# Workshop on Computability, Randomness, and Structure

# University of Auckland, Dept. of Computer Science

## April 22-23, 2010

Organized by André Nies.

All talks take place in the CS Department, Room 561.

## April 22

## 16:00 -18:00

TITLE: Interactions of randomness and computability I

SPEAKER: André Nies, The University of Auckland

ABSTRACT: This is an updated version of a lecture series I gave at the ASL summer meeting in Sofia, 2009.

The first hour is targeted at a general scientific audience. We discuss our intuition on randomness, and how it translates into mathematical definitions. We give examples of random sequences of bits, and random continuous functions.

Thereafter, we study the complexity and randomness aspects of sets of natural numbers in more depth. Traditionally, computability theory is concerned with the complexity aspect. However, computability-theoretic tools can also be used to introduce mathematical definitions of randomness of a set; further, once defined, these notions can be studied by considering their interplay with the complexity aspect of a set. There is also an interaction in the converse direction: concepts and methods from randomness enrich computability theory. This converse interaction is the subject of much recent research and a main topic of my 2009 Oxford Book ``Computability and randomness''.

## April 23

## 14:00 -14:50

TITLE: The Effective Algebra of Uncountable Linear Orders

SPEAKER: *Asher Kac*h, Victoria University of Wellington (is joint work with Noam Greenberg, Steffen Lempp, and Daniel Turetsky)

ABSTRACT: Countable linear orders are fairly well understood in the effective setting. In contrast, the study of *uncountable* linear orders in an effective setting is rather recent. After motivating and introducing computability theory within the context of larger domains, we will compare and contrast the effective algebra of countable and uncountable (mainly, size \aleph\_1) linear orders. In particular, we will discuss computable categoricity, degree spectra, and externa relations (such as successor, being infinitely-far-apart, and being uncountably-far-apart).

### 15:10 -16:00

TITLE: The complexity of neutral measures.

SPEAKER: *Adam Day*, Victoria University of Wellington (joint work with Joseph Miller)

ABSTRACT: There have been different approaches taken to studying randomness for non-computable probability measures. We will explain the approach taken by Reimann and Slaman, along with the uniform test approach first introduced by Levin and also used by Gacs, Horyup and Rojas. We will outline why these approaches are fundamentally equivalent. Levin showed that there exists a non-computable probability measure for which all sequences are random. This he called a neutral measure. We will show that a neutral measure has no least Turing degree representation. We will explain why Miller's framework of continuous degrees can be used to determine the computational complexity of neutral measures.

### 16:00 Coffee Break

### 16:30 - 18:00

TITLE: Interactions of randomness and computability II

SPEAKER: André Nies, The University of Auckland