

A Survey of Intelligent Digital Ink Tools use in STEM Education

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ABSTRACT

Digital ink affords numerous opportunities to broaden the use of computer supported learning in science, technology, engineering and mathematics (STEM) education. This review of the recent literature on digital ink tools demonstrates that they offer significant potential, and although there are relatively few studies on intelligent digital ink tools, it is notable that they span the STEM disciplines. In this review, we identify the major challenges for applications of digital ink in STEM education, and discuss the primary areas of use, feedback mechanisms and interaction methods.

CCS Concepts

H5.2. Information interfaces and presentation (e.g., HCI): User Interfaces. - Graphical user interfaces.

Keywords

Digital Ink; Educational Tools; STEM Education; Intelligent Tools.

1. INTRODUCTION

Diagrams are frequently used to model, or visualize complex information in fields such as Mathematics, Chemistry and Computer Science [9, 10, 29]. In order to provide automated feedback and learning support for activities that use diagrams, the diagrams must be drawn digitally. Unfortunately, using a traditional mouse and keyboard to draw diagrams is difficult, and is a modality known to impose a higher extrinsic cognitive load compared with pen and paper [4, 29].

Increasingly, technology is used to support learning activities outside the classroom [5]. Technology used in this way needs to include sufficient guidance for a student to work independently, without relying on direct contact with a teacher. Furthermore, technology can facilitate active learning [8] by allowing students to interact with the material while the system demonstrates the effect of their changes.

Interactive tutoring environments guide the student through a set of interactive tasks to help them learn a concept. The majority of such tools are WIMP-based (Windows, Icons, Menus, Pointers) resulting in low fluidity and steep learning curves [13, 15, 17, 23], which may distract a student from their learning task. The rising

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popularity of devices that support digital ink provides an opportunity for users to experience interactive tutoring environments without the high cognitive load associated with WIMP input modalities.

In this study, we analyse published literature on intelligent digital ink tools used in STEM (Science, Technology, Engineering and Mathematics) education. We describe the current state of research in this field, identify gaps in the literature and suggest avenues of future research.

2. BACKGROUND

Digital Ink, with its many advantages, has the potential to play an important role in the future of educational software. In this section, we briefly describe interactive learning, visual languages, constructivist learning and how digital ink supports these approaches.

2.1 Two major learning strategies

Learning strategies can be categorized into two types: active learning and passive learning. Passive learning is a more traditional pedagogy, in which the student is considered to be a bucket into which the teacher pours information. It can also be said that the student is merely an observer trying to imbibe what the teacher demonstrates.

Whereas in active learning, students are directly engaged in the learning process, they are not mere observers. This approach is more hands-on and students are engaged in activities and exercises. Each activity is followed by feedback, which leads to a better understanding of the topic. Active learning has consistently resulted in better understanding and knowledge retention [8, 12].

Many intelligent tutoring systems that can help students learn interactively in the absence of a human teacher have been developed (e.g. [7, 11]). Typically, these systems use mouse and keyboard for interaction. The few existing digital ink based intelligent tools are discussed in the next section.

2.2 Visual Languages in learning

Visual languages (e.g. diagrams) have been used extensively in STEM education. An early example being Euler's use of diagrams in the 1760s [16] to visualize set theories. Diagrams are a convenient way of describing complex phenomenon and abstract concepts. They help us both understand and analyse the concepts. They can also increase student engagement, make learning more interactive and improve understanding of a topic [22, 36].

With technology, there are a number of ways to create and use visualisations. Broadly, they can be classified into static (e.g. sketching using paper and pen) and dynamic (e.g. movies, animations, simulations). Sketching on paper is the simplest form

of visualisation and may help a student to understand a problem in areas like mathematics and algorithms. However, static visualisation is often insufficient for an in-depth analysis and comprehension [10].

Among the dynamic techniques, simply watching an animation or video is passive. If the dynamic technique involves interaction with visualized objects, then it is an interactive dynamic visualisation. However, the interaction is typically through mouse and keyboard, which involves a steep learning curve, and may distract a student from the main task of problem-solving [22, 29]. This is where the advantage of digital ink based tutoring tools come into play.

2.3 Constructivist Learning and Digital Ink

One of the popular theories of learning, the Constructivist Theory states that to get a deep understanding of a topic, it is important for the learner to be actively engaged in activities concerning it. One way to do so is through active learning and visualisations [6].

Digital Ink can be a pivotal technology in facilitating deep learning because of the many advantages it has over other approaches. Though slightly more cognitively demanding than simple paper-pen sketching, users can still intuitively and swiftly use digital ink based tools to sketch. The additional advantage is that digital ink sketches can be dynamic with interactive animation and hence are more suitable for deep learning than paper-pen sketches [23, 29]. Moreover, digital ink has less cognitive load than mouse and keyboard based visualisation tools [4, 29]. Additionally, many studies have shown that a majority of users prefer digital ink based tools over keyboard and mouse, as it allows them to spend more time on their task rather than learning to use the tool [18, 29].

In constructivist learning the teacher and students play close and symbiotic roles, with students doing the activities and the teacher providing continuous feedback [6]. With the increasing use of software tools in education, constructivism can be implemented when there is little contact between the teacher and student. This is possible with the use of intelligent tutoring systems. Such a system provides feedback to students as they solve a problem and informs them of their mistakes and successful approaches. Thus, students can almost have the same level of interactivity without the presence of teachers [38].

3. METHODOLOGY

To identify relevant research articles we undertook a systematic literature review [42]. The databases used to search for existing

literature were – ACM Digital Library, IEEE XPLORE, Science Direct, Springer Link and Scopus.

All search results contained the phrase ‘digital ink’ AND one of the following terms: ‘education’, ‘STEM’, ‘student’, ‘teacher’ ‘intelligent tool’, or ‘intelligent’. The initial search resulted in 1678 publications.

To select the final publications, the following criteria were used-

- We defined an intelligent digital ink tool as - "*Intelligent digital ink tools are those that use machine learning techniques to not only recognize sketch and gesture but interpret the meaning of the drawn symbols and provide feedback to the user and guide them in fixing the issues if required*". This is based on the description of intelligent digital ink tools by Stahovich [33] and Wang et al [38]. Only tools that fit the above definition were selected.
- Only tools in STEM areas were selected.
- Only studies which had an actual tool with some empirical evaluation and not just a proposed one, were selected.
- Only studies that used the tools for actually teaching the concepts of the subjects were selected. Tools which used digital ink only for classroom facilitation like presentation, sharing of notes etc., were excluded.

After applying these strict inclusion criteria, only 24 studies were selected for further analysis. In the next section, we present the analysis of these papers.

4. RESULTS

The 24 studies were from the years 2004 to 2016 from peer-reviewed conferences, journals and one doctoral thesis. Table 1 gives a summary of all the studies. The tools are grouped according to their key feature. The tool name, its underlying technology, intended education level, the area of STEM that it encompasses, and the tool’s pedagogical purpose are noted in this table.

Figure 1, which follows Table 1, shows how the research in intelligent digital ink tools for STEM education has progressed through the years. From 2004 the number of studies has increased, but it is not a steady increase, and in 2013 there was no publication in this area.

Table 1. Summary of Intelligent Digital Ink Tools used in STEM Education

Study	Tool Name	Underlying Technology	Intended Education Level	STEM Coverage	Pedagogy Purpose
Key Feature: Educational Data Mining					
Stahovich et al. [34]	No Name	Digitizer	University	S&M	Assessment/Administration
Koile et al. [20]	No Name	Tablet PC	High School	M	Assessment/Administration
Tool Key Feature: Sketching & interpreting diagrams only					
Williford et al. [41]	PerSketchTivity	Tablet PC	University	E	Tutoring -Teaching/Learning
Valentine et al. [37]	Mechanix	Tablet PC	University	S	Tutoring -Teaching/Learning
Wetzel et al. [40]	Design Coach	Tablet PC	University	E	Tutoring -Teaching/Learning
Key Feature: Sketching & interpreting equations only					
Taranta et al. [35]	Math Boxes	Tablet PC	University/High School	M	Tutoring -Teaching/Learning
Labahn et al. [21]	MathBrush	Tablet PC	University/High School	M	Tutoring -Teaching/Learning
Key Feature: Sketching & interpreting diagrams and texts					
Alvarado et al. [3]	LogiSketch	Tablet PC	University	T	Tutoring -Teaching/Learning
Bryfczynski et al. [9]	BeSocratic	Tablet PC	University	STEM	Tutoring, Assessment, and Administration
Buchanan et al. [10]	CSTutor	Tablet PC	University	T	Tutoring -Teaching/Learning
Adamchik [1]	No Name	Tablet PC	University	T	Tutoring -Teaching/Learning
Wang et al. [39]	SketchSet	Tablet PC	University	T	Tutoring -Teaching/Learning
Ouyang et al. [28]	ChemInk	Tablet PC	University	S	Tutoring -Teaching/Learning
Yuan et al. [42]	No Name	Whiteboard	University	T	Tutoring- Teaching only
Liu et al. [25]	PIGP	Tablet PC	High School	M	Tutoring -Teaching/Learning
Liwicki et al. [26]	Smart Chalklets	Whiteboard	University	T	Tutoring- Teaching only
Key Feature: Sketching & interpreting diagrams and equations					
Cheema et al. [13]	No Name	Tablet PC	University	S&M	Tutoring -Teaching/Learning
Cossairt et al. [14]	SetPad	Tablet PC	University	T	Tutoring -Teaching/Learning
Jiang et al. [17]	PenProof	Tablet PC	High School	M	Tutoring -Teaching/Learning
Meyer et al. [27]	Smart Chalklets	Whiteboard	University	T	Tutoring- Teaching only
Lee et al. [24]	Newton's Pen	Digitizer	University	S&M	Tutoring -Teaching/Learning
De Silva et al. [15]	Kirchhoff's Pen	Tablet PC	University	E	Tutoring -Teaching/Learning
LaViola Jr et al. [23]	Math Pad2	Tablet PC	University/High School	S&M	Tutoring -Teaching/Learning
Key Feature: Sketching & interpreting diagrams, texts & speech					
Sabisch [32]	CoMo	Whiteboard	University	T	Tutoring -Teaching/Learning

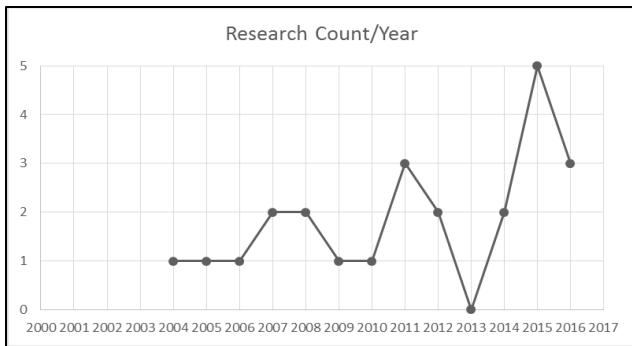


Figure 1. Research Count Per Year

4.1 Education Areas

STEM is composed of a wide range of subjects and usage of digital ink in the subjects may vary as each subject may require a different set of notations and symbols. Figure 2 shows tool distributions in different STEM areas. Most of the tools cater to the Technology area. Figure 3 gives a finer breakdown of number of tools as per specific subjects in STEM. We can see that none of the subjects have many studies.

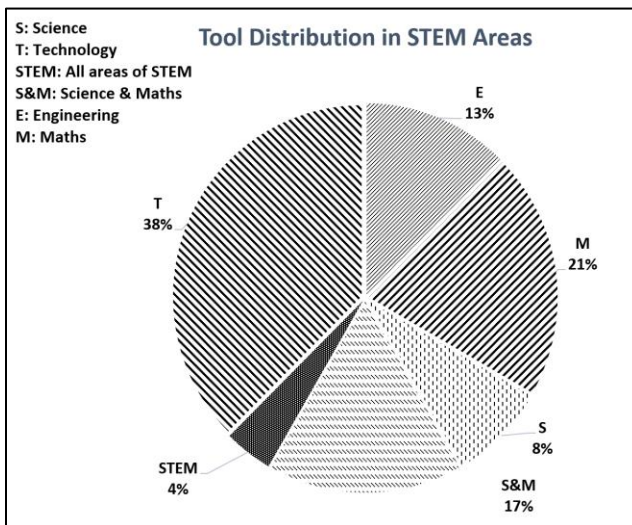


Figure 2. Tools Distribution in STEM Areas

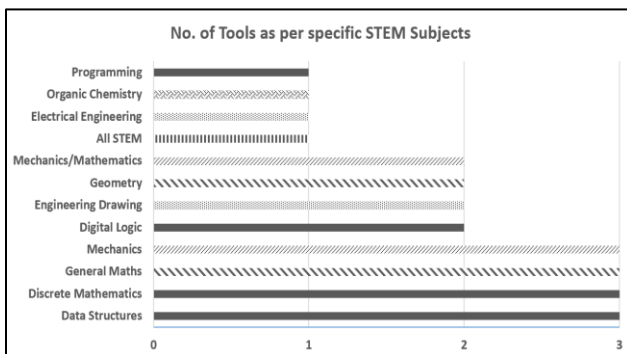


Figure 3. Tools as per specific STEM subjects

4.2 Pedagogy Areas

Pedagogy consists of three main facets, namely - tutoring, assessment, and administration. These facets are interdependent,

sometimes working simultaneously and other times one after the other. In Figure 4, studies have been grouped together as per the pedagogical use and their target education level. The most numbers of studies have been for teaching/learning and for university level.

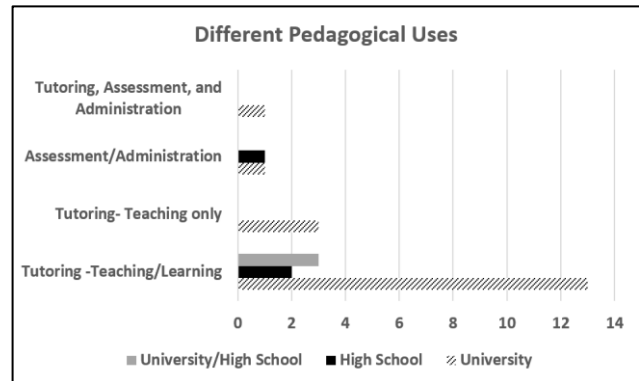


Figure 4. Tool pedagogical use

4.3 Tool Functional Purpose & Mode

In general, digital ink tools can be grouped by their functional abilities. They can be used for diagrams only, general text only, equations only, or for gathering information. Some can perform a combination of these functions. Some tools need the user to specify what type of sketch they are sketching. This helps the tool by constraining the recognition of the sketch. So, a tool which allows both diagramming and writing equations might require the user to specify if they are sketching a diagram or equation; this is called mode dependency. A tool that doesn't require the user to specify the type of sketch is modeless.

From Figure 5 we can see that most tools are modeless and have diagram and text sketching and interpretation functionality. The tool CoMo [32] is the only one which supports diagram, text and speech input.

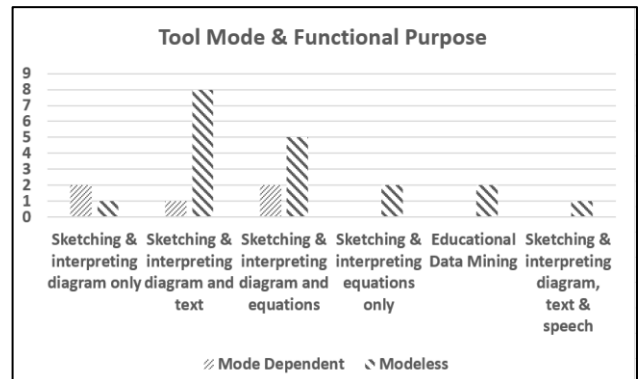


Figure 5. Tool Mode and Functional Scope

4.4 Digital Ink Recognition

The user gesturing on the interface creates the digital ink. The same input mode can also be used for functional gestures such as undo. Some tools have only sketch recognition: others have both sketch and gesture recognition. These may support functional gestures like scribble for delete and tapping for right-click. The aim of these gestures is to make the use of pen-based software as natural as possible. From Figure 6 it can be seen that there is a similar number of studies between the two groups. Here too, the tool CoMo is the only one which allows sketch, gesture and speech.

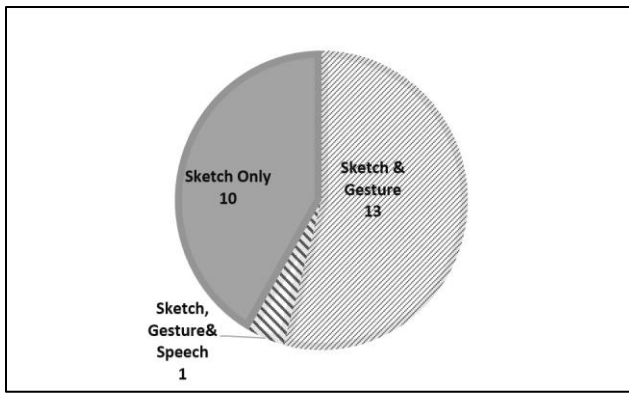


Figure 6. Tools as per recognition ability

4.5 Interaction and visualisation

The key ability that makes digital ink beneficial for learning is its potential to support intelligent interaction and visualisation features. In Table 2 studies have been grouped by their interaction method and recognition feedback mechanisms. The number of studies that have used interactive animation or colour/text based feedback is the same. Just one study that uses audio feedback to inform the students of their progress, it was conducted on Anoto paper. Another study uses both interactive animation and audio feedback [32]. Two studies [20, 34] are intelligent educational data mining tools used for marking and hence didn't need to provide recognition feedback to the students.

The tools are further divided into the frequency of feedback. It can either be at each step, i.e. stepwise or after the user has completed the whole task. Stepwise feedback tools are slightly more in number than non-stepwise.

Table 2. Studies involving interaction and visualisation

Interaction Style	Recognition Feedback method	Studies	Total Studies
Interactive Animation	Colour based animation and/or associative changing of diagram, equation or text -stepwise	[1, 10, 14, 23, 27, 35]	6
	Colour based animation and/or associative changing of diagram, equation or text – non-stepwise	[3, 13, 25, 26]	4
Audio Feedback	Stepwise audio feedback	[24]	1
Text/Colour Coded feedback	Stepwise	[24, 28, 37, 39]	4
	Non-stepwise	[9, 17, 21, 40-42]	6
Interactive Animation and Audio	Stepwise	[32]	1

5. DISCUSSION

In our background section, we discussed the importance of interaction and visualisation in learning. We also described how a digital ink based intelligent tool could promote constructivist learning. The presence of only 24 studies from 2004 to 2016 in itself is a precursor enough to promote more research in this area.

The technology used in most of the studies is tablet PCs or their ilk, which is not a surprise with the increasing usage of portable computing devices. Two studies that used a digitizer pen on paper based their decision on studies [4, 29] which claim that there is more cognitive load when using any kind of computer equipment compared to paper and pen for problem-solving. Nevertheless, since technology offers benefits that cannot be provided by paper and pen they trade-off ease of drawing for interactivity. A counter argument to this is that even though the cognitive load for tablet PC based software is slightly more than a digitizer pen, that the value of interactive feedback may outweigh the ease of paper. Furthermore, displays and styli are continuing to improve.

Four of the studies used a whiteboard and hence these tools are primarily for the use of a teacher except for the tool CoMo which is also intended to be used as a collaborative tool among the students.

5.1 The scenario as per education area and pedagogical use

The educational scenario has changed in the past few decades and in the future a good deal of education could be online. The main drawback from such an education is the lack of interaction between students and teachers, hence giving such initiatives a low score in constructivism. One way forward is that the “tool” itself becomes intelligent enough to act as a teacher by providing necessary guidance and feedback. However, the benefits of such an educational setup are debatable and beyond the scope of the discussion of this paper. Nevertheless, a hybrid scenario where the student uses such tools for learning when away from classes, along with classroom learning seems more ideal.

The results show that most of the tools are intended for university students. Just three specifically for high school students and three for both high school and university students. Studies [19] in HCI have shown that tool use is often dependent on the cognition and motor capability of the user. A student at primary school will have different skills than someone at high school level and so on. A tool to be efficient in teacher independent tutoring needs to specifically cater to the user's cognitive and motor skills. Hence, tools need to be developed for the different education levels.

Pedagogy can be viewed from three sides tutoring, assessment, and administration. A tool suited for assessment might not be suited for tutoring. Most of the tools in the studies are for teaching and learning but three tools which used whiteboards are more suited for use by a teacher to teach.

There are very few tools for assessment and administration. These tools are educational data mining tools that can record and analyse student freehand answers. The tools can identify student response patterns that can, in turn, be used by the teachers to improve their teaching. For a holistic technology driven education, along with intelligent tutoring systems, there is also a need for intelligent assessment and administration tools.

Considering the distribution of usage in different STEM areas and subjects. There is no subject with more than three tools, which is a small number in 12 years of research. A major percentage of tools are for science and mathematics areas, followed by technology

(computer science and electronics) and engineering. There is one tool, BeSocratic [9] which covers broadly all areas of STEM.

BeSocratic can also be used for all the three facets of pedagogy. However, the range and type of STEM topics that it can deal with are mostly basic. Having subject specific tools is important because as the number of possible symbols are reduced recognition becomes more accurate. Every subject has its own set of notations and diagrams. For example, in a tool for electrical engineering circuit diagrams sketching a resistor which is represented by a zig-zag line could conflict with the scribble gesture. But, this conflict will not happen for a tool used for tutoring data structures.

Another challenge is to develop semantics and heuristics to make sense of a whole diagram or set of diagrams, which is another big area to explore. Here again developing tools specific to a particular subject area is more feasible as making sense out of a large possibility of combinations of diagrams is computationally expensive.

As most areas of STEM utilize diagramming there is a need to develop intelligent digital ink tools for the many subjects.

5.2 Functionality and fluidity

Natural user interfaces stress fluidity and a small learning curve and try to closely mimic their non-technological counterpart. Limiting the natural way of solving a problem, in the way that WIMP tools do, hampers usability and eventually cognition [30]. The ideal situation would be if a student could write as they would on paper, with accurate, timely feedback. However, this requires much better digital ink recognition than is currently available. There are several areas where this impacts STEM tools.

Presently many tools do not provide enough support. For example, tools which are intended for mathematical/engineering education but only allow the entering of diagrams, e.g. [41]. This is an issue, as a student using the tool would also be expected to use equations.

Differentiating writing and drawing is difficult for recognizers. Some tools required the user to change modes e.g. [15, 26], but this impacts usability. Both modality and poor recognition impact tool usability [23]. Researchers should keep a tight balance between recognition rates and making the tool more natural.

Likewise, support for familiar functional gestures, for example cross-out, increase naturalness but also increase the demand on the recognizer.

Single stroke recognition is more advanced than multi-stroke recognition. Some tools force users to draw figures in single strokes to improve recognition. Others tools overcome this by using a lasso gesture or circling gesture around the multi-stroke figure, e.g. [23]. These solutions decrease the fluidity of use of the tool.

The key for better digital ink tools is better recognition. With the ongoing research in this area recognition should improve further.

5.3 Interactive feedback

When a digital ink tool can provide interactive feedback it wins comfortably as an interactive learning environment. Digital ink scores above paper and pen in creating a more visually interactive environment. With only slightly more cognitive effort, users can easily create interactive solutions with intelligent tutoring support. This should aid better problem comprehension, analysis and solution building.

Some of the studies reported [2, 9] scored highly on interactivity and visualisation. The common denominator for these studies is the facility of interactive animation that they provide. An animation is in itself attention grabbing but it becomes better when it is

interactive and can be altered. These tools have associative mutation features where mutating a diagram or the animation of a process also changes the corresponding text or equation and vice versa [10, 23]. Colour changing animation is utilized by some studies for feedback [3]. Ghosting, another feedback technique in which a shadow of the correct diagram is shown below the wrong diagram is also used. Apart from this, tools in this category also provide detailed text based feedback and error messages. Another highly engaging tool [32], and the only of its kind, used a multimodal approach supporting both sketching and speech input. It utilized interactive animation and audio for feedback.

The next category of tools rate medium on interactivity and visualisation. These tools do not have interactive animation but still interacted with the users using colour change, text, and pop-up messages. Tools of this type verify the correctness of a user's input text, equation or diagram and respond providing suggestions or corrections. The most interesting tool of this category is BeSocratic [9]. It is unique because apart from system evaluation of a user's answer it also evaluates it against a solution entered by the teacher. Thus, it has a two-step verification of student solutions and provides very detailed textual feedback and suggestions.

In the last category with minimum points for visualisation and interaction come tools that use a digitizer pen. A digitizer is an inherent disadvantage as it can only create static ink visualisations. However, such tools are simpler to use and apart from the digitizer, no special equipment is required. The study [24] that utilizes this technology compensated the lack of visual feedback with audio feedback to guide the student at each step. The audio feedback it generates may help the users but it is unlikely to be as clear as a visual representation.

The other important point of discussion is the timing and frequency of feedback. From the results, it can be seen that there is no real difference between the number of studies for stepwise and non-stepwise feedback. The proponents of stepwise feedback say that it provides feedback after each step thus acting like a tutor who verifies student's work at each step while teaching them. The opponents argue that stepwise feedback irritates the user and lowers their concentration [2].

An interesting result is presented in [31] regarding stepwise and non-stepwise feedback. The study found a relationship between student performance and preference between stepwise and non-stepwise feedback. It indicates that students who prefer stepwise feedback performed lower than those who preferred non-stepwise feedback. There are numerous possible explanations for this and it is worthy of more investigation.

6. CONCLUSION AND FUTURE WORK

In this study, we extracted and analysed available literature on intelligent digital ink tools used in STEM Education.

It is evident that there is very little work in this area with only 24 studies in 12 years and these are spread across many subjects and learning levels. Furthermore, some are focused more on student activity where others are more teacher or administration tools. There are many underexplored questions such as in what situations stepwise or non-stepwise feedback should be provided and how animation can be most effectively used.

Most telling of the state of research in this field is that none of the studies have addressed the big question – “Does using intelligent digital ink tool really improve learning?” Studies are needed which compare learning gain between traditional pen and paper, WIMP tools and digital ink tools.

7. REFERENCES

- [1] Adamchik, V., 2011. Data structures and algorithms in pen-based computing environments. In Global Engineering Education Conference (EDUCON), 2011 IEEE IEEE, 1211-1214.
- [2] Alvarado, C. and Davis, R., 2004. SketchREAD: a multi-domain sketch recognition engine. In Proceedings of the 17th annual ACM symposium on User interface software and technology ACM, 23-32.
- [3] Alvarado, C., Kearney, A., Keizur, A., Loncaric, C., Parker, M., Peck, J., Sobel, K., and Tay, F., 2015. LogiSketch: A Free-Sketch Digital Circuit Design and Simulation System LogiSketch. In *The Impact of Pen and Touch Technology on Education* Springer, 83-90.
- [4] Anthony, L., Yang, J., and Koedinger, K.R., 2005. Evaluation of multimodal input for entering mathematical equations on the computer. In CHI'05 Extended Abstracts on Human Factors in Computing Systems ACM, 1184-1187.
- [5] Bates, A.T., 2005. Technology, e-learning and distance education. Routledge.
- [6] Ben-Ari, M., 1998. Constructivism in computer science education. In *Acm sigcse bulletin* ACM, 257-261.
- [7] Billingsley, W., Robinson, P., Ashdown, M., and Hanson, C., 2004. Intelligent tutoring and supervised problem solving in the browser. In ICWI, 806-810.
- [8] Bonwell, C.C. and Eison, J.A., 1991. Active Learning: Creating Excitement in the Classroom. 1991 ASHE-ERIC Higher Education Reports. ERIC.
- [9] Bryfczynski, S., Pargas, R.P., Cooper, M.M., Klymkowsky, M., Hester, J., and Grove, N.P., 2015. Classroom Uses for BeSocratic. In *The Impact of Pen and Touch Technology on Education* Springer, 127-136.
- [10] Buchanan, S., Ochs, B., and Laviola Jr, J.J., 2012. CSTutor: a pen-based tutor for data structure visualization. In Proceedings of the 43rd ACM technical symposium on Computer Science Education ACM, 565-570.
- [11] Butz, B.P., Duarte, M., and Miller, S.M., 2006. An intelligent tutoring system for circuit analysis. *Education, IEEE Transactions on* 49, 2, 216-223.
- [12] Caceffo, R., Da Rocha, H.V., and Azevedo, R., 2015. A Ubiquitous, Pen-Based And Touch Classroom Response System Supported By Learning Styles. In *The Impact of Pen and Touch Technology on Education* Springer, 373-380.
- [13] Cheema, S. and Laviola Jr, J.J., 2010. Applying mathematical sketching to sketch-based physics tutoring software. In *Smart Graphics* Springer, 13-24.
- [14] Cossairt, T.J. and Laviola Jr, J.J., 2012. SetPad: a sketch-based tool for exploring discrete math set problems. In Proceedings of the International Symposium on Sketch-Based Interfaces and Modeling Eurographics Association, 47-56.
- [15] De Silva, R., Bischel, D.T., Lee, W., Peterson, E.J., Calfee, R.C., and Stahovich, T.F., 2007. Kirchhoff's Pen: a pen-based circuit analysis tutor. In Proceedings of the 4th Eurographics workshop on Sketch-based interfaces and modeling ACM, 75-82.
- [16] Euler, L., 1761. Lettres a une princesse d'Allemagne. Sur divers sujets de physique et de philosophie, 1761. Letters No. 102-108 vol 2 Basel Birkhauser.
- [17] Jiang, Y., Tian, F., Wang, H., Zhang, X., Wang, X., and Dai, G., 2010. Intelligent understanding of handwritten geometry theorem proving. In Proceedings of the 15th international conference on Intelligent user interfaces ACM, 119-128.
- [18] Johnson, G., Gross, M.D., Hong, J., and Yi-Luen Do, E., 2009. Computational support for sketching in design: a review. *Foundations and Trends in Human-Computer Interaction* 2, 1, 1-93.
- [19] Kim, H.-H., Valentine, S., Taelle, P., and Hammond, T., 2015. EasySketch: A Sketch-based Educational Interface to Support Children's Self-regulation and School Readiness. In *The Impact of Pen and Touch Technology on Education* Springer, 35-46.
- [20] Koile, K., Rubin, A., Chapman, S., Kliman, M., and Ko, L., 2016. Using machine analysis to make elementary students' mathematical thinking visible. In Proceedings of the Sixth International Conference on Learning Analytics & Knowledge ACM, 524-525.
- [21] Labahn, G., Maclean, S., Marzouk, M., Rutherford, I., and Tausky, D., 2006. A preliminary report on the MathBrush pen-math system. In *Maple 2006 Conference*, 162-178.
- [22] Laviola Jr, J.J., 2011. Mathematical sketching: An approach to making dynamic illustrations. In *Sketch-based Interfaces and Modeling* Springer, 81-118.
- [23] Laviola Jr, J.J. and Zeleznik, R.C., 2007. MathPad 2: a system for the creation and exploration of mathematical sketches. In *ACM SIGGRAPH 2007 courses* ACM, 46.
- [24] Lee, W., De Silva, R., Peterson, E.J., Calfee, R.C., and Stahovich, T.F., 2008. Newton's Pen: A pen-based tutoring system for statics. *Computers & Graphics* 32, 5, 511-524.
- [25] Liu, Y., Lin, Q., and Dai, G., 2007. PIGP: A pen-based intelligent dynamic lecture system for geometry teaching. In *Technologies for E-Learning and Digital Entertainment* Springer, 381-390.
- [26] Liwicki, M. and Knipping, L., 2005. Recognizing and simulating sketched logic circuits. In *Knowledge-Based Intelligent Information and Engineering Systems* Springer, 588-594.
- [27] Meyer, B., Marriott, K., Bickerstaffe, A., and Knipping, L., 2009. Intelligent diagramming in the electronic online classroom. In *Human System Interactions, 2009. HSI'09. 2nd Conference on IEEE*, 177-183.
- [28] Ouyang, T.Y. and Davis, R., 2011. ChemInk: a natural real-time recognition system for chemical drawings. In Proceedings of the 16th international conference on Intelligent user interfaces ACM, 267-276.
- [29] Oviatt, S., Arthur, A., and Cohen, J., 2006. Quiet interfaces that help students think. In Proceedings of the 19th annual ACM symposium on User interface software and technology ACM, 191-200.
- [30] Plimmer, B. and Apperley, M., 2003. Software to sketch interface designs. In *Ninth International Conference on Human-Computer Interaction*, 73-80.
- [31] Polsley, S., Ray, J., Nelligan, T., Helms, M., Linsey, J., and Hammond, T., 2016. Leveraging trends in student interaction

- to enhance the effectiveness of sketch-based educational software. In *Revolutionizing Education with Digital Ink* Springer, 103-114.
- [32] Sabisch, A.T.C., 2014. CoMo: a whiteboard that converses about code Massachusetts Institute of Technology.
- [33] Stahovich, T.F., 2011. Pen-based interfaces for engineering and education. In *Sketch-based Interfaces and Modeling* Springer, 119-152.
- [34] Stahovich, T.F. and Lin, H., 2016. Enabling data mining of handwritten coursework. *Computers & Graphics* 57, 31-45.
- [35] Taranta, E.M. and Laviola Jr, J.J., 2015. Math Boxes: A Pen-Based User Interface for Writing Difficult Mathematical Expressions. In *Proceedings of the 20th International Conference on Intelligent User Interfaces* ACM, 87-96.
- [36] Tversky, B., 2015. Keynote Address: Tools for Thinking. In *The Impact of Pen and Touch Technology on Education* Springer, 1-4.
- [37] Valentine, S., Lara-Garduno, R., Linsey, J., and Hammond, T., 2015. *Mechanix: A Sketch-Based Tutoring System that Automatically Corrects Hand-Sketched Statics Homework*. In *The Impact of Pen and Touch Technology on Education* Springer, 91-103.
- [38] Wang, G., Bowditch, N., Zeleznik, R., Kwon, M., and Laviola Jr, J.J., 2016. A Tablet-Based Math Tutor For Beginning Algebra. In *Revolutionizing Education with Digital Ink* Springer, 91-102.
- [39] Wang, M., Plimmer, B., Schmieder, P., Stapleton, G., Rodgers, P., and Delaney, A., 2011. SketchSet: Creating Euler diagrams using pen or mouse. In *Visual Languages and Human-Centric Computing (VL/HCC), 2011 IEEE Symposium on IEEE*, 75-82.
- [40] Wetzel, J. and Forbus, K., 2015. Increasing Student Confidence in Engineering Sketching via a Software Coach. In *The Impact of Pen and Touch Technology on Education* Springer, 107-118.
- [41] Williford, B., Taele, P., Nelligan, T., Li, W., Linsey, J., and Hammond, T., 2016. Persketchtivity: an intelligent pen-based educational application for design sketching instruction. In *Revolutionizing Education with Digital Ink* Springer, 115-127.
- [42] Kitchenham, B., 2004. Procedures for performing systematic reviews. *Keele, UK, Keele University* 33, 2004, 1-26.