Computer Science 773

Robotics and Real-time Control

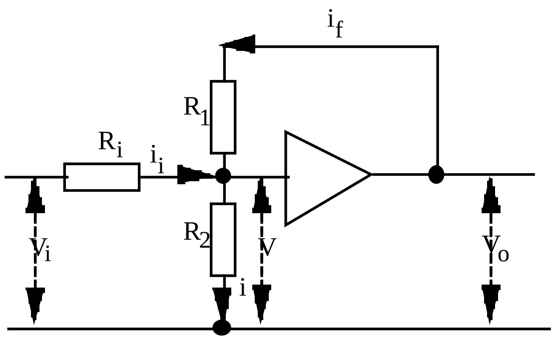
ANALOGUE HARDWARE

Analogue hardware itself isn't specially important for the course, but it's a convenient example of how devices which incorporate continuously varying quantities behave. As quite a bit of control computing is devoted to controlling various sorts of machines with continuous motions, that's of interest. The similarity between electrical machines and other sorts (mechanical, hydraulic, etc.) is very strong, and in many cases it is customary to express a mechanical or hydraulic system in terms of its electrical "equivalent circuit" for analysis.

I've restricted this discussion to things which you might use to build analogue computers. Again, it isn't a very significant choice, but they're the same things that you might use to simulate other sorts of machine, and we have a couple of little analogue computers in the laboratory if you'd like to play with them.

OPERATIONAL AMPLIFIERS.

The operational amplifier is the basic electronic analogue computing element. It is typically a high-gain DC amplifier with high input impedance and a feedback connection from output to input.



If the current into the amplifier is negligible (recall the high input impedance),

$$i = i_i + i_f$$

From Ohm's law :

$$V = iR_2$$
$$V_i = V + i_iR_i$$
$$V_o = V + i_fR_1$$

If the amplifier's gain is G :

$$V_o = GV.$$

In practice, if G is large (say, thousands) and V_o is still of the same order of magnitude as V_i, V must be very much smaller. If we assume that V = 0 (which it isn't, but it's close), there are two consequences : as R_2 is in practice not particularly small, i must be small; and, if i is small, then i_i is approximately equal to $-i_f$. Accepting these approximations, some obvious manipulation gives :

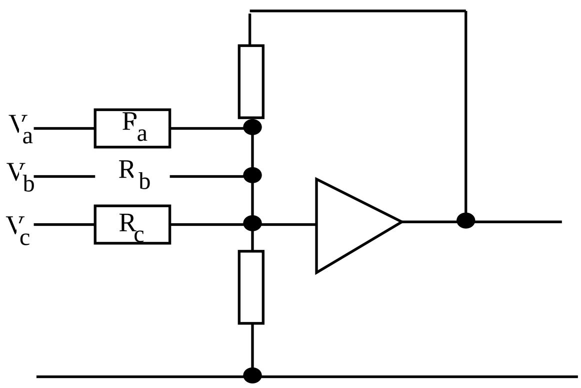
$$V_o / V_i = - R_1 / R_i.$$

The important fact about that equation is that the gain of the amplifier with feedback is determined by the ratio of two resistances, which can be determined very precisely, and is constant; the original amplifier gain was G, which can easily be made large but is very difficult to make precise and stable. We have traded high gain and low quality for moderate gain (in practice, the values of R_1 and R_i are comparable) and high quality.

To be more precise, we should specify that the "moderate" gain is a *voltage* gain. Our analysis has said nothing about power gains, and operational amplifiers are frequently used as means of increasing the power of a signal. Recall that in the BACKGROUND sheet, we remarked that the lack of precise power amplification was a severe hindrance to the development of mechanical control systems; with electrical systems, the problem is overcome.

A SUMMING AMPLIFIER.

Consider a modification of the circuit shown above, with the single input and input resistor replaced by several :



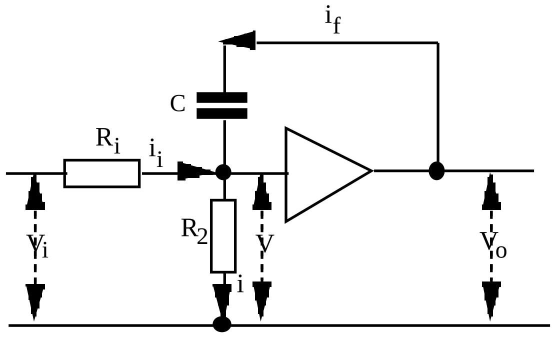
By just the same argument,

$$V_o = (V_a / R_a + V_b / R_b + V_c / R_c) R_1.$$

For the special case where $R_a = R_b = R_c$, the output is proportional to the input; the circuit is an analogue adder.

AN INTEGRATOR.

Now suppose that the feedback resistor, R_1, in the original operational amplifier is replaced by a capacitor with capacitance C.



The feedback current is now determined by :

$$i_f = C\ d(V_o - V)/dt,$$

but the rest of the argument still holds. Following similar manipulations,

$$C\ dV_o/dt = - V_i / R_i,$$

or

$$V_o = - (\int V_i\ dt) / CR_i.$$

The output is proportional to the integral of the input with respect to time. The quantity CR_i has the dimensions of time; it is the circuit's time constant.

ANALOGUE COMPUTATION.

Other analogue computation circuits can be constructed, but the summing amplifier and the integrator are the major components of analogue systems. They can be used to construct analogue computers; and the analogue computers can be used to calculate the control signals required in an automatic control system. Summation, integration, and scaling (multiplication by a constant) are sufficient for linear systems.

Alan Creak,
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