons.
- Several VRML files of visualizations of the brain data set.

# APPENDIX C

# *Mathematical Background*

## C.1 Polar Decomposition of a Symmetric 2D Second-Order Tensor

While the eigenvalues and eigenvectors of a three-dimensional symmetric tensor are computed numerically it is possible to compute the eigenvalues and eigenvectors of a two-dimensional symmetric tensor exactly as shown below. We use this technique for the computation of the 2D tensor topology.

Let

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{pmatrix}$$

be a symmetric 2D second-order tensor.

We want to compute the eigenvalues $\lambda_{1,2}$ and eigenvectors $\mathbf{e}_{1,2}$ of $T$. The eigenvalues $\lambda$ of $T$ are the roots of the characteristc polynomial [Fis86]

$$\det(T - \lambda I) = (T_{11} - \lambda)(T_{22} - \lambda) - T_{12}^2 = \lambda^2 - \lambda(T_{11} + T_{22}) + T_{11}T_{22} - T_{12}^2$$

which are

$$\begin{aligned} \lambda_{1,2} &= \frac{T_{11} + T_{22}}{2} \pm \sqrt{\left(\frac{T_{11} + T_{22}}{2}\right)^2 - T_{11}T_{22} + T_{12}^2} \\ &= \frac{T_{11} + T_{22}}{2} \pm \sqrt{\left(\frac{T_{11} - T_{22}}{2}\right)^2 + T_{12}^2} \end{aligned}$$

Setting

$$\begin{aligned} a &= \frac{1}{2}(T_{11} + T_{22}) \\ b &= \frac{1}{2}(T_{11} - T_{22}) \\ c &= \sqrt{b^2 + T_{12}^2} \end{aligned}$$

we obtain $\lambda_{1,2} = a \pm c$.

It remains to compute the eigenvectors. Note that an eigenvector multiplied with a scalar is again an eigenvector. Therefore it is sufficient to compute the angle $\phi \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$ of the eigenvector $\mathbf{e}_1$ with the x-axis. Since $\mathbf{e}_2$ must be orthogonal to $\mathbf{e}_1$ we obtain

$$\mathbf{e}_1 = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \qquad \mathbf{e}_2 = \begin{pmatrix} \sin\phi \\ -\cos\phi \end{pmatrix}$$

Assuming that $c \neq 0$ we can use the fact that

$$T = S^T \Lambda S$$

where $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2)$ and $S = (\mathbf{e}_1, \mathbf{e}_2)$ [Fis86] and we obtain

$$\begin{aligned} \begin{pmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{pmatrix} &= \begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{pmatrix} \\ &= \begin{pmatrix} \lambda_1\cos^2\phi + \lambda_2\sin^2\phi & \lambda_1\sin\phi\cos\phi - \lambda_2\sin\phi\cos\phi \\ \lambda_1\sin\phi\cos\phi - \lambda_2\sin\phi\cos\phi & \lambda_1\sin^2\phi + \lambda_2\cos^2\phi \end{pmatrix} \end{aligned}$$

This yields the equations

$$\begin{aligned} T_{11} &= \lambda_1\cos^2\phi + \lambda_2\sin^2\phi && \text{(C.1)} \\ T_{12} &= (\lambda_1 - \lambda_2)\sin\phi\cos\phi && \text{(C.2)} \\ T_{22} &= \lambda_1\sin^2\phi + \lambda_2\cos^2\phi && \text{(C.3)} \end{aligned}$$

Subtracting equation C.3 from equation C.1 gives

$$(\lambda_1 - \lambda_2)(cos^2\phi - \sin^2\phi) = T_{11} - T_{22} \tag{C.4}$$

Using this result and equation C.2 we derive

$$\frac{2T_{12}}{T_{11} - T_{22}} = \frac{2\sin\phi\cos\phi}{\cos^2\phi - \sin^2\phi} = \frac{2\frac{\sin\phi}{\cos\phi}}{1 - \frac{\sin^2\phi}{\cos^2\phi}} = \frac{2\tan\phi}{1 - \tan^2\phi} = \tan 2\phi$$

and hence

$$\phi = \frac{1}{2}\arctan\left(\frac{T_{12}}{\frac{1}{2}(T_{11} - T_{22})}\right) = \frac{1}{2}\arctan\left(\frac{T_{12}}{b}\right)$$

Note the usage of $\arctan(\frac{x}{y}) = \arctan(y, x)$ which gives the arc tangent of $\frac{x}{y}$ taking into account in which quadrant the point $(x, y)$ is, i.e., let $z = \frac{x}{y}$ then

$$\arctan\left(\frac{x}{y}\right) = \begin{cases} \arctan(z) & \text{if } y >= 0 \\ \arctan(z) + \pi & \text{if } y < 0 \end{cases}$$

In order to see that $2\phi = \arctan(b, T_{12})$ gives the correct result consider equation C.4. We chose $\lambda_1 = b + c$ and $\lambda_2 = b - c$, hence $\lambda_1 - \lambda_2 = 2c > 0$. Also note that $T_{11} - T_{22} = 2d$ and $\cos^2\phi - \sin^2\phi = 1 - 2\sin^2\phi = \cos 2\phi$. We see that in order for equation C.4 to be true we require

$$-\frac{\pi}{2} \le \phi \le \frac{\pi}{2} \quad \text{if} \quad d \ge 0 \qquad \text{and}$$
$$\frac{\pi}{2} \le \phi \le \frac{3\pi}{2} \quad \text{if} \quad d < 0$$

which is fulfilled by the definition of $\arctan(y, x)$.

## C.2 Cylindrical Coordinates

Section 2.2 introduced the concept of curvilinear coordinates. In this section we illustrate the concepts and principles presented previously by using as an example cylindrical coordinates. Cylindrical coordinates are popular in fluid dynamics and are therefore frequently used in bioengineering, e.g., for modelling blood flow.

Cylindrical coordinates $(r, \theta, z)$ are obtained from rectangular Cartesian (world) coordinates $(x, y, z)$ by using the transformation

$$r = \sqrt{x^2 + y^2}\ , \qquad \theta = \arccos\frac{x}{\sqrt{x^2+y^2}} \left( = \arcsin\frac{y}{\sqrt{x^2+y^2}} \right)\ , \qquad z = z \qquad \text{(C.5)}$$

where arccos is chosen such that a unique $\theta$ in $0 \le \theta \le 2\pi$ exists such that $\cos\theta = x/\sqrt{x^2+y^2}$ and $\sin\theta = y/\sqrt{x^2+y^2}$ .

Figure C.1 shows an example of a point $P$ in world coordinates $(x, y, z)$ and cylindrical coordinates $(r, \theta, z)$. Using the notation from section 2.2 the Cartesian coordinates $(x, y, z)$ correspond to $(r_1, r_2, r_3)$ and the cylindrical coordinates $(r, \theta, z)$ correspond to $(q_1, q_2, q_3)$.

The Jacobian of the tranformation C.5 is[1]

$$\mathbf{J} = \frac{\partial(r,\theta,z)}{\partial(x,y,z)} = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} & \frac{\partial r}{\partial z} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} & \frac{\partial \theta}{\partial z} \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} & \frac{\partial z}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} & 0 \\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and its determinant is

$$|\mathbf{J}| = \frac{1}{r}$$

Thus the inverse transformation exists at all points except of the origin and is given by

$$x = r\cos\theta\ , \quad y = r\sin\theta\ , \quad z = z$$

[1]Good online sources for differentation formulas and other mathematical concepts are [Unia] and [Wei].

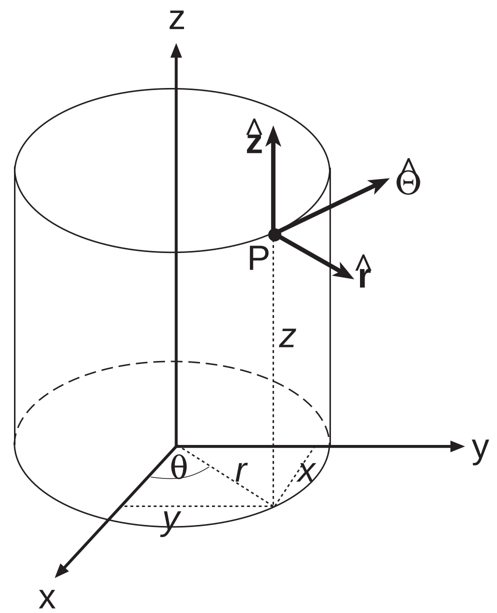


**Figure C.1.** *A point $P$ in world coordinates $(x, y, z)$ and cylindrical coordinates $(r, \theta, z)$ together with its unitary basis $\{\hat{\mathbf{r}}, \hat{\theta}, \hat{\mathbf{z}}\}$.*

The Jacobian of the inverse transformation is

$$\mathbf{J}^{-1} = \frac{\partial(x,y,z)}{\partial(r,\theta,z)} = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It can be easily seen that $|\mathbf{J}^{-1}| = r$ and that the product of the two Jacobians is 1.

The coordinate curves of the cylindrical coordinate system are obtained by varying one coordinate and fixing the other two. The coordinate curves for $r$, $\theta$ and $z$ are

$$\begin{pmatrix} r\cos\theta_c \\ r\sin\theta_c \\ z_c \end{pmatrix}, \quad \begin{pmatrix} r_c\cos\theta \\ r_c\sin\theta \\ z_c \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} r_c\cos\theta_c \\ r_c\sin\theta_c \\ z \end{pmatrix}$$

and have a radial, circumferential and vertical direction, respectively. The subscript "$c$" indicates that the corresponding coordinate is fixed.

In order to determine the unitary basis for the cylindrical coordinate system consider a point $P = P(r, \theta, z)$. Using equation 2.7 the local basis vectors are computed as

$$\hat{\mathbf{r}} \;=\; \frac{\partial \mathbf{p}}{\partial r} = \frac{\partial x}{\partial r}\mathbf{e}_1 + \frac{\partial y}{\partial r}\mathbf{e}_2 + \frac{\partial z}{\partial r}\mathbf{e}_3 = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$\hat{\theta} = \frac{\partial \mathbf{p}}{\partial \theta} = \frac{\partial x}{\partial \theta}\mathbf{e}_1 + \frac{\partial y}{\partial \theta}\mathbf{e}_2 + \frac{\partial z}{\partial \theta}\mathbf{e}_3 = \begin{pmatrix} -r\sin\theta \\ r\cos\theta \\ 0 \end{pmatrix}$$

$$\hat{\mathbf{z}} = \frac{\partial \mathbf{p}}{\partial z} = \frac{\partial x}{\partial z}\mathbf{e}_1 + \frac{\partial y}{\partial z}\mathbf{e}_2 + \frac{\partial z}{\partial z}\mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and are illustrated in figure C.1.

### C.2.1 Coordinate Transformation of a Vector Quantity

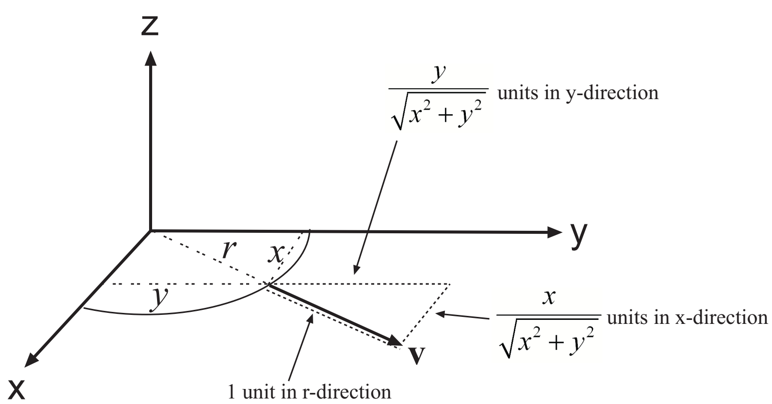


**Figure C.2.** *A vector* $\mathbf{v}$ *in world coordinates* $(x, y, z)$ *and cylindrical coordinates* $(r, \theta, z)$.

Using equation 2.8 it is now possible to express a vector in both coordinate systems. For example, consider the vector

$$\mathbf{v}_{cylindrical\ coordinates} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

in radial direction expressed in cylindrical coordinates. The same vector in world coordinates is

$$\mathbf{v}_{world\ coordinates} = \mathbf{J}^{-1}\mathbf{v}_{cylindrical\ coordinates} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} \\ \frac{y}{\sqrt{x^2+y^2}} \\ 0 \end{pmatrix}$$

The two representations of the vector are illustrated in figure C.2.

# APPENDIX D

# *The Finite Element Method - Examples*

Subsection 2.4.5 gave previously an overview of the FEM solution process. In summary the steps are:

**Step 1:** Choose element basis functions and discretize the domain.

**Step 2:** Create Galerkin Residual equations (obtained by rearranging the governing equations such that their right hand side is zero).

**Step 3:** Use the divergence theorem to create an integral equation involving the internal unknowns and the boundary normal derivatives (e.g., internal forces and external loads).

**Step 4:** Apply the finite element approximation to the integral equation and substitute the element trial solutions and weighting functions.

**Step 5:** Evaluate the element systems.

**Step 6:** Assemble the element systems to a global system.

**Step 7:** Insert the boundary conditions.

**Step 8:** Solve the global system.

**Step 9:** Compute the fluxes and other derived quantities.

The solution processes for FE problems usually only differ in the formulation of the Galerkin residual equations and the corresponding integral equations. Hence all steps save for step 2 and 3 proceed in a similar fashion. This appendix demonstrates the solution process using as examples the problems of 2D heat conduction and linear elasticity.

# D.1 2D Heat Conduction

The heat conduction of an object with thermal conductivity $k$ is described by the quasi-harmonic equation

$$-\nabla^T(k\nabla T) = Q$$

which in two dimensions is

$$-\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = Q$$

where $T$ is the temperature and $Q$ the internally generated energy. Given a homogeneous 2D domain $\Omega$ with boundary $\Gamma$ and a given set of temperature and flux boundary condition the 2D heat conduction FE problem is now stated as finding the resulting steady-state temperature distribution $T(\mathbf{x})$ over the domain $\Omega$ [1]. The following computations use the vector form and apply therefore also to the equivalent 3D problems.

### Step 1: Element Basis Functions and Discretization of the Domain

Figure D.1 shows the domain $\Omega$ of a 2D heat conduction problem approximated using six nodes and two bilinear finite elements.

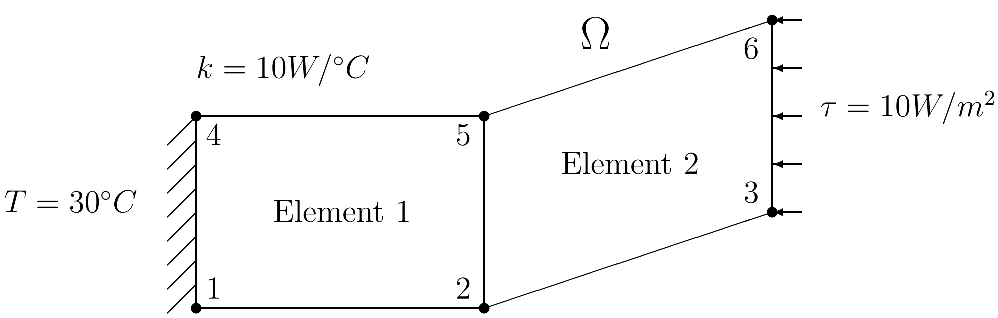


**Figure D.1.** *Mathematical description of the 2D heat conduction problem.*

[1] The program on the accompanying CD contains a 2D and a 3D implementation of the slightly more general governing equation

$$-\frac{\partial}{\partial x}\left(\alpha_x\frac{\partial u(x,y)}{\partial x}\right) - \frac{\partial}{\partial x}\left(\alpha_y\frac{\partial u(x,y)}{\partial x}\right) + \beta u(x,y) = h(x,y) \tag{D.1}$$

or in vector form

$$-\nabla^T\left(\boldsymbol{\alpha}\nabla u\right) + \beta u = h \tag{D.2}$$

The equation is sometimes referred to as the *Sturm-Liouville boundary problem* and is also used to describe 2D electrostatic fields, 2D flows of incompressible inviscid fluid, and 3D convection problems [Bur87].

**Step 2: Create Galerkin Residual Equations**

In order to keep with our general notation for the unknown variable we denote the temperature distribution $T$ with $u(\mathbf{x})$ and the internally generated heat $Q$ with $h(\mathbf{x})$. The governing equation is rewritten by replacing $u$ with the FE approximation function $\tilde{u}$, i.e.,

$$-\frac{\partial}{\partial x}\left(k\frac{\partial \tilde{u}}{\partial x}\right) - \frac{\partial}{\partial y}\left(k\frac{\partial \tilde{u}}{\partial y}\right) = h \tag{D.3}$$

and in vector form

$$-\nabla^T(k\nabla\tilde{u}) = h \tag{D.4}$$

Using this equation the *Galerkin residual*, which is a measure of the error in our solution, is given by

$$R = R(\mathbf{x}) = -\nabla^T\left(k\nabla\tilde{u}\right) - h \tag{D.5}$$

The residual is weighted using a weighting function $\omega = \omega(\mathbf{x})$ and is integrated over the domain, to obtain a total residual which, ideally, should be zero, i.e.,

$$\int_\Omega R\omega d\Omega = 0 \tag{D.6}$$

By substituting equation D.5 this equation becomes

$$\int_\Omega \left(-\nabla^T\left(k\nabla\tilde{u}\right) - h\right)\omega d\Omega = 0 \tag{D.7}$$

For the *Galerkin FEM* the weighting function will be replaced in the FE approximation step (step 4) with each of the basis functions in turn, yielding a set of equations. Solving that set of equations yields a solution for $\tilde{u}(\mathbf{x})$ in which the error is minimal in an integral sense with respect to all trial functions of the form D.10 [BP91, Pro96].

**Step 3: Divergence Theorem**

Using the product rule

$$\nabla^T\left(\nabla v w\right) = \nabla^T\nabla v w + (\nabla v)^T\nabla w$$

rearranging it

$$-\nabla^T\nabla v w = -\nabla^T\left(\nabla v w\right) + (\nabla v)^T\nabla w$$

and applying it to equation D.7 yields

$$-\int_\Omega \nabla^T\left(k\nabla\tilde{u}\omega\right)d\Omega + \int_\Omega \left(\left(k\nabla\tilde{u}\right)^T\nabla\omega - h\omega\right)d\Omega = 0 \tag{D.8}$$

The first integral is transformed by the Gauss-Green theorem (divergence theorem, e.g., [Heu81]) as

$$-\int_\Omega \nabla^T\left(k\nabla\tilde{u}\omega\right)d\Omega \quad = \quad \oint_\Gamma \left(-k\nabla\tilde{u}\omega\right)^T\mathbf{n}d\Gamma$$

$$
\begin{aligned}
&= \oint_\Gamma (-k\nabla\tilde{u})^T \mathbf{n}\omega d\Gamma \\
&= \oint_\Gamma \boldsymbol{\tau}^T \mathbf{n}\omega d\Gamma \\
&= \oint_\Gamma \tau_{\mathbf{n}}\omega d\Gamma \\
&= -\oint_\Gamma \tau_{-\mathbf{n}}\omega d\Gamma
\end{aligned}
$$

so that equation D.8 becomes

$$\int_\Omega (k\nabla\tilde{u})^T \nabla\omega = \int_\Omega h\omega d\Omega + \oint_\Gamma \tau_{-\mathbf{n}}\omega d\Gamma \tag{D.9}$$

Here $\mathbf{n} = (n_x, n_y)^T$ is the outward normal of the element boundary $\Gamma$, $\oint d\Gamma$ is the integral around the element boundary in counter-clockwise direction, $\boldsymbol{\tau} = -k\nabla\tilde{u}$ is the flux, and $\tau_n = \tau_x n_x + \tau_y n_y$ is the boundary flux in the normal direction. Since the heat flux is usually specified as going into the domain (i.e., energy is added) the inward normal $-\mathbf{n}$ is used.

**Step 4: Substitute Element Trial Solutions and Weighting Functions**

For each element $e$ the residual equation D.9 has to be fulfilled by an *element trial solution* $\tilde{u}^{(e)}$ of the form

$$\tilde{u}^{(e)} = \tilde{u}^{(e)}(\mathbf{x}) = \sum_{j=1}^{n} u_j^{(e)}\phi_j(\mathbf{x}) \tag{D.10}$$

where $\phi_j$ are the element basis functions (see subsection 2.4.2).

The element equations of the Galerkin FEM are then formed by substituting the element trial solution D.10 into the residual equation D.9 and by replacing the weighting function with each element basis function, i.e.,

$$
\begin{aligned}
&\sum_{j=1}^{n}\left(\iint^{(e)}\left[k\frac{\partial\phi_j}{\partial x}\frac{\partial\phi_i}{\partial x} + k\frac{\partial\phi_j}{\partial y}\frac{\partial\phi_i}{\partial y}\right]dxdy\right)u_j^{(e)} \\
&= \iint^{(e)} h\phi_i dxdy + \oint^{(e)} \tau_{-\mathbf{n}}^{(e)}\phi_i ds \qquad i = 1,\ldots,n
\end{aligned}
\tag{D.11}
$$

or in matrix form

$$\mathbf{K}^{(e)}\mathbf{u}^{(e)} = \mathbf{f}^{(e)} \tag{D.12}$$

where $\mathbf{K}^{(e)}$ is the *element stiffness matrix* with components

$$K_{ij}^{(e)} = \iint^{(e)}\left(k\frac{\partial\phi_j}{\partial x}\frac{\partial\phi_i}{\partial x} + k\frac{\partial\phi_j}{\partial y}\frac{\partial\phi_i}{\partial y}\right)dxdy \tag{D.13}$$

$\mathbf{u}^{(e)} = (u_1^{(e)},\ldots,u_n^{(e)})^T$ is the solution vector, and $\mathbf{f}^{(e)}$ is the *element load vector* with the components

$$f_i^{(e)} = \iint^{(e)} h\phi_i dxdy + \oint^{(e)} \tau_{-\mathbf{n}}^{(e)}\phi_i ds \tag{D.14}$$

**Step 5: Evaluate Element Systems**

In order to evaluate equations D.13–D.14 they are rewritten in terms of the material coordinates $\boldsymbol{\xi} = (\xi, \mu)$ using the substitution rule for multidimensional integration ([Heu81])

$$K_{ij}^{(e)} = \int_0^1\int_0^1 \left( k\frac{\partial \phi_j}{\partial x}\frac{\partial \phi_i}{\partial x} + k\frac{\partial \phi_j}{\partial y}\frac{\partial \phi_i}{\partial y} \right) \left|\det \mathbf{J}^{(e)}\right| d\xi d\mu \tag{D.15}$$

$$f_i^{(e)} = \int_0^1\int_0^1 h\phi_i \left|\det \mathbf{J}^{(e)}\right| d\xi d\mu + \oint^{(e)} \tau_{-\mathbf{n}}^{(e)}\phi_i ds = f_{1,i}^{(e)} + f_{2,i}^{(e)} \tag{D.16}$$

where the Jacobian $\mathbf{J}^{(e)}$ and the partial derivatives with respect to the world coordinates are computed as shown in subsection 2.4.3.

The two dimensional integrals are evaluated by gauss quadrature

$$K_{ij}^{(e)} = \sum_{k=1}^{m}\sum_{l=1}^{m} w_k w_l \left[ \left( k\frac{\partial \phi_j}{\partial x}\frac{\partial \phi_i}{\partial x} + k\frac{\partial \phi_j}{\partial y}\frac{\partial \phi_i}{\partial y} \right) \left|\det \mathbf{J}^{(e)}\right| \right]_{(\xi_k, \mu_l)} \tag{D.17}$$

$$f_{1,i}^{(e)} = \sum_{k=1}^{m}\sum_{l=1}^{m} w_k w_l \left[ h\phi_i \left|\det \mathbf{J}^{(e)}\right| \right]_{(\xi_k, \mu_l)} \tag{D.18}$$

Since bilinear elements and a second degree governing equation was used $m = 2$ gauss points in each dimension are sufficient ([Bur87, page 617]). The gauss points are

$$\xi_1 = \mu_1 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{3}}\right) \quad , \quad \xi_2 = \mu_2 = \frac{1}{2}\left(1 + \frac{1}{\sqrt{3}}\right)$$

It remains to evaluate the boundary flux integral $f_{2,i}^{(e)}$ in equation D.16. Burnett shows [Bur87, page 617ff.] that it is sufficient to consider only element boundaries that are domain boundaries. The flux boundary integral therefore represents the addition of the natural boundary conditions to the global system and is introduced in step 7.

**Step 6: Assembly of the Global System**

The element systems as given by equation D.12 are assembled into a global system

$$\mathbf{Ku} = \mathbf{f} \tag{D.19}$$

by inserting the entries of the element stiffness matrices $\mathbf{K}^{(e)}$ and element load vectors $\mathbf{f}^{(e)}$ into the global stiffness matrix $\mathbf{K}$ and into the global load vector $\mathbf{f}$, respectively. If a node is shared by several elements each element contributes a term to the corresponding position in the system and the terms are added together.

To demonstrate the process consider figure D.1. The global nodes 1,2,4, and 5 are the local nodes 1-4 of element $e_1$ and the global nodes 2,3,5, and 6 are the local nodes 1-4 of element $e_2$. The element equations for element 1 and 2 then have the form

$$\begin{pmatrix} K_{11}^{(e_i)} & K_{12}^{(e_i)} & K_{13}^{(e_i)} & K_{14}^{(e_i)} \\ K_{21}^{(e_i)} & K_{22}^{(e_i)} & K_{23}^{(e_i)} & K_{24}^{(e_i)} \\ K_{31}^{(e_i)} & K_{32}^{(e_i)} & K_{33}^{(e_i)} & K_{34}^{(e_i)} \\ K_{41}^{(e_i)} & K_{42}^{(e_i)} & K_{43}^{(e_i)} & K_{44}^{(e_i)} \end{pmatrix} \begin{pmatrix} u_1^{(e_i)} \\ u_2^{(e_i)} \\ u_3^{(e_i)} \\ u_4^{(e_i)} \end{pmatrix} = \begin{pmatrix} f_1^{(e_i)} \\ f_2^{(e_i)} \\ f_3^{(e_i)} \\ f_4^{(e_i)} \end{pmatrix} \qquad i = 1, 2$$

Assembling these two systems into a global system yields

$$\begin{pmatrix} K_{11}^{(e_1)} & K_{12}^{(e_1)} & & K_{13}^{(e_1)} & K_{14}^{(e_1)} & \\ K_{21}^{(e_1)} & K_{22}^{(e_1)}+K_{11}^{(e_2)} & K_{12}^{(e_2)} & K_{23}^{(e_1)} & K_{24}^{(e_1)}+K_{13}^{(e_2)} & K_{14}^{(e_2)} \\ & K_{21}^{(e_2)} & K_{22}^{(e_2)} & & K_{23}^{(e_2)} & K_{24}^{(e_2)} \\ K_{31}^{(e_1)} & K_{32}^{(e_1)} & & K_{33}^{(e_1)} & K_{34}^{(e_1)} & \\ K_{41}^{(e_1)} & K_{42}^{(e_1)}+K_{31}^{(e_2)} & K_{32}^{(e_2)} & K_{43}^{(e_1)} & K_{44}^{(e_1)}+K_{33}^{(e_2)} & K_{34}^{(e_2)} \\ & K_{41}^{(e_2)} & K_{42}^{(e_2)} & & K_{43}^{(e_2)} & K_{44}^{(e_2)} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} = \begin{pmatrix} f_1^{(e_1)} \\ f_2^{(e_1)}+f_1^{(e_2)} \\ f_2^{(e_2)} \\ f_3^{(e_1)} \\ f_4^{(e_1)}+f_3^{(e_2)} \\ f_4^{(e_2)} \end{pmatrix}$$

### Step 7: Inserting Boundary Conditions

The governing equation D.3 of the heat conduction problem is a partial differential equation of degree two and therefore contains two types of boundary conditions (BCs): *Essential boundary conditions* specify $\tilde{u}(\mathbf{x})$ along the boundary and *natural boundary conditions* specify the flux $\tau_{-\mathbf{n}}(\mathbf{x})$ normal to the boundary, where

$$\tau_{-\mathbf{n}} = -\boldsymbol{\tau}^T \mathbf{n} = k\frac{\partial u}{\partial x}n_x + k\frac{\partial u}{\partial y}n_y \tag{D.20}$$

In order to get a unique solution one boundary condition must be specified for each boundary node. At least one essential BC must be specified since otherwise the solution is indeterminate with respect to a constant additive term.

It remains to apply the natural and essential boundary conditions to the global system, i.e., the global system must be modified in a way such that the solution fulfills the given boundary conditions. Natural boundary conditions are given as point fluxes and can be directly inserted into the load vector as $f_i = \tau_{-\mathbf{n},i}$.

Essential boundary conditions are applied by inserting them directly into the solution vector. If an essential boundary condition $\hat{u}$ is given for node $k$ then the solution vector must fulfill $u_k = \hat{u}$. To achieve this set $K_{kk} = 1$ and $K_{ki} = 0\ ,\ \ i = 1, \ldots, n,\ i \neq k$ in the global stiffness matrix and set $f_k = \hat{u}$ in the global load vector. In order to keep the global stiffness matrix symmetric set $K_{ik} = 0\ ,\ \ i = 1, \ldots, n,\ i \neq k$ and subtract $\hat{u}K_{ik}$ from $f_i$ for $i = 1, \ldots, n,\ i \neq k$.

Again the process is best demonstrated by an example. Below we show the modifications to a global system with three nodes and the essential boundary condition $u_2 = \hat{u}$.

$$\begin{pmatrix} K_{1,1} & K_{1,2} & K_{1,3} \\ K_{2,1} & K_{2,2} & K_{2,3} \\ K_{3,1} & K_{3,2} & K_{3,3} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ 0 & 1 & 0 \\ K_{31} & K_{32} & K_{33} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} f_1 \\ \hat{u} \\ f_3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} K_{1,1} & 0 & K_{1,3} \\ 0 & 1 & 0 \\ K_{3,1} & 0 & K_{3,3} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} f_1 - \hat{u}K_{12} \\ \hat{u} \\ f_3 - \hat{u}K_{32} \end{pmatrix}$$

### Step 8: Solving the Global System

Since basis functions are local to elements the global system of the FEM is usually sparse, i.e., most of the matrix elements are zero. Furthermore the global matrix usually has a band structure (depending on the chosen numbering of nodes) so that the global system can be efficiently solved using band matrix solvers [Bur87]. Alternative methods employed in FE problems include the preconditioned conjugate gradient method, Cholesky decomposition and iterative methods such as the Gauss-Seidel iteration [BP91].

### Step 9: Computing Fluxes

In many applications the boundary nodal fluxes resulting from the solution to the global system are of special interest. The easiest way to compute them is as

$$\tilde{\mathbf{f}} = \mathbf{K}\mathbf{u} \tag{D.21}$$

where $\mathbf{u}$ is the solution vector resulting from solving the global system after insertion of the boundary conditions and $\mathbf{K}$ is the global stiffness matrix before insertion of the boundary conditions as in equation D.19. The boundary nodal fluxes are then the components of $\tilde{\mathbf{f}}$ corresponding to boundary nodes, i.e., $\tau_{-\mathbf{n},i} = f_i$ if $i$ is the index of a boundary node [HP02]. An alternative way to compute the normal fluxes is by averaging the element fluxes [Bur87, page 620].

# D.2 Linear Elasticity

The theory of linear elasticity is used to predict the deformation of a (nearly) rigid body under applied loads. An introduction to the topic was given in chapter 2. This section explains the steps 2-4 of the FE modelling of a linear elastic solid. The other solution steps are analogous to the ones for the 2D heat conduction problem (see section D.1).

### Step 2: Create Galerkin Residual Equations

The governing equation for the problem of a $d$-dimensional linear elastic solid under an applied load was given by equation 2.12 as

$$\sum_{j=1}^{d} \frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0 \qquad \text{i=1,...,d}$$

or in matrix form

$$\nabla^T \boldsymbol{\sigma} + \mathbf{f} = \mathbf{0}$$

where, for $d = 3$, $\mathbf{u}^T = (u_x, u_y, u_z)$ is the displacement vector, $\mathbf{f}^T = (f_x, f_y, f_z)$ is the internal load vector,

$$\boldsymbol{\sigma}^T = \begin{pmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \sigma_{xy} & \sigma_{yz} & \sigma_{xz} \end{pmatrix}$$

is the stress vector and

$$\nabla^T = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix}$$

is the generalized divergence operator.

The residual $\mathbf{R}$ is then defined as

$$\mathbf{R} = \mathbf{R}(\mathbf{x}) = \nabla^T \boldsymbol{\sigma} + \mathbf{f} = \mathbf{0}$$

which multiplied by a weighting function and integrated gives the residual function

$$\int_\Omega \mathbf{R} \omega d\Omega = \int_\Omega (\nabla^T \boldsymbol{\sigma} + \mathbf{f}) \omega d\Omega = 0 \tag{D.22}$$

### Step 3: Divergence Theorem

Rearranging terms and applying the product rule yields

$$\int_\Omega \nabla^T \omega \boldsymbol{\sigma} d\Omega = \int_\Omega \nabla^T (\boldsymbol{\sigma} \omega) d\Omega + \int_\Omega \mathbf{f} \omega d\Omega$$

Using the following generalization of the divergence theorem[2]

$$\int_\Omega \nabla^T (\boldsymbol{\sigma} \omega) d\Omega = \int_\Gamma (\boldsymbol{\sigma} \omega)^T \mathbf{n} d\Gamma \tag{D.23}$$

[2] A proof of equation D.23 is given in section D.3.

with the matrix form of $\boldsymbol{\sigma}$ on the right-hand side gives

$$\begin{aligned}\int_\Omega \nabla^T \omega \boldsymbol{\sigma} d\Omega &= \int_\Gamma (\boldsymbol{\sigma}\omega)^T \mathbf{n} d\Gamma + \int_\Omega \mathbf{f}\omega d\Omega \\ &= \int_\Gamma \boldsymbol{\sigma}^T \mathbf{n}\omega d\Gamma + \int_\Omega \mathbf{f}\omega d\Omega \\ &= \int_\Gamma \boldsymbol{\tau}_\mathbf{n} \omega d\Gamma + \int_\Omega \mathbf{f}\omega d\Omega \qquad \text{(D.24)}\end{aligned}$$

where $\Gamma$ is the boundary of the domain $\Omega$, $\mathbf{n}$ is its normal, $\nabla$ is the generalized gradient operator, which is adjoint to the generalized divergence operator $\nabla^T$, and

$$\boldsymbol{\tau}_\mathbf{n} = \boldsymbol{\sigma}^T\mathbf{n} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx}n_x + \sigma_{xy}n_y + \sigma_{xz}n_z \\ \sigma_{xy}n_x + \sigma_{yy}n_y + \sigma_{yz}n_z \\ \sigma_{xz}n_x + \sigma_{yz}n_y + \sigma_{zz}n_z \end{pmatrix}$$

is the boundary normal stress or *traction*.

### Step 4: Substitute Element Trial Solutions

For each element the solution for the unknown displacement field has the form

$$\tilde{\mathbf{u}}^{(e)} = \sum_{j=1}^{n} \mathbf{u}_j^{(e)} \phi_j$$

The strain vector hence is

$$\begin{aligned}\boldsymbol{\epsilon}^{(e)} &= \begin{pmatrix} \epsilon_{xx}^{(e)} \\ \epsilon_{yy}^{(e)} \\ \epsilon_{zz}^{(e)} \\ \epsilon_{xy}^{(e)} \\ \epsilon_{yz}^{(e)} \\ \epsilon_{xz}^{(e)} \end{pmatrix} = \begin{pmatrix} \frac{\partial \tilde{u}_x^{(e)}}{\partial x} \\ \frac{\partial \tilde{u}_y^{(e)}}{\partial y} \\ \frac{\partial \tilde{u}_z^{(e)}}{\partial z} \\ \frac{1}{2}\left(\frac{\partial \tilde{u}_x^{(e)}}{\partial y} + \frac{\partial \tilde{u}_y^{(e)}}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial \tilde{u}_y^{(e)}}{\partial z} + \frac{\partial \tilde{u}_z^{(e)}}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial \tilde{u}_x^{(e)}}{\partial z} + \frac{\partial \tilde{u}_z^{(e)}}{\partial x}\right) \end{pmatrix} \\ &= \begin{pmatrix} \frac{\partial \phi_1}{\partial x} & 0 & 0 & \cdots & \frac{\partial \phi_n}{\partial x} & 0 & 0 \\ 0 & \frac{\partial \phi_1}{\partial y} & 0 & \cdots & 0 & \frac{\partial \phi_n}{\partial y} & 0 \\ 0 & 0 & \frac{\partial \phi_1}{\partial z} & \cdots & 0 & 0 & \frac{\partial \phi_n}{\partial z} \\ \frac{1}{2}\frac{\partial \phi_1}{\partial y} & \frac{1}{2}\frac{\partial \phi_1}{\partial x} & 0 & \cdots & \frac{1}{2}\frac{\partial \phi_n}{\partial y} & \frac{1}{2}\frac{\partial \phi_n}{\partial x} & 0 \\ 0 & \frac{1}{2}\frac{\partial \phi_1}{\partial z} & \frac{1}{2}\frac{\partial \phi_1}{\partial y} & \cdots & 0 & \frac{1}{2}\frac{\partial \phi_n}{\partial z} & \frac{1}{2}\frac{\partial \phi_n}{\partial y} \\ \frac{1}{2}\frac{\partial \phi_1}{\partial z} & 0 & \frac{1}{2}\frac{\partial \phi_1}{\partial x} & \cdots & \frac{1}{2}\frac{\partial \phi_n}{\partial z} & 0 & \frac{1}{2}\frac{\partial \phi_n}{\partial x} \end{pmatrix} \begin{pmatrix} u_{1,x}^{(e)} \\ u_{1,y}^{(e)} \\ u_{1,z}^{(e)} \\ \vdots \\ u_{n,x}^{(e)} \\ u_{n,y}^{(e)} \\ u_{n,z}^{(e)} \end{pmatrix} \\ &= \mathbf{B}\mathbf{u}^{(e)} \qquad \text{(D.25)}\end{aligned}$$

and substituting the expression into the constitutive relation 2.15 yields[3]

$$\boldsymbol{\sigma}^{(e)} = \mathbf{C}\mathbf{B}\mathbf{u}^{(e)} \qquad \text{(D.26)}$$

[3] Note that if the material is inhomogeneous the constitutive matrix $\mathbf{C}$ is not constant, i.e., $\mathbf{C} = \mathbf{C}(\mathbf{x}) = \mathbf{C}^{(e)}(\boldsymbol{\xi})$.

Rewriting equation D.24 $n$ times with the weighting function replaced by the basis functions $\phi_i$ gives

$$\int_{\Omega^{(e)}} \mathbf{B}^T \boldsymbol{\sigma}^{(e)} d\Omega = \int_{\Gamma^{(e)}} \boldsymbol{\Phi}^T \boldsymbol{\tau}_{\mathbf{n}}^{(e)} d\Gamma + \int_{\Omega^{(e)}} \boldsymbol{\Phi}^T \mathbf{f}^{(e)} d\Omega$$

where

$$\boldsymbol{\Phi} = \begin{pmatrix} \phi_1 & 0 & 0 & \cdots & \phi_n & 0 & 0 \\ 0 & \phi_1 & 0 & 0 & \cdots & \phi_n & 0 \\ 0 & 0 & \phi_1 & 0 & 0 & \cdots & \phi_n \end{pmatrix}$$

and by inserting equation D.26

$$\int_{\Omega^{(e)}} \mathbf{B}^T \mathbf{C} \mathbf{B} \mathbf{u}^{(e)} d\Omega = \int_{\Gamma^{(e)}} \boldsymbol{\Phi}^T \boldsymbol{\tau}_{\mathbf{n}}^{(e)} d\Gamma + \int_{\Omega^{(e)}} \boldsymbol{\Phi}^T \mathbf{f}^{(e)} d\Omega$$

The resulting $3n$ equations can be written in the usual condensed matrix form

$$\mathbf{K}^{(e)} \mathbf{u}^{(e)} = \mathbf{b}^{(e)}$$

where

$$\mathbf{K}^{(e)} = \int_{\Omega^{(e)}} \mathbf{B}^T \mathbf{C} \mathbf{B} \mathbf{u}^{(e)} d\Omega$$

is the element stiffness matrix and

$$\mathbf{b}^{(e)} = \mathbf{b}_f^{(e)} + \mathbf{b}_\tau^{(e)} = \int_{\Omega^{(e)}} \boldsymbol{\Phi}^T \boldsymbol{\tau}_{\mathbf{n}}^{(e)} d\Gamma + \int_{\Omega^{(e)}} \boldsymbol{\Phi}^T \mathbf{f}^{(e)} d\Omega$$

is the element load vector consisting of the external loads $\mathbf{b}_\tau^{(e)}$ and the internal loads $\mathbf{b}_f^{(e)}$.

Note that the resulting element systems have the same form as equation D.12 in the previous example and can therefore be evaluated and assembled analogously. The essential boundary conditions are now displacement vectors and the natural boundary conditions are surface traction vectors. The global system can be solved resulting into the unknown displacement values at the mesh nodes. Stresses and strain are computed by evaluating equations D.25 and D.26, respectively, using the solution vector for the unknown displacement.

## D.3 Divergence Theorem for a Symmetric Tensor

Claim:

$$\int_\Omega \nabla^T(\boldsymbol{\sigma}\omega)d\Omega = \int_\Gamma (\boldsymbol{\sigma}\omega)^T \mathbf{n} d\Gamma$$

where

$$\nabla^T = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix}$$

and the tensor $\boldsymbol{\sigma}$ is in vector form on the left hand side, i.e.,

$$\boldsymbol{\sigma}^T = \begin{pmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \sigma_{xy} & \sigma_{yz} & \sigma_{xz} \end{pmatrix}$$

and in matrix form on the right-hand side, i.e.,

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}$$

Proof:

Without loss of generality omit $\omega$ (otherwise define $\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}\omega$). Using the divergence theorem (Green-Gauss theorem) [Heu81, p.522]

$$\int_\Omega \nabla^T \mathbf{F} d\Omega = \int_\Gamma \mathbf{F}^T \mathbf{n} d\Gamma$$

which in component form in 3D is

$$\int_\Omega \left( \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) d\Omega = \int_\Gamma \left( F_x n_x + F_y n_y + F_z n_z \right) d\Gamma$$

gives

$$\begin{aligned}
\int_\Omega \nabla^T \boldsymbol{\sigma} d\Omega &= \int_\Omega \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{pmatrix} d\Omega \\
&= \int_\Omega \begin{pmatrix} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \\ \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} \\ \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial x} \end{pmatrix} d\Omega \\
&= \int_\Gamma \begin{pmatrix} \sigma_{xx} n_x + \sigma_{xy} n_y + \sigma_{xz} n_z \\ \sigma_{xy} n_x + \sigma_{yy} n_y + \sigma_{yz} n_z \\ \sigma_{xz} n_x + \sigma_{yz} n_y + \sigma_{zz} n_z \end{pmatrix} d\Gamma
\end{aligned}$$

$$
\begin{aligned}
&= \int_\Gamma \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} d\Gamma \\
&= \int_\Gamma \boldsymbol{\sigma}^T \mathbf{n} d\Gamma
\end{aligned}
$$

This completes the proof.

# APPENDIX E

# *Abbreviations*

**A-V** atrioventricular

**AMLIC** anisotropy modulated line integral convolution

**AS** aortic stenosis

**AVS** Application Visualization System [AVS]

**BPI** blood pool imaging

**CAD** computer aided design

**CAM** computer aided manufacturing

**CCD** charge-coupled devices

**CFD** computational fluid dynamics

**CHF** congestive heart failure

**CIE** Commission International de l'Eclairage [=International Commision on Illumination] (also used as a name of a common colour space)

**CSF** cerebral spinal fluid

**CSPAMM** complementary spatial modulation of magnetization (tagged MRI pulse sequence) [SSF+99]

**CT** computed tomography

**DANTE** delays alternating with nutations for tailored excitation (MRI pulse sequence) [MCD+96]

**DDA** digital differential analysis

**DENSE** fast displacement encoding with stimulated echoes (MRI pulse sequence) [ABW99]

**DNA** deoxyribonucleic acid

**DTI** diffusion tensor imaging

**DVR** direct volume rendering

**DWI** diffusion-weighted (MRI) imaging

**EBCT** electron beam computed tomography [POCD99]

**ECG** electrocardiogram/echocardiography

**ED** end-diastole

**EEG** electroencephalograph/electroencephalography

**EF** ejection fraction

**EKG** electrocardiogram

**ES** end-systole

**FE** finite element

**FEA** finite element analysis

**FEM** finite element method

**FIESTA** fast imaging employing steady-state acquisition (MRI pulse sequence) [LSM+02]

**FLASH** fast low flip-angle shot (MRI pulse sequence) [MCD+96]

**FLIC** fast line integral convolution [SH95]

**FLTK** Fast Light Toolkit [Spi]

**fMRI** functional magnetic resonance imaging

**FROLIC** fast rendering of oriented line integral convolution [WG97]

**GLU** OpenGL utility library

**GLUT** OpenGL utility toolkit

**GUI** graphical user interface

**HARP** harmonic phase (MRI pulse sequence) [OKMP99]

**HLS** hue-lightness-saturation (colour space)

**HSB** hue-saturation-brightness (colour space)

**HSV** hue-saturation-value (colour space)

**IDL** Interactive Data Language [Res]

**LA** long-axis

**LV** left ventricle/left ventricular

**LAD** left anterior descending coronary artery

**LIC** line integral convolution [CL93]

**LVH** left ventricular hypertrophy

**MI** myocardial infarction

**MEG** magnetoencephalograph/magnetoencephalography

**MIP** maximum intensity projection

**MPI** myocardial perfusion imaging

**MR** magnetic resonance

**MRA** magnetic resonance angiography

**MRI** magnetic resonance imaging

**MRS** magnetic resonance spectroscopy [TM01]

**NURBS** non-uniform rational B-Spline

**OCT** optical coherence tomography

**ODE** ordinary differential equation

**OLIC** oriented line integral convolution [WG97]

**PC-MRI** phase-contrast MRI

**PACS** picture archiving and communication system

**PCA** phase-contrast angiography

**PD** Parkinson disease

**PET** positron emission tomography

**PLIC** pseudo line integral convolution [VKP99]

**PVL** periventricular leukomalacia

**RGB** red-green-blue (colour space)

**RNA** ribonucleic acid

**RV** right ventricle/right ventricular

**SA** short-axis

**S-A** sino-atrial

**SD** standard deviation

**SPAMM** spatial modulation of magnetization (tagged MRI pulse sequence) [YICA94]

**SPECT** single-photon emission computed tomography

**SRI** strain rate imaging

**SV** stroke volume

**SVC** superior vena cava

**SVCS** superior vena cava syndrome

**SVD** singular value decomposition

**TEE** transesophogeal echocardiography

**UFLIC** unsteady flow line integral convolution [SK97b, SK98]

**UML** unified modeling language

**US** ultrasonography

**VEC-MRI** velocity-encoded cine MRI

**VR** Virtual Reality

**VRML** Virtual Reality Modelling Language

**VTK** Visualization Toolkit [Kit]

**XML** extended markup language

# APPENDIX F

# *Glossary*

This appendix contains an explanation of technical terms used in this thesis which might not be known to the reader. Since we assume that the reader is not a medical specialist this glossary contains mostly medical terms though some engineering and mathematical terms are listed where deemed appropriate. Good medical online dictionaries are [Hey95, Har, Web02].

**±2 standard deviation** a deviation of $2\sigma$ from either side of the mean of a standard normal distribution. The area under the Gaussian curve within these boundaries is

$$area = \int_{-2}^{2} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx = 0.954$$

i.e., 95.4% of all data values for a standard normal distribution lie within $2\sigma$ of the mean [Lar82].

**A-V node** see *atrioventricular node*

**aberrant** an atypical group, individual, or structure, especially one with an aberrant chromosome number [McG].

**active contour model** (also *snake*) is an energy-minimizing spline guided by external constraint forces that pull it towards features in a set of images such as lines and edges.

**acoustic impedance** absorption of sound in a medium, equal to the ratio of the sound pressure at a boundary surface to the sound flux through the surface [Enc].

**akinesia** loss or impairment of voluntary activity (as of a muscle) [Web02].

**akinetic** of, relating to, or affected by *akinesia* [Web02].

**anatomy** 1: a branch of morphology that deals with the structure of organisms (comp. *physiology*).
2: the art of separating the parts of an organism in order to ascertain their position, relations, structure, and function.
3: structural makeup especially of an organism or any of its parts [Web02].

**aneurysm** (Gr. *aneurysma* a widening) a sac formed by the dilation of the wall of an artery, a vein, or the heart. The chief signs of arterial aneurysm are the formation of a pulsating tumour, and often a bruit (aneurysmal bruit) heard over the swelling. Sometimes there are symptoms from pressure on contiguous parts [Hey95].

**angiogenesis** formation and differentiation of blood vessels [Web02].

**angiography** 1: the roentgenographic visualization of the blood vessels after injection of a radiopaque substance [Web02].
2: phase-contrast angiography: see *phase-contrast MRI*.

**angiocardiography** an X-ray examination of the heart and great vessels following the injection of a radiopaque contrast medium into a blood vessel or one of the cardiac chambers [Har].

**anterior** towards the abdomial surface of the body (Brain anatomy: towards the face) [EW91].

**aorta** the large arterial trunk that carries blood from the heart to be distributed by branch arteries through the body [Web02].

**apex** a narrowed or pointed end of an anatomical structure: as (a) the narrow somewhat conical upper part of a lung extending into the root; (b) the lower pointed end of the heart situated in humans opposite the space between the cartilages of the fifth and sixth ribs on the left side; (c) the extremity of the root of a tooth [Web02].

**apical** of, relating to, or situated at an apex [Web02].

**artery** any of the tubular branching muscular- and elastic-walled vessels that carry blood from the heart through the body [Web02].

**atheroma** (adj. *atheromatous*) 1: fatty degeneration of the inner coat of the arteries.
2: an abnormal fatty deposit in an artery [Web02].

**atherosclerosis** narrowing of an artery; characterized by *atheromatous* deposits in and fibrosis of the inner layer of arteries [Web02].

**atrioventricular node** a small mass of tissue that is situated in the wall of the right atrium adjacent to the septum between the atria, passes impulses received from the sinoatrial node to the ventricles by way of the bundles of His, and in some pathological states replaces the sinoatrial node as pacemaker of the heart [Web02].

**auscultation** the act of listening to sounds arising within organs (as the lungs or heart) as an aid to diagnosis and treatment [Web02].

**autonomic** 1: (a) acting or occuring involuntary <*autonomic* reflexes> (b) relating to, affecting, or controlled by the *autonomic nervous system* <*autonomic* dysfunction>.
2: having an effect upon tissue supplied by the *autonomic nervous system* <*autonomic* drugs> [Web02].

**autonomic nervous system** (also *vegetative nervous system*) a part of the vertebrate nervous system that innervates smooth and cardiac muscle and glandular tissues and governs involuntary actions (as secretion, vasoconstriction, or peristalsis) and that consists of the *sympathetic nervous system* and the *parasympathetic nervous system* (compare *central nervous system*, *peripheral nervous system*) [Web02].

**axon** a usually long and single nerve-cell process that usually conducts impulses away from the cell body [Web02].

**basal** 1: relating to, situated at, or forming the base.
2: of, or relating, or essential for maintaining the fundamental vital activities of an organism (as respiration, heartbeat, or excretion) [Web02].

**basal ganglion** (also *basal nucleus*) any of four deeply placed masses of gray matter within each cerebral hemisphere comprising the *caudate nucleus*, the lentiform nucleus, the amygdala, and the claustrum - usually used in plural [Web02].

**cardiac** (L. *cardiacus* from Gr. *kardiakos*) pertaining to the heart [Hey95].

**cardiac blood pool imaging** this noninvasive test uses radioactive tracers to delineate the heart's chambers and major vessels. It may be used to detect a heart attack, heart muscle function, and coronary artery disease. The patient receives a radioactive tracer by injection (into a vein) and then the heart is imaged using a gamma camera. The heart is imaged before and after exercise. This test may be used to detect and evaluate atrial septal defect, *congestive heart failure*, *cardiomyopathy*, *Lyme disease* (secondary), *mitral stenosis*, and *superior vena cava syndrome* [Meda].

**cardiac output** the volume of blood ejected from the left side of the heart in one minute - called also *minute volume* [Web02].

**cardiomyopathy** disease of the heart muscle that impairs the ability of the heart to pump [Maya].

**cardiovascular** pertaining to the heart and blood vessels [Hey95].

**caudate nucleus** (also *caudate*) the one of the four *basal ganglia* in each cerebral hemisphere that comprises a mass of gray matter in the corpus striatum, forms part of the floor of the lateral ventricle, and is separated from the lentiform nucleus by the internal capsule [Web02].

**central nervous system** the part of the nervous system which in vertebrates consists of the brain and spinal cord, to which sensory impulses are transmitted and from which motor impulses pass out, and which supervises and coordinates the activity of the entire nervous system (compare *autonomic nervous system*, *peripheral nervous system*) [Web02].

**cerebellum** portion of the brain responsible for coordinating movements [Maya].

**cerebral cortex** the superficial layer of the cerebral hemispheres, composed of gray matter and concerned with coordination of higher nervous activity [McG].

**cerebrum** largest portion of the brain, consisting of two hemispheres; responsible for thinking, feelings and voluntary movement [Maya].

**cholangiopancreatography** technique for imaging the *pancreatobiliary tree* [Mat].

**chorda tendinea** any of the delicate tendinous cords that are attached to the edges of the atrioventricular valves of the heart and to the papillary muscles and serve to prevent the valves from being pushed into the atrium during the ventricular contraction [Web02].

**ciliary ganglion** a *ganglion* which lies directly behind the eye [Unic].

**cine** a series of rapidly recorded multiple images taken at sequential cycles of time and displayed on a monitor in a dynamic movie display format [Fon].

**cognition** 1: *cognitive* mental processes.
2: a conscious intellectual act [Web02].

**cognitive** of, relating to, or being conscious intellectual activity (as thinking, reasoning, remembering, imagining, or learning words) [Web02].

**collagen** (Gr. *kolla* glue + *gennan* to produce) the protein substance of the white fibres (collagenous fibres) of skin, tendon, bone, cartilage, and all other connective tissue; composed of molecules of tropocollagen (q.v.), it is converted into gelatin by boiling [Hey95].

**collagenous** pertaining to collagen; forming or producing collagen [Hey95].

**collateral vessel** can be pre-existing vessels that normally have little or no blood flow. Accute occlusion of normal vessels (e.g., thrombosis of a large artery) can cause a redristribution of pressures within the vascular bed thereby causing blood flow to occur in collateral vessels. Conditions of chronic stress (e.g., endurance exercise training) can cause new blood vessels to form by *angiogenesis* [Kla].

**collateralization** structural development of existing vessels which provide alternative blood supply routes after occlusion of another artery [Unt03].

**computed tomography (CT)** also called CT or CAT scan: X-ray technique that uses a computer to construct images of the body [Maya].

**congenital** (L. *congenitus* born together) existing at, and usually before, birth; referring to conditions that are present at birth, regardless of their causation [Hey95].

**congestion** (L. *congestio*, from *congerere* to heap together) excessive or abnormal accumulation of blood in a part [Hey95].

**congestive heart failure (CHF)** occurs when muscle cells in the heart die or no longer function properly, causing the heart to lose its ability to pump enough blood through the body. Heart failure usually develops gradually, over many years, as the heart becomes less and less efficient. Systolic heart failure occurs when the heart's ability to contract decreases. The heart cannot pump with enough force to push a sufficient amount of blood through the body. Blood coming into the heart from the lungs may back up and cause fluid to leak into the lungs, a condition known as pulmonary congestion. Diastolic heart failure occurs when the heart has difficulty relaxing (it's too efficient) and cannot properly fill with blood because the muscle has become stiff. This may lead to fluid accumulation, especially in the feet, ankles, legs and lungs [Medb].

**constrictive pericarditis** when the *pericardium* is scarred or thickened, the heart has difficulty contracting. This is because the pericardium has shrunken or tightened around the heart, constricting the muscle's heart movement. This usually occurs as a result of tuberculosis, which now is rarely found in the United States, except in immigrant, AIDS, and prison populations [Medb].

**corticospinal tract** any of four columns of motor fibers of which two run on each side of the spinal cord and which are continuations of the pyramids of the medulla oblongata: (a) *lateral corticospinal tract* (b) *ventral corticospinal tract* [Web02].

**cranium** portion of the skull that houses the brain [Maya].

**cytoplasm** the organized complex of inorganic and organic substances external to the nuclear membrane of a cell and including the cytosol and membrane-bound organelles (as mitochondria or chloroplasts) [Web02].

**dendrite** (Gr. *dendron* tree) the highly branched tree-like process of a neuron that serves as a receptive field and conducts impulses toward the cell body [Bru].

**deoxyribonucleic acid (DNA)** the molecule that encodes genetic information. DNA is a double-stranded molecule held together by weak bonds between base pairs of nucleotides. The four nucleotides in DNA contain the bases: adenine (A), guanine (G), cytosine (C), and thymine (T). In nature, base pairs form only between A and T and between G and C; thus the base sequence of each single strand can be deduced from that of its partner [Sch].

**dependent variable** variable defined over the data domain [WLG97].

**diabetes** any of various abnormal conditions characterized by the secretion and excretion of excessive amounts of urine; especially: *diabetes mellitus* [Web02].

**diabetes mellitus** a variable disorder of carbohydrate metabolism caused by a combination of hereditary and environmental factors and usually characterized by inadequate secretion or utilization of insulin, by excessive urine production, by excessive amounts of sugar in the blood and urine, and by thirst, hunger, and loss of weight [Web02].

**diastole** Period during the heart cycle in which the muscle relaxes, followed by contraction (systole). At end-diastole the heart has achieved maximum filling [Maya, GZM97].

**diffusion** 1: the process whereby particles of liquids, gases, or solids intermingle as the result of their spontaneous movement caused by thermal agitation and in dissolved substances move from a region of higher to one of lower concentration. 2 (a) reflection of light by a rough reflecting surface (b) transmission of light through a translucent material [Web02].

**dilated cardiomyopathy** a condition in which the heart's ability to pump blood is reduced because the left ventricle (one of the two pumping chambers of the heart) is enlarged (dilated) [Hea].

**Doppler effect** a change in the frequency with which waves (as sound, light, or radio waves) from a given source reach an observer when the source and the observer are in motion with respect to each other so that the frequency increases or decreases according to the speed at which the distance is decreasing or increasing [Web02].

**dysplasia** (dys- + Gr. *plassein* to form) abnormality of development; in pathology, alteration in size, shape, and organization of adult cells [Hey95].

**echocardiography** ultrasonography applied to the heart.

**edema** an abnormal excess accumulation of *serous* fluid in connective tissue or in a serous cavity [Web02].

**ejection fraction** a measure of ventricular contractility, equal to normally 65.8%; lower values indicate ventricular dysfunction [Hey95].

**electrocardiogram (ECG or EKG)** a graphical record of the variations in electrical potential caused by electrical activity of the heart muscle and detected at the body surface, as a method for studying the action of the heart muscle [Hey95].

**electrocardiography** the making of an *electrocardiogram* [Hey95].

**electroencephalogram** a graphic record of minute changes in the electric potential associated with the activity of the cerebral cortex, as picked up by electrodes placed on the scalp [Har].

**electroencephalography** a method of graphically recording brain wave activity, used to diagnose seizure disorders, brainstem disorders, tumors, or clots [Har].

**embolus** an abnormal particle (as an air bubble) circulating in the blood [Web02].

**endocrine** (endo- + Gr. *krinein* to separate) pertaining to internal secretions; hormonal [Hey95].

**endothelium** cell layer forming the interface between the blood and vessel walls. The endothelium plays a critical role in the mechanics of blood flow, vascular smooth muscle cell growth, and serves as a barrier to the transvascular diffusion of liquids and solutes [URL: http://hsc.virginia.edu/medicine/basic-sci/biomed/ley/endothelium.htm].

**endocarditis** exudative and proliferative inflammatory alterations of the endocardium, characterized by the presence of vegetations on the surface of the endocardium or in the endocardium itself, and most commonly involving a heart valve, but sometimes affecting the inner lining of the cardiac chambers or the endocardium elsewhere. It may occur as a primary disorder or as a complication of or in association with another disease [Hey95].

**endocardium** the thin, inner membrane that lines the heart [Maya].

**enzyme** any of numerous complex proteins that are produced by living cells and catalyze specific biochemical reactions at body temperatures [Web02].

**epicardium** the thin membrane on the surface of the heart [Maya].

**esophagus** a muscular tube that in adult humans is about nine inches (23cm) long and passes from the pharynx down the neck between the trachea and the spinal column and behind the left bronchus where it pierces the diaphragm slightly to the left of the middle line and joins the cardiac end of the stomach [Web02].

**excitation** delivering (inducing, transferring) energy into a “spinning” nuclei via radio-frequency pulse(s), which puts the nuclei into a higher energy state. By producing a net transverse magnetization an MRI system can observe a response from the excited system [Fon].

**fasciculus** (diminutive of Lat. *fascic* = bundle) bundle of nerve or muscle fibers [AB].

**fibrillation** a small, local, involuntary contraction of muscle, invisible under the skin, resulting from spontaneous activation of single muscle cells or muscle fibres [Hey95].

**fibrosis** the formation of *fibrous* tissue; fibroid or fibrous degeneration [Hey95].

**fibrous** 1: containing, consisting of, or resembling fibers <collagen is a *fibrous* protein>.
2: characterized by fibrosis [Web02].

**fluoroscopy** observing the internal structure of opaque objects (as the living body) by means of the shadow cast by the object examined upon a fluorescent screen when placed between the screen and a source of X rays [Web02].

**Fragile X syndrome** (also *Martin-Bell syndrome*, *Marker X syndrome*, *FRAXA syndrome*) is the most common form of inherited mental retardation. Individuals with this condition have developmental delay, variable levels of mental retardation, and behavioral and emotional difficulties. They may also have characteristic physical traits. Generally, males are affected with moderate mental retardation and females with mild mental retardation. Fragile X syndrome is caused by a mutation in the FMR-1 gene, located on the X chromosome [Medb].

**functional imaging** can identify the kinds of molecular structures/ receptors that cover the surface of e.g., a tumor, information that potentially can predict how it may behave and respond to certain treatments. By providing a picture of glucose utilization in tumor cells, imaging can demonstrate without the need for a biopsy how a tumor is responding to a recently administered treatment [CHIa].

**ganglion** a collection of nerve cell bodies outside the central nervous system. They can be associated with the autonomic nervous system, with cranial nerves, or spinal nerves [Unic].

**Gaussian curvature** an intrinsic property of space independent of the coordinate system used to describe it. The Gaussian curvature of a regular surface in $\mathbb{R}^3$ is

$$K = \kappa_1 \kappa_2$$

where $\kappa_1$ and $\kappa_2$ are the *principal curvatures* of the surface [Wei].

**gene expression** the process by which a gene's coded information is converted into the structures present and operating in the cell. Expressed genes include those that are transcribed into mRNA (messenger RNA) and then translated into protein and those that are transcribed into RNA but not translated into protein [Sch].

**heat flux** the flow of heat (transfer of energy from one substance to another as a result of a temperature difference) across a surface of unit area in a unit amount of time; commonly expressed in units of $cal/(cm^2 sec)$ or $W/m^2$ [Har].

**His-Purkinje system** (also *bundle of His, atrioventricular bundle, His bundle*) a slender bundle of modified cardiac muscle that passes from the atrioventricular node in the right atrium to the right and left ventricles by way of the septum and that maintains the normal sequence of the heartbeat by conducting the wave of excitation from the right atrium to the ventricles [Web02].

**hypertension** (hyper- + tension) persistently high arterial blood pressure. Various criteria for its threshold have been suggested, ranging from $140mm$ Hg systolic and $90mm$ Hg diastolic to as high as $200mm$ Hg systolic and $110mm$ Hg diastolic. Hypertension may have no known cause (essential or idiopathic h.) or be associated with other primary diseases (secondary h.) [Hey95].

**hypertensive** marked by a rise in blood pressure: suffering or caused by hypertension [Web02].

**hypertrophic** relating to or affected by *hypertrophy* [Har].

**hypertrophy** (hyper- + Gr. *troph* nutrition) the enlargement or overgrowth of an organ or part due to an increase in size of its constituent cells [Hey95].

**hypokinetic** characterized by, associated with, or caused by decreased motor activity [Web02].

**hypothalamus** a basal part of the forebrain that lies beneath the thalamus on each side, forms the floor of the third ventricle, and includes vital autonomic regulatory centers (as for the control of food intake) [Web02].

**idiopathic** arising spontaneously or from an obscure or unknown cause [Cru].

**independent variable** 1: a variable that is deliberately varied or changed in a controlled manner in an experiment to observe its effects on the response variable; sometimes caused *causal variable* [ERC].
2: a variable representing the data domain [WLG97].

**infarct** an area of tissue that dies because of lack of blood supply [Maya].

**infarction** 1: the process of forming an *infarct*.
2: see *infarct* [Web02].

**inferior** towards the feet (Brain anatomy: towards the base of the skull) [EW91].

**inotropic** relating to or influencing the force of muscular contractions <digitalis is a positive *inotropic* agent> [Web02].

**intima** the innermost coat of an organ (as a blood vessel) consisting usually of an endothelial layer backed by connective tissue and elastic tissue - called also *tunica intima* [Web02].

**in vitro** within a glass; observable in a test tube; in an artificial environment [Hey95].

**in vivo** within the living body [Hey95].

**ischemia** deficiency of blood flow within an organ or part of an organ [Maya].

**ischemic** oxygen-starved [GZM97].

**isoparametric element** finite element for which the basis functions and the geometry functions coincide.

**kinetic** (Gr. *kintikos*) pertaining to or producing motion [Hey95].

**lateral** of or relating to the side; especially of a body part: lying at or extending toward the right or left side: lying away from the median axis of the body [Web02].

**lateral corticospinal tract** (also *crossed pyramidal tract*) a band of nerve fibers that descends in the posterolateral part of each side of the spinal cord and consists mostly of fibers arising in the motor cortex of the contralateral side of the brain and crossing over in the decussation of pyramids with some fibers arising in the motor cortex of the same side [Web02].

**ligament** 1: a tough band of tissue that serves to connect the articular extremeties of bones or to support or retain an organ in place and is usually composed of coarse bundles of dense white fibrous tissue parallel or closely interlaced, pliant, and flexible, but not extensible.
2: any of various folds or bands of membranes connecting parts or organs [Web02].

**Lyme disease** (also called *Lyme*, *Lyme borreliosis*) an acute inflammatory disease that is usually characterized initially by the skin lesion erythema migrans and by fatigue, fever, and chills and if left untreated may later manifest itself in cardiac and neurological disorders, joint pain, and arthritis and that is caused by a spirochete of the genus Borrelia (B. burgdorferi) transmitted by the bite of a tick especially of the genus Ixodes (I. scapularis syn. I. dammini in the eastern and midwestern U.S., I. pacificus especially in some parts of the Pacific coastal states of the U.S., and I. ricinus in Europe) [Web02].

**magnetic haemodynamic effect** see *magnetohydrodynamic effect*.

**magnetic resonance imaging (MRI)** a noninvasive diagnostic technique that produces computerized images of internal body tissues and is based on nuclear magnetic resonance of atoms within the body induced by the application of radio waves [Web02].

**magnetic resonance spectroscopy (MRS)** provides a measure of metabolic differences in various brain areas, as in detecting foci of acute cerebral ischemia and stroke [TM01].

**magnetohydrodynamic effect** (also *magnetic haemodynamic effect*) generation of an electric current when a conducting fluid (blood) moves in a magnetic field. It causes an elevation of the T wave of a patient's ECG during MRI [SB99, WK98].

**magnetoencephalograph** an instrument that records magnetic signals proportional to electroencephalographic waves emanating from electrical activity in the brain [Har].

**magnetoencephalography** the making of an *magnetoencephalograph* [Har].

**mean curvature** defined as

$$H = \frac{1}{2}(\kappa_1 + \kappa_2)$$

where $K = \kappa_1$ and $\kappa_2$ are the *principal curvatures* [Wei].

**metabolic** of, relating to, or based on metabolism [Web02].

**metabolism** the sum of the processes in the builtup and destruction of protoplasm; specifically: the chemical changes in living cells by which energy is provided for vital processes and activities and new material is assimilated [Web02].

**metastasis** change of position, state, or form: as
1: a secondary metastatic growth of a malignant tumor.
2: transfer of a disease-producing agency (as cancer cells or bacteria) from an original site of disease to another part of the body with development of a similar lesion in the new location [Web02].

**mitral stenosis** a condition usually the result of disease in which the mitral valve is abnormally low [Web02].

**molecular imaging** involves the use of molecular probes or tracers. Molecular Imaging fuses the disciplines of molecular biology, genetic engineering, immunology, cytology, and biochemistry with imaging. Advances in MRI/MRS, MR microscopy, cellular tags, PET and SPECT are used to evaluate normal and abnormal tissue metabolism and perfusion in response to genetic, physiological, or therapeutic challenges [CHIa].

**multiple sclerosis** a demyelinating disease marked by patches of hardened tissue in the brain or the spinal cord and associated especially with partial or complete paralysis and jerking muscle tremor [Web02].

**myelin** (also *white substance of Schwann*) the substance of the cell membrane of Schwann's cells that coils to form the myelin sheath [Har].

**myelin sheath** a layer of fatty material (*myelin*) that surrounds certain nerve fibers; it has a high proportion of lipid to protein, and serves as an electrical insulator [Har].

**myocarditis** (myo- + Gr. *kardia* heart + -itis) inflammation of the myocardium; inflammation of the muscular walls of the heart [Hey95].

**myocardial infarction** heart attack; death of an area of heart muscle due to lack of blood supply [Maya].

**myocardial perfusion imaging** a nuclear medicine technique designed to depict the blood flow to the heart muscle noninvasively. The radioactive drugs (radiopharmaceuticals) used for this study are extracted by the heart muscle in proportion to the local blood flow in the heart [UNM01].

**myocardium** the heart muscle [Maya].

**myocyte** 1: any contractile cell.
2: a muscle cell [Har].

**myopathy** (myo- + -pathy) any disease of a muscle [Hey95].

**myxoma** (also *myxoblastoma*) a soft, jellylike tumor that is composed of connective and mucoid tissue and that may grow to over 30 cm in diameter [Har].

**near-infrared fluorescence imaging** imaging modality which uses a probe emitting a strong near-infrared signal after activation (e.g., by tumour proteases) which is registered by a photodetector (e.g., CCD camera) [WTMB99].

**necrosis** (Gr. *nekrosis* deadness) the sum of the morphological changes indicative of cell death and caused by the progressive degradative action of enzymes; it may affect groups of cells or part of a structure or an organ [Hey95].

**neuro** (also *neurological*) of or relating to the nervous system especially in respect to its structure, functions and abnormalities [Web02].

**neuron** a nerve cell: the functional unit of the nervous system. Structurally, the neuron is made up of a cell body (soma) and one or more long processes: a single axon and dendrites [McG].

**neuroglia** supporting tissue that is intermingled with the essential elements of nervous tissue especially in the brain, spinal cord, and ganglia, is of ectodermal origin, and is composed of a network of fine fibrils and of flattened stellate cells with numerous radiating fibrillar processes [Web02].

**non-uniform rational B-Spline (NURBS)** NURBS curves and surfaces are the industry standard for geometry description in CAD/CAM or Computer Graphics. NURBS are piecewise rational; they are the logical extension of B-Splines or Bezier entities [Far95].

**occipital** of, relating to, or located within or near the occiput or the occipital bone [Web02].

**optic radiation** any of several neural radiations concerned with the visual function; especially: one made up of fibers from the pulvinar and the lateral geniculate body to the cuneus and other parts of the occipital lobe [Web02].

**optical coherence tomography (OCT)** is a new imaging technique that utilizes photonics and fiber optics to obtain images and tissue characterization. OCT uses infrared light waves that reflect off the internal microstructure within the biological tissues. The frequencies and bandwidths of infrared light are orders of magnitude higher than medical ultrasound signals resulting in greatly increased image resolution [Lig].

**optical imaging** the branch of in vivo diagnostics that generates images by using photons of light in the wavelength range from ultraviolet to near-infrared, including the range of wavelengths visible to the human eye. The propagation of light through tissue is mainly influenced by absorption and scattering in the tissue itself or by a fluorescent contrast agent [WWH01].

**orthogonal transformation** a linear transformation $T : V \to V$ which preserves a symmetric inner product. In particular, an orthogonal transformation (technically, an *orthonormal transformation*) preserves lengths of vectors and angles between vectors,

$$\langle v, w \rangle = \langle Tv, Tw \rangle$$

In addition, an orthogonal transformation is either a rigid rotation or a rotoinversion (a rotation followed by a flip). Orthogonal transformations correspond to and may be represented using orthogonal matrices [Wei].

**palpation** examination by touching [Maya].

**pancreatobiliary tree** consists of the bile ducts, which are tubes that carry bile from the liver to the gallbladder and small intestine, and the pancreas which is a large gland that produces chemicals that help with digestion [URL: http://www.niddk.nih.gov/health/digest/pubs/diagtest/ercp.htm].

**papillary muscle** one of the small muscular columns attached at one end to the chordae tendineae and at the other to the wall of the ventricle and that maintain tension on the chordae tendineae as the ventricle contracts [Web02].

**paracellular** surrounding the cells [DBK01].

**parasympathetic nervous system** the part of the autonomic nervous system that contains chiefly cholinergic fibers, that tends to induce secretion, to increase the tone and contractility of smooth muscle, and to slow the heart rate, and that consists of (1) a cranial part made up of preganglionic fibers leaving and passing the midbrain by the oculomotor nerves and the hindbrain by the facial, glossopharyngeal, vagus, and accessory nerves and passing to the ciliary, sphenopalatine, submandibular, and optic ganglia of the head or to ganglionated plexuses of the thorax and abdomen and postganglionic fibers passing from these ganglia to end organs of the head and upper trunk and (2) a sacral part made up of preganglionic fibers emerging and passing in the sacral nerves and passing to ganglionated plexuses of the lower trunk and postganglionic fibers passing from these plexuses chiefly to the viscera of the lower abdomen and the external genital organs [Web02].

**parietal** 1: of or relating to the walls of a part or cavity.
2: of, relating to, or located in the upper posterior part of the head; specifically: relating to the parietal bones [Web02].

**Parkinson disease (PD)** is a progressive movement disorder marked by tremors, rigidity, slow movements (bradykinesia), and posture instability. It occurs when cells in the *substantia nigra*, one of the movement-control centers of the brain, begin to die for unknown reasons. Most cases of PD are sporadic. This means that there is a spontaneous and permanent change in nucleotide sequences (the building blocks of genes). PD was first noted by British physician James Parkinson in the early 1800s [Medb].

**pathology** branch of medicine that treats the essential nature of the disease, especially the structural and functional changes in tissues and organs of the body caused by the disease [Hey95].

**pathophysiologic** 1: the physiology of abnormal states; specifically
2: the functional changes that accompany a particular syndrome or disease [Web02].

**perfusion** 1: the act of pouring over or through, especially the passage of a fluid through the vessels of a specific organ.
2.: a liquid poured over or through an organ or tissue [Hey95].

**perception** awareness of the elements of environment through physical sensation [Web02].

**pericardium** the conical sac of *serous* membrane that encloses the heart and the roots of the great blood vessels of vertebrates and consists of an outer fibrous coat that loosely invests the heart and is prolonged on the outer surface of the great vessels except the inferior vena cava and a double inner serous coat of which one layer is closely adherent to the heart while the other lines the inner surface of the outer coat with the intervening space being filled with pericardial fluid [Web02].

**pericarditis** is an inflammation of the two layers of the *pericardium*, the thin, sac-like membrane that surrounds the heart [Medb].

**peripheral nervous system** the part of the nervous system that is outside the *central nervous system* and comprises the cranial nerves excepting the optic nerve, the spinal nerves, and the *autonomic nervous system* [Web02].

**periventricular leukomalacia (PVL)** damage or softening of the white matter near the ventricles [Chib].

**phase-contrast MRI (PC-MRI)** a magnetic resonance imaging (MRI) scan performed after the injection of a contrast medium (a dye that makes blood vessels more visible). PC-MRI may be better than an angiogram at detecting whether a vessel has closed after an angioplasty [Hea].

**physiology** 1: a branch of biology that deals with the functions and activities of life or of living matter (as organs, tissues, or cells) and of the physical and chemical phenomena involved.
2: the organic processes and phenomena of an organism or any of its parts or of a particular bodily process (e.g., the physiology of the thyroid gland) [Web02].

**polarization** [particle physics] property of a collection of particles with spin, in which the majority have spin components pointing in one direction, rather than at random [McG].

**positron emission tomography (PET)** tomography in which an in vivo, noninvasive, cross-sectional image of regional metabolism is obtained by a usually color-coded cathode-ray tube representation of the distribution of gamma radiation given off in the collision of electrons in cells with positrons emitted by radionuclides incorporated into metabolic substances

**posterior** towards the back of the body (brain anatomy: towards the occipital region) [EW91].

**principal curvature** the maximum and minimum of the normal curvature $\kappa_1$ and $\kappa_2$ at a given point on a surface are called the principal curvatures. The normal curvature $\kappa_n$ is the curvature of the curve created by cutting the surface at the given point with a plane which contains the normal at that point. Formulas for the computation of the principal curvatures are given in [Wei, HL92b].

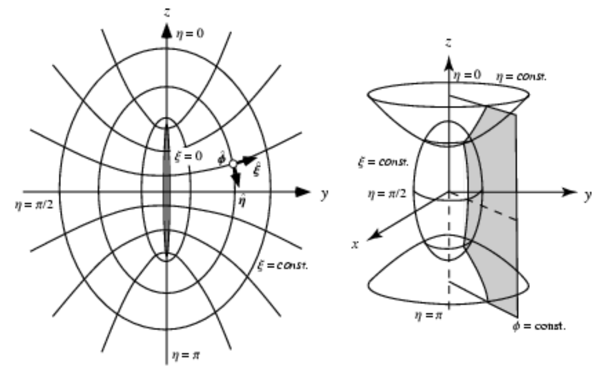


**Figure F.1.** *Prolate spheroidal coordinates (©2002 Eric W. Weisstein [Wei]).*

**prolate spheroidal coordinates** a system of curvilinear coordinates in which two sets of coordinate surfaces are obtained by revolving the curves of elliptic cylindrical coordinates about the x-axis, which is relabeled the z-axis. The third set of coordinate planes consists of planes passing through this axis:

$$\begin{aligned} x &= a \sinh \xi \sin \nu \cos \theta \\ y &= a \sinh \xi \sin \nu \sin \theta \\ z &= a \cosh \xi \cos \nu \end{aligned}$$

where $\xi \in [0, \infty)$, $\nu \in [0, \pi]$, and $\theta \in [0, 2\pi)$ [Wei].

**protease** any of numerous enzymes that hydrolyze proteins and are classified according to the most prominent functional group (as serine or cysteine) at the active site - called also *proteinase* [Web02].

**psychology** 1: the science of mind and behaviour.
2: the mental or behavioral characteristics typical of an individual or group or a particular form of behavior; the study of mind and behavior in relation to a particular field of knowledge or activity [Web02].

**pulmonary artery** an arterial trunk or either of its two main branches that carry blood to the lungs: (a) a large arterial trunk that arises from the conus arteriosus of the right ventricle, ascends in front of the aorta, and branches into the right and left pulmonary arteries – called also *pulmonary trunk* (b) a branch of the pulmonary trunk that passes under the arch of the aorta to the right lung where it divides into branches – called also *right pulmonary artery* (c) a branch of the pulmonary trunk that passes to the left in front of the descending part of the aorta, gives off the ductus arteriosus in the fetus which regresses to the ligamentum arteriosum in the adult, and passes to the left lung where it divides into branches - called also *left pulmonary artery* [Web02].

**pulmonary vein** any of usually four veins comprising two from each lung that return oxygenated blood from the lungs to the superior part of the left atrium, that may include three veins from the right lung if the veins from all three lobes of the right lung remain separate, and that may include a single trunk from the left lung if its major veins unite before emptying into the left atrium [Web02].

**pyramidal tract** see *corticospinal tract*

**radiography** (radio- + Gr. *graphein* to write) the making of film records (radiographs) of internal structures of the body by passage of x-rays or gamma rays through the body to act on specially sensitized film [Hey95].

**radiopaque** being opaque to radiation and especially X-rays [Web02].

**regurgitation** (re- + L. *gurgitare* to flood) a backward flowing, as the casting up of undigested food, or the backward flowing of blood into the heart, or between the chambers of the heart when a valve is incompetent [Hey95].

**remodelling** an enlargement and thinning out of the heart's left ventricle as it tries to adapt to the effects of heart failure. This leads to further damage to heart cells, reduced cardiac output of blood and more severe heart disease [Hea].

**reperfusion** restoration of the flow of blood to a previously ischemic tissue or organ (as the heart) [Web02].

**revascularization, transmyocardial** technique, in which microscopic holes are made into the heart muscle using a laser. It is speculated that such holes improve blood flow to the heart by promoting *angiogenesis* [Mayb].

**ribonucleic acid (RNA)** a chemical found in the nucleus and cytoplasm of cells; it plays an important role in protein synthesis and other chemical activities of the cell. The structure of RNA is similar to DNA. There are several classes of RNA molecules, including messenger RNA, transfer RNA, ribosomal RNA, and other small RNAs, each serving a different purpose [Sch].

**S-A node** see *sino-atrial node*

**sagittal** 1: of, relating to, or being the *sagittal suture* of the skull.
2: of, relating to, situated in, or being the median plane of the body or any plane parallel to it [Web02].

**sagittal suture** the deeply serrated articulation between the two parietal bones in the median plane of the top of the head [Web02].

**saltatory conduction** electrical conduction by jumping action potentials. Occurs in myelinated axons which have Schwann cells wraped so tightly around the axonal membrane that no extracellular space is underneath them. Therefore, the only place that an action potential can occur is at the node of Ranvier– the space between the Schwann cells [http://lls.stcc.mass.edu/tamarkin/AP/AP1 pages/saltator.htm].

**sarcomere** the basic contractile unit of muscle (skeletal and cardiac) that structurally is the portion of a myofibril between two adjacent Z lines, the unit repeated along the entire length of the myofibril [Har].

**schizophrenia** a psychotic disorder characterized by loss of contact with the environment, by noticeable deterioration in the level of functioning in everyday life, and by disintegration of personality expressed as disorder of feeling, thought, and conduct [Web02].

**scintigraphy** producing a graphic record of the gamma rays emitted by a radioisotope, revealing its relative concentration in various tissues [Har].

**septal** of or relating to a *septum* [Web02].

**septal defect** hole in the wall separating the two atria or the two ventricles [Maya].

**septum** a wall dividing two cavities or compartments [Maya].

**serous** of, relating to, producing, or resembling *serum*; especially: having a thin watery constitution <a *serous* exudate> [Web02].

**serum** the watery portion of an animal fluid remaining after coagulation: (a) the clear yellowish fluid that remains from blood plasma after fibrinogen, prothrombin, and other clotting factors have been removed by clot formation - called also blood serum (b) a normal or pathological serous fluid (as in a blister) [Web02]

**single photon emission computed tomography (SPECT)** a medical imaging technique that is used especially for mapping brain function and that is similar to *positron-emission tomography* in using the photons emitted by the agency of a radioactive tracer to create an image but that differs in being able to detect only a single photon for each nuclear disintegration and in generating a lower-quality image [Web02].

**sinus node** see *sino-atrial node*

**sino-atrial node** a small mass of tissue that is made up of Purkinje fibers, ganglion cells, and nerve fibers, that is embedded in the musculature of the right atrium of higher vertebrates, and that originates the impulses stimulating the heartbeat [Web02].

**sonomicrometry** measurement of distances using ultra-sound and transducers made from piezo-electric ceramic material to transmit and receive sound energy. Offers a resolution of up to 0.015mm [http://www.sonometrics.com/sono_101.htm].

**stenosis** the narrowing or closure of an opening or passageway in the body (aortic stenosis = narrowing of the valve opening between the left ventricle and the aorta; mitral stenosis = narrowing of the valve between the left atrium and ventricle) [Maya].

**sternal** of or relating to the sternum [Web02].

**sternum** the breastbone [Maya].

**stethoscope** an instrument that is used in medical observation to transmit low-volume internal bodily sounds, such as the heartbeat, or intestinal, venous, or fetal sounds, to the ear of the observer; it consists of two earpieces connected by means of flexible tubing to a diaphragm, which is placed against the patient's skin [Har].

**strain rate imaging (SRI)** uses ultrasound to measure the rate of deformation of tissue using a method based on the Doppler technique. In cardiac imaging SRI can, unlike tissue Doppler, differentiate between actively deforming and passively drawn muscle segments [Hei].

**stunned myocardium** reversible postischemic mechanical dysfunction that occurs after reperfusion despite the absence of irreversible damage [SSW+96].

**subendocardial** situated or occuring beneath the *endocardium* or between the *endocardium* and *myocardium* [Web02].

**subepicardial** situated or occuring beneath the *epicardium* or between the *epicardium* and *myocardium* [Web02].

**substantia nigra** a layer of deeply pigmented gray matter situated in the midbrain and containing the cell bodies of a tract of dopamine-producing nerve cells whose secretion tends to be deficient in Parkinson's disease [Web02].

**superior** towards the head (Brain anatomy: towards the top of the head) [EW91].

**superior vena cava** large vein returning blood from the head and arms to the heart [Maya].

**superior vena cava syndrome (SVCS)** (also *superior mediastinal syndrome*, *superior vena cava obstruction*) is a partial occlusion of the superior vena cava. This leads to a lower than normal blood flow through this major vein. More than 95% of all cases of SVCS are associated with cancers involving the upper chest. The cancers most commonly associated with SVCS are advanced lung cancers, which account for nearly 80% of all cases of SVCS, and lymphoma [Medb].

**sympathetic nervous system** the part of the autonomic nervous system that is concerned especially with preparing the body to react to situations of stress or emergency, that contains chiefly adrenergic fibers and tends to depress secretion, decrease the tone and contractility of smooth muscle, increase heart rate, and that consists essentially of preganglionic fibers arising in the thoracic and upper lumbar parts of the spinal cord and passing through delicate white rami communicantes to ganglia located in a pair of sympathetic chains situated one on each side of the spinal column or to more peripheral ganglia or ganglionated plexuses and postganglionic fibers passing typically through gray rami communicantes to spinal nerves with which they are distributed to various end organs [Web02].

**synapses** the place at which a nervous impulse passes from one neuron to another [Web02].

**systemic** pertaining to or affecting the body as a whole [Hey95].

**sytole** the portion of the heart cycle during which the heart muscle is contracting. At end-systole the heart has ejected the maximum amount of blood [Maya, GZM97].

**tachycardia** (tachy- + Gr. *kardia* heart) excessive rapidity in the action of the heart; the term is usually applied to a heart rate above 100 per minute and may be qualified as atrial, junctional (nodal), or ventricular, and as paroxysmal [Hey95].

**tendom** connective tissue that joins muscle to bone [Maya].

**tensor** an $n$th-rank tensor (or tensor of *order* $n$) in $m$-space is a mathematical object in $m$-dimensional space that has $n$ indices and $m^n$ components and obeys certain transformation rules. Each index of a tensor ranges over the number of dimensions of space. However, the dimension of the space is largely irrelevant in most tensor equations (with the notable exception of the contracted Kronecker delta). The notation for a tensor is similar to that of a matrix, except that a tensor $a_{i,j,k,...}$ may have an arbitrary number of indices [Wei].

**thorax** the portion of the anatomy below the neck and above the diaphragm; the chest [Maya].

**thalamus** (L.; Gr. *thalamos* inner chamber) either of two large, ovoid masses, consisting chiefly of grey substance, situated one on each side of and forming part of the lateral wall of the third ventricle. It is divided into two major parts: dorsal and ventral, each of which contains many nuclei [Hey95].

**thrombus** blood clot [Maya].

**tomography** a method of producing a three-dimensional image of the internal structures of a solid object (as the human body) by the observation and recording of the differences in the effects on the passage of waves of energy impinging on those structures - called also stratigraphy [Web02].

**transesophogeal** passing through or performed by way of the *esophagus* [Web02].

**transmural** 1: passing or administered through an anatomical wall.
2: involving the whole thickness of a wall [Web02].

**ultrasonography** the use of ultrasound as a diagnostic aid. Ultrasound waves are directed at the tissues, and a record is made, as on an oscilloscope, of the waves reflected back through the tissues, which indicate interfaces of different acoustic densities and thus differentiate between solid and cystic structures [Hey95].

**valve** 1: a structure especially in a vein or lymphatic that closes temporarily a passage or orifice or permits movement of fluid in one direction only
2: any of various mechanical devices by which the flow of liquid (as blood) may be started, stopped, or regulated by a movable part that opens, shuts, or partially obstructs one or more ports or passageways; also: the movable part of such a device [Web02].

**vein** any of the tubular branching vessels that carry blood from the capillaries toward the heart and have thinner walls than the arteries and often valves at intervals to prevent reflux of the blood which flows in a steady stream and is in most cases dark-colored due to the presence of reduced hemoglobin [Web02].

**velocity-encoded cine MRI** MRI technique for production of (flow) velocity maps [SB99].

**ventral corticospinal tract** (also *anterior corticospinal tract*, *direct pyramidal tract*) a band of nerve fibers that descends in the ventrolateral part of the spinal cord and consists of fibers arising in the motor cortex of the brain on the same side of the body and not crossing over in the decussation of pyramids [Web02].

**ventricle** a cavity of a bodily part or organ: as
1: a chamber of the heart which receives blood from a corresponding atrium and from which blood is forced into the arteries.
2: one of the system of communicating cavities in the brain that are continuous with the central canal of the spinal cord, that like it are derived from the medullary canal of the embryo, that are lined with an epithelial ependyma, and that contain a serous fluid.
3: a fossa or pouch on each side of the larynx between the false vocal cords above and the true vocal cords below [Web02].

**ventricular** of, relating to, or being a ventricle especially of the heart or brain [Web02].

**ventriculography** the act or process of making an X-ray photograph of the ventricle of the heart after injection of a radiopaque substance [Web02].

**X ray** 1: any of the electromagnetic radiations of the same nature as visible radiation but of an extremely short wavelength less than 100 angstroms that is produced by bombarding a metallic target with fast electrons in vacuum or by transition of atoms to lower energy states and that has the properties of ionizing a gas upon passage through it, of penetrating various thicknesses of all solids, of producing secondary radiations by impinging on material bodies, of acting on photographic films and plates as light does, and of causing fluorescent screens to emit light – called also roentgen ray.

2 : a photograph obtained by use of X rays <a chest X ray> [Web02].