UC 2008 WORKSHOP ON PHYSICS AND COMPUTATION

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May 20, 2008

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- Andrew ADAMATZKY

  University of the West England, BRISTOL, U.K.

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  FROM REACTION-DIFFUSION TO PHYSARUM COMPUTING

  ABSTRACT. A reaction–diffusion chemical computer is a spatially extended non–stirred reaction where data are represented by concentration profiles of disturbances, information is transferred by traveling wave–patterns, computation is done in collision of wave–fronts, and final concentration, or excitation, profile represents results of the computation. Reaction–diffusion computers are massively parallel non–silicon computers capable of efficiently solving a wide range of problems of image processing, computational geometry, and robot control. They can also realize functionally complete logical systems, thus computationally universal. However reaction–diffusion computers cannot compute proximity graphs and lack ability to realize tree–like machines (ancestors of random access machines).

  There is a wonderful model of a reaction–diffusion (excitable) medium encapsulated in an elastic membrane: vegetative state, or plasmodium, of Physarum polycephalum. We experimentally demonstrate that computation of spanning trees, and other types of proximity graphs, and implementation of storage-modification devices (e.g. Kolmogorov–Uspensky machine) can be executed by plasmodium.

- Udi BOKER

  Tel Aviv University

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  THE INFLUENCE OF THE DOMAIN INTERPRETATION ON COMPUTATIONAL MODELS

  ABSTRACT. Computational models are usually defined over specific domains. For example, Turing machines are defined over strings, and the recursive functions over the natural numbers. Nevertheless, one often uses one computational model to compute functions over another domain, in which case, one is obliged to employ a representation, mapping elements of one domain into the other. For instance, Turing machines (or modern computers) are interpreted as computing numerical functions, where strings are interpreted as numbers, via binary or decimal representation, say.

  We ask: Is the choice of the domain interpretation important? Clearly, complexity is influenced, but does the representation also affect computability? Can it be that the same model computes strictly more functions via one representation than another? We show that the answer is "yes," and further analyze the influence of domain interpretation on the

extensionality of computational models (that is, on the set of functions computed by the model).

We introduce the notion *interpretation–complete* for computational models that are basically unaffected by the choice of domain interpretation, and prove that Turing machines and the recursive functions are interpretation–complete, while two–counter machines are incomplete. We continue by examining issues based on model extensionality, which are influenced by the domain interpretation. We provide a notion for comparing computational power of models operating over arbitrary domains, as well as an interpretation of the Church–Turing Thesis over arbitrary domains.

- Olivier BOURNEZ

  LORIA

  `http://www.loria.fr/~bournez/`

  ON THE CONVERGENCE OF A POPULATION PROTOCOL WHEN POPULATION GOES TO INFINITY

  ABSTRACT. Population protocols have been introduced as a model of sensor networks consisting of very limited mobile agents with no control over their own movement. A population protocol corresponds to a collection of anonymous agents, modeled by finite automata, that interact with one another to carry out computations, by updating their states, using some rules.

  Their computational power has been investigated under several hypotheses but always when restricted to finite size populations. In particular, predicates stably computable in the original model have been characterized as those definable in Presburger arithmetic.

  In this lecture, we study mathematically a particular population protocol that we show to compute in some natural sense some algebraic irrational number, whenever the population goes to infinity. Hence, we show that these protocols, seem to have a rather different computational power when considered as computing functions, and when a huge population hypothesis is considered. (Talk based on joint work with Philippe CHASSAING, Johanne COHEN, Lucas GERIN and Xavier KOEGLER.)

- Časlav BRUKNER

  Fakultät für Physik, Universität Wien, Austria

  `http://homepage.univie.ac.at/Caslav.Brukner/`

  HOW QUANTUM MECHANICAL EXPERIMENTS CAN TEST THE (UN)DECIDABILITY OF MATHEMATICAL STATEMENTS?

  ABSTRACT. The mathematics of the early twentieth century was concerned with the question whether a complete and consistent set of axioms for all of mathematics is conceivable. In 1931 Gödel showed that this

is fundamentally impossible. In every consistent theory capable of expressing elementary arithmetic there are propositions which can neither be proven nor disproven within the axiomatic system, i.e. they are *undecidable*. It was argued that mathematical undecidability arises whenever a proposition to be proven requires more information than it is contained in a given set of axioms. On the other hand, quantum mechanics puts a fundamental upper limit on how much information can be encoded in a quantum system. I will show that this allows us to use quantum measurements to test the (un)decidability of certain mathematical propositions. To this end, I will show that elementary quantum systems are capable of encoding axioms, and quantum measurements are capable of verifying truth values of propositions. If a mathematical proposition is undecidable in the deductive system of the axioms encoded in a quantum system, then the system will give random results in the measurement that is associated to the proposition. This sheds some new light on the long–standing discussion of whether or not quantum randomness is irreducible.

- S. Barry COOPER

  University of Leeds

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  EMERGENCE AS A COMPUTABILITY–THEORETIC PHENOMENON

  ABSTRACT. In dealing with emergent phenomena, a common task is to identify useful descriptions of them in terms of the underlying atomic processes, and to extract enough computational content from these descriptions to enable predictions to be made. Generally, the underlying atomic processes are quite well understood, and (with important exceptions) captured by mathematics from which it is relatively easy to extract algorithmic content.

  A widespread view is that the difficulty in describing transitions from algorithmic activity to the emergence associated with chaotic situations is a simple case of complexity outstripping computational resources and human ingenuity. Or, on the other hand, that phenomena transcending the standard Turing model of computation, if they exist, must necessarily lie outside the domain of classical computability theory.

  In this talk we suggest that much of the current confusion arises from conceptual gaps and the lack of a suitably fundamental model within which to situate emergence. We examine the potential for placing emergent relations in a familiar context based on Turing's 1939 model for interactive computation over structures described in terms of reals. The explanatory power of this model is explored, formalising informal descriptions in terms of mathematical definability and invariance, and relating a range of basic scientific puzzles to results and intractable problems in computability theory.

- Jean-Charles DELVENNE

Imperial College

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WHAT IS A UNIVERSAL COMPUTING MACHINE?

ABSTRACT. Computation has been initially defined for a specific class of dynamical systems: Turing machines. Among them, the most powerful ones are called "universal." Later on, many dynamical systems, such as cellular automata or neural networks, have been proved to be Turing–universal, in various meanings. We review some definitions of "Turing–universal computation" for dynamical systems, and the relations between Turing universality and dynamical properties (e.g., chaos). The emphasis in on general results rather than particular examples.

- Francisco António DORIA

Universidade Federal do Rio de Janeiro

http://buscatextual.cnpq.br/buscatextual/visualizacv.jsp?id=K4783152D2
http://famadoria.googlepages.com/franciscoantoniodoria

HOW TO BUILD A HYPERCOMPUTER

ABSTRACT. We claim that the theoretical hypercomputation problem has already been solved, and that what remains is an engineering problem. We review our construction of the Halting Function (the function that settles the Halting Problem) and then sketch possible blueprints for an actual hyper-computer.

- Jérôme DURAND–LOSE

LIFO, Université d'Orléans, France

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BLACK HOLE COMPUTATION: IMPLEMENTATIONS WITH SIGNAL MACHINES

ABSTRACT. The so-called Black Hole model of computation involves a non Euclidean space-time where one device is infinitely "accelerated" on one world-line but can send some limited information to an observer working at "normal pace". The key stone is that after a finite duration, the observer has received the information or knows that no information was ever sent by the device. This allows to decide semi-decidable problems and clearly falls out of classical computability.

A setting in a continuous Euclidean space-time that mimics this is presented. The computations generate line-segments in a continuous space-time. Not only is Zeno effect possible but it is used to unleash the black hole power. Both discrete (classical) computation and analog computation (in the understanding of Blum, Shub and Smale) are considered. Moreover, using nested singularities (which are built), it is shown how to decide higher levels of the corresponding arithmetical hierarchies.

- Jerzy GÓRECKI

  Institute of Physical Chemistry (Polish Academy of Science) and Faculty of Mathematics and Natural Sciences (Cardinal Stefan Wyszynski University), Warsaw, Poland

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  INFORMATION PROCESSING WITH STRUCTURED EXCITABLE MEDIUM

  ABSTRACT. There are many ways in which a nonlinear chemical medium can be used for information processing. Here we are concerned with an excitable medium and a straightforward method of information coding: a single excitation pulse represents a bit of information and a group of excitations forms a message. On the basis of such assumptions information can be coded or in the number of pulses or in the times between subsequent excitations.

  The properties of excitable medium provide us with a pleasant environment for information processing. Pulses of excitation appear as the result of external stimuli and they propagate in a homogeneous medium with a constant velocity and a stationary shape. This is achieved by dissipating medium energy.

  Our attention is focused on a quite specific type of nonhomogeneous medium that has an intentionally introduced geometrical structure of regions characterized by different excitability levels. In information processing applications the geometry plays equally important role as the dynamics of the medium and allows one to construct devices that perform complex signal processing operations even for a relatively simple kinetics of reactions involved. The ideas of information processing with structured excitable medium are tested in numerical simulations based on simple reaction-diffusion models and in experiments with Bielousov–Zhabotinsky reaction. Considering a chemical signal diode as an example we demonstrate a kind of balance between the geometry and the chemical kinetics. A diode action can be observed for a wide class of reactions if a complex geometry of excitable and nonexcitable areas is set. On the other hand, the geometrical construction of the diode can be simplified, but at the cost of restricted class of reaction parameters allowed.

  We present chemical realizations of simple information processing devices like logical gates, signal comparers or memory cells and we show that by combining these devices as building blocks the medium can perform complex signal processing operations like for example counting of arriving excitations. We also discuss a few ideas for programming a structured information processing medium with excitation pulses. (Talk based on join work with J. N. Gorecka and Y. Igarashi.)

- Daniel GRAÇA

  University of Algarve, Portugal

http://w3.ualg.pt/~dgraca/english.htm

COMPUTATIONAL BOUNDS ON POLYNOMIAL DIFFERENTIAL EQUATIONS

ABSTRACT. In this talk we study from a computational perspective some properties of the solutions of polynomial ordinary differential equations or, what is equivalent, of functions generated by Shannon's General Purpose Analog Computer.

We consider elementary (in the sense of Analysis) discrete-time dynamical systems satisfying certain criteria of robustness. We show that those systems can be simulated with elementary and robust continuous-time dynamical systems which can be expanded into fully polynomial ordinary differential equations in $\mathbf{Q}[\pi]$. This sets a computational lower bound on polynomial ODEs / GPACs since the former class is large enough to include the dynamics of arbitrary Turing machines.

We also apply the previous methods to show that the problem of determining whether the maximal interval of definition of an initial-value problem defined with polynomial ODEs is bounded or not is in general undecidable, even if the parameters of the system are computable and comparable and if the degree of the corresponding polynomial is at most 56.

Combined with earlier results on the computability of solutions of polynomial ODEs / functions generated by GPACs, one can conclude that there is from a computational point of view a close connection between these systems and Turing machines. (Talk based on joint work with Jorge BUESCU and Manuel CAMPAGNOLO.)

• Mark HOGARTH

University of Cambridge

http://www.girton.cam.ac.uk/staff/fellows/byefellows/m_hogarth/

A NEW PROBLEM FOR RULE FOLLOWING

ABSTRACT. While it goes without saying that computing devices need space and time (spacetime), nothing in Turing's theory of idealised computation prepares one for the rich nature of a computer's dependence on the underlying spacetime geometry. Nor does that theory prepare one for the vast array of non–Turing computers (and one computer à la Turing) obtained by painting ordinary Turing machine hardware on a variety of different geometries. Different geometries yield radically different computers, and the range of computers extends far beyond Turing's machine. These discoveries suggest that computability is a two–sided concept like geometry, with pure models on one side and physics on the other. The Church–Turing thesis, which is an attempt to single–out fundamentally one particular computer, is seen as misguided as those nineteenth century attempts to discredit the non–Euclidean geometries.

• István NÉMETI & Hajnal ANDRÉKA

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Budapest.

http://www.renyi.hu/~nemeti/

http://www.renyi.hu/~andreka/

GENERAL RELATIVISTIC HYPERCOMPUTING AND FOUNDATION OF MATHEMATICS

ABSTRACT. Looking at very recent developments in spacetime theory, we can wonder whether these results exhibit features of hypercomputation that traditionally seemed impossible or absurd. Namely, we describe a physical device in relativistic spacetime which can compute a non-Turing computable task, e.g. which can decide the halting problem of Turing machines or decide whether ZF set theory is consistent. Starting from this, we will discuss the impact of recent breakthrough results of relativity theory, black hole physics and cosmology to well established foundational issues of computability theory as well as logic. We find that the unexpected, revolutionary results in the mentioned branches of science force us to reconsider the status of the physical Church Thesis and to consider it as being seriously challenged. We will outline the consequences of all this for the foundation of mathematics (e.g. Hilbert's programme).

Observational, empirical evidence will be quoted to show that the statements above do not require any assumption of some physical universe outside of our one: in our specific physical universe there seem to exist regions of spacetime supporting potential non–Turing computations.

Additionally, new *engineering* ideas will be outlined for solving e.g. the so–called blue–shift problem of GR–computing.

Connections with related talks at the meeting, e.g. those of Jerome Durand–Lose, Mark Hogarth and Martin Ziegler, will be indicated.

- Mike STANNETT

  University of Sheffield

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  COMPUTABLE, UNCOMPUTABLE, NEITHER OR BOTH? — A FINITARY COMPUTATIONAL FORMULATION OF QUANTUM THEORY

  ABSTRACT. What is the relationship between physics and computation? We describe a finitary reformulation of the (non-relativistic) path-integral model which is equivalent to the standard version, but in which particles never follow continuous trajectories; instead, motion comprises a finite sequence of distinct "hops," each of which takes the particle directly from one discrete location to another. The model treats space and time symmetrically, in that particles may hop both forwards and backwards in time as well as space. Despite the finitary nature of the model, it is mathematically equivalent to the standard continuous-path formulation, whence standard assumptions (e.g. time having an arrow) arise as "unavoidable

quantum illusions," Each finitary path generates a computational state machine drawn on "spacetime paper," and this machine computes the trajectory's amplitude. Each machine describes a family of observationally equivalent paths, and the path-integral approach is therefore equivalent to an intrinsically computational "machine-integral" formulation. In a fairly deep sense, then, the quantum world has an inherently computational structure, but one which is defined over a spacetime that could nonetheless allow uncomputable values to be observed.

- Karl SVOZIL

  Institut für Theoretische Physik, Technische Universität Wien, Austria

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  ON THE SOLUTION OF TRIVALENT DECISION PROBLEMS BY QUANTUM STATE IDENTICATION

  ABSTRACT. The trivalent functions of a trit can be grouped into equipartitions of three elements. We discuss the separation of the corresponding functional classes by quantum state identifications.

- John V. TUCKER

  University of Wales Swansea

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  COMPUTATIONS VIA NEWTONIAN AND RELATIVISTIC KINEMATIC SYSTEMS

  ABSTRACT. In this lecture I will survey some old and new results that try to answer these questions:

  1.What are the functions computable by experimental procedures applied to different physical systems? 2.How do they compare with the functions computable by algorithms? 3.Do there exist physical systems that exhibit behaviour is not algorithmically computable? 4.What are the physical limits to computation?

  First, I will give some brief background on computability with continuous data, such as real numbers, which will link with recent work with Jeffery I. Zucker (McMaster) on modelling technologies by networks of physical components.

  Second, I will give a progress report on my new research programme with Edwin J. Beggs (Swansea) investigating notions of experimental computation using physical theories. One aim is to study the embedding of computational models and non–computable behaviours into small simple subtheories of physical theories in order to explore formally the boundary between the computable and non–computable inside physical theories. See, for example, Proc. Royal Society, Series A, 463 (2007), 1541-1561.

  Some theorems show there exists simple kinematic systems that can compute algorithmically non–computable data. These results give technical

insight into the questions above, and show they are painfully subtle. Although we have a methodology that enables us to make dramatic technical progress, we are also aware that we are stockpiling mathematical and philosophical problems. These problems concern the nature of both physical theories and computing technologies.

- Kumaraswamy VELUPILLAI

  John E. Cairnes Professor of Economics, National University of Ireland, and University of Trento

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  UNCOMPUTABILITY AND UNDECIDABILITY IN ECONOMIC THEORY

  ABSTRACT. Interpreting the core of economic theory to be defined by General Equilibrium Theory and Game Theory, a general analysis of the computable and decidable content of the implications of these two areas is investigated. The point of view adopted in that of classical recursion theory and varieties of constructive mathematics. In particular, all the main computable and constructive claims of the practitioners of so–called Computable General Equilibrium Theory are shown to be false. The constructive content of the Hahn–Banach theorem is also dissected to reveal the uncomputability of fundamental theorems of welfare economics. The role of decidability in economic theory is investigated by returning to the Zermelo–Banach tradition of game theory, which was subverted by the von Neumann–Nash approach via fixed-point theorems.

- Damien WOODS

  University of Seville

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  OPTICAL COMPUTING

  ABSTRACT. We consider optical computers that encode data using images and compute by transforming such images. We give an overview of a number of such optical computing architectures, including descriptions of the type of hardware commonly used in optical computing, as well as some of the computational efficiencies of optical devices. We go on to discuss optical computing from the point of view of computational complexity theory, with the aim of putting some old, and some very recent, results in context. Finally, we focus on a particular optical model of computation called the continuous space machine. We describe some results for this model including characterisations in terms of well-known complexity classes. (This is joint work with Thomas J. NAUGHTON from the National University of Ireland, Maynooth University of Oulu.)

- Martin ZIEGLER

  University of Paderborn, GERMANY

http://www.upb.de/cs/ziegler.html

PHYSICALLY-RELATIVIZED CHURCH-TURING HYPOTHESES

ABSTRACT. The Church–Turing Hypothesis is a claim involving both computer science and physics. Therefore, any meaningful formalization of it must refer to a physical theory: e.g. to Newtonian mechanics of point masses, to continuum mechanics of solids, to electrostatics, to classical optics of light–rays, to Maxwell's electrodynamics, or to one of the many flavors of quantum mechanics and/or theories of relativity. Several examples from literature reveal that the status of the Church–Turing Hypothesis may indeed depend on the physical theory under consideration.

- Jeffery ZUCKER

  McMaster University, Canada

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  THE SEMANTICS OF CLASSICAL PHYSICAL NETWORKS

  ABSTRACT. Analog computation is based on networks of components, which, traditionally, are either mechanical or electrical in nature. We construct mathematical models of such networks. A major question that interests us is: Can we decide whether, or within what limits, our models represent physical systems that are stable w.r.t. perturbations? We proceed by case studies, considering two broad categories of analog systems, involving continuous vs discrete data streams, corresponding (respectively) to continuous vs discrete global clocks.

  We also relate our models of analog systems to (digital) computability theories for topological data types. (Talk based on joint work with John TUCKER from the University of Wales Swansea.)

4 TIMETABLE

The time schedule is shown in the following table.
(TO BE INCLUDED)