WORKING NOTE AC67

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MIXING DISCRETE AND CONTINUOUS SIMULATION.

THE PROBLEM.

Discrete simulation is commonly implemented by setting up some sort of queue of operations to be performed. An operation can insert other operations into the queue for immediate execution (appropriate when the consequences of changes to output values must be evaluated immediately), or it can schedule operations for execution at some later time. Systems for discrete simulation are most naturally described in imperative terms :

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Operation 23 :
    When activated :
        Do certain things;
        If somecondition schedule operation 17 after 0 ticks;
        Schedule operation 3 after 25 ticks;
        Schedule operation 23 after 10 ticks.
```

Continuous simulation normally requires that each operation be performed at every “tick” of a system clock. While it is possible that some operations may be unnecessary in some cycles because their inputs have not changed, this is an exceptional case. Systems for continuous simulation are most naturally described in declarative terms :

$$dm/dt = 2LA - k_2mA - 2k_3m^2 - k_4mr;$$
$$dr/dt = k_2mA - k_4mr - 2k_5r^2.$$

Our choice of description is not completely determined by the system under investigation alone; in making the choice, we also take into account the amount of detail we wish to reproduce in the simulation. The example given above for continuous simulation is a model of the behaviour of a certain chemical reaction system; at a very fine level of detail, the same system could be described as a set of colliding, and sometimes reacting, molecules, when a discrete simulation would be more natural.

(The choice of implementation technique is even less dependent on the system. Any continuous model, if evaluated on a conventional digital computer, is in fact evaluated discretely; while, if we choose to inspect every operation of a discrete simulation during every computing cycle, it is in effect being evaluated continuously.)

If we intend to provide a programme which can simulate anything whatever, we have to provide means of describing both of these forms, both separately (for systems which are unambiguously of one type or the other), and together (for systems with properties naturally handled in both ways).

Can we devise a language in which these descriptions can be formulated naturally ? Perhaps it would be easier if we had a model which would help us to imagine the relationships between the different parts of the system.

A MODEL.

We live in a world of events which have unfolding consequences. If I put a ball onto a sloping table, it will begin to roll. To simulate that system, it would be natural to think of the ball’s arrival at the table as an event, and to handle the subsequent motion by continuous techniques. But there is a sense in which the continuous part of the behaviour - rolling along the table - is “always there”; whenever a ball appears on the table, it will roll subject to the same laws and constraints.

Can we generalise this view ? Can we say that the description of a system which we expect to simulate continuously is analogous to the “natural law” of our little universe, which by its nature always applies and must therefore be taken into account continuously; while the events are means of resetting the universe to a new configuration, and, once accomplished, can be forgotten ?

Consider the example again. What happens when the ball reaches the edge of the table ? We want to switch from the old “natural law” appropriate to rolling along a table to a new one appropriate to falling through the air. Clearly (?) the arrival at the edge of the table must constitute an event. Is it reasonable to describe the effect of the event as transferring the action from one universe (the top of the table) to another (the space between table and floor) ? If we do, we move towards a picture of a problem space split up into a set of disjoint universes, each having its own set of natural laws. If a universe is not at equilibrium, it needs continuous attention; but events can disturb the state of equilibrium of universes (where "disturb" may include "restore" : consider the table-top universe after the ball has rolled over the edge).

A description of the ball and table system :

```
IN UNIVERSE A :
Laws :
     Equations of motion for rolling along the table.
     If coordinates at edge of table, Schedule Event A2
     now.
Event A1 :
     Add a ball at coordinates x, y;
     Add Universe A to the "continuous attention" set.
Event A2 :
     Remove the ball;
     Remove Universe A from the "continuous attention"
     set;
     Schedule Event B1 now.

IN UNIVERSE B :
Laws :
     Equations of motion for falling.
     If coordinates at floor …
Event B1 :
     Add a ball at coordinates x, y, z;
     Add Universe B to the "continuous attention" set.
```

In the chemical system of the continuous example, m and r represent concentrations of two chemical substances. The point of interest in the system is the behaviour of these concentrations as the light intensity (L) is switched on and off at different frequencies. The complete system can be described in terms something like this :

Laws :

$$dm/dt = 2LA - k_2mA - 2k_3m^2 - k_4mr;$$

$$dr/dt = k_2mA - k_4mr - 2k_5r^2.$$

Event 1 :

Set L to L_{max}.

Schedule Event 2 after time t_{on}.

Event 2 :

Set L to 0.

Schedule Event 1 after time t_{off}.

QUESTIONS.

- Is this a useful view ? How general is it ? Does it help with implementation ?
- Can universes be nested ? It may be useful to inherit natural laws from superuniverses.
- If it's not very good, how could it be improved ? Or should it just be thrown away ?
- What sort of communications are needed between events, universes … ?