Dear Cris,

I am thinking over our talk, especially the first part of it, which appeared to be very much in the same philosophy as our papers on the trespassing of the Turing barrier. You probably recall, that we actually re-opened the question on the Turing barrier, attempting to formulate the question in such a way , that it becomes ”environment-dependent”, that is can be classified as a Gödel-type question, which can be answered in different ways, depending on the structure we have in reality. According to your intuition, we wanted to find a structure of the probability space, where the probability of non-clicking of the control device would go to zero, when the number ”l” of trials goes to ∞, once a “ wrong stack” is present in the system. We were not able to get this result for Gaussian probability distribution in the infinite-dimensional Hilbert space. This subtle result appeared to be connected with the impossibility of Lebesgues continuation of the gaussian measure defined by the density $\pi^{-n/2} \exp[-\sum_{s=1}^{n} |x_s|^2]$ on all finite-dimensional cylindrical sets. Your hypothesis on ”natural ” behavior of the probability of non- clicking of the controlling device became right, if the Hilbert space is supplied with appropriate Wiener measure. This was our result.

In our talk, announced on the conference program, the question on the decay is treated similarly. All experiments reveal an exponential decay in all quantum experiments, while the standard quantum mechanics based on the Schrödinger equation gives formally a non-exponential decay. In our talk we explain this phenomenon based on assumption, that separation of the fast variable in the process , to derive the Schrödinger equation from the second order (in time) Klein-Gordon-Fock equation, and treat the wave- function based on probabilistic interpretation, brings a non reparable error into quantum evolution, while returning to exact model (without splitting of the fast variable) permits to get the exponential decay in the corresponding *energy-normed space*. Unfortunately this is not for free- the probabilistic interpretation of the wave function is possible only for Schrödinger type equation. So the exponential decay and the probabilistic interpretation of the wave function can’t be fit into a common physical picture.

While thinking on on all of that I noticed that imperfections of the gaussian distribution can be noticed even before passing to the limit of the infinite-dimensional space. I am planning to show you a naive calculation which we never noticed before, to reveal the imperfection, even without discussion of the Lebesgues continuation of the Gaussian measure, which would be easier to see for general mathematical audience.

Let us introduce in the Hilbert space $\{X\}$ the operator $Q = P^{\|} + (1+\gamma)P^{\perp},\ 0 < \gamma << 1$ with two complementary orthogonal projections $P^{\|}, P^{\perp}$. Presence of the term $\gamma)P^{\perp}$ corresponds, in our quantum version of the Turing problem, the presence of a stuck of false coins. In our “quantum” experiment we observe the quadratic forms of powers of the operator $\langle Q^l X, X\rangle = |X|^2 + [(1+\gamma)^l - 1]|X^{\perp}|^2$. Once for given X the inequality $\langle Q^l X, X\rangle < (1+\varepsilon)|X|^2$ is fulfilled, the control device does not click, as if $Q = I$, thus providing us an ”erroneous information” in the case when $\gamma > 0$. But if the Hilbert space $\{X\}$ is finite-dimensional, then for large l the probability of non-clicking becomes small. Let us assume now, that *the dimension of the quantum space is finite for each experiment , but not bounded uniformly.* It appears that the probability of clicking /non clicking of the control device depends on how fast the dimension of the observed part of the quantum space growth in course of the experiment.

Indeed, consider the set of vectors X defined by the ”non clicking condition” for Q^l

$$\frac{|X^{\perp}|^2}{|X|^2} \leq \frac{\varepsilon}{(1+\gamma)^l - 1}.$$

The gaussian measure of the corresponding ”non-click” set $G_{non-click}$ is calculated as an integral with the gaussian density on the 2m-dimensional Hilbert space:

$$G_{non-click} = \int_{\frac{|X^{\perp}|^2}{|X|^2} \leq \frac{\varepsilon}{(1+\gamma)^l - 1}} \pi^{-n/2} \exp[-\sum_{s=1}^{n} |x_s|^2] dx^n \approx$$

$$\pi^{-m} \int_0^{\infty} e^{-r^2} r^{2m-1}\, dr \frac{\varepsilon}{(1+\gamma)^l - 1} \Sigma_{2m-1},\ n = 2m. \tag{1}$$

Where Σ_{2m-1} is an area of the unit sphere in the $2m-1$ dimensional Hilbert space. Calculating the n-dimensional integral via separation variables we obtain the relations between the area of the unit sphere and 1D the Gaussian integrals:

$$label(Eq_G auss) \int_0^{\infty} e^{-r^2} r^{2m-1}\, dr \Sigma_{2m} = \pi^m,\ \int_0^{\infty} e^{-r^2} r^{2m-2}\, dr \Sigma_{2m-1} = \pi^{m-1/2}. \tag{2}$$

This implies an expression for the non-click probability in terms of the 1D gaussian integrals

$$G_{non-click} = \frac{\varepsilon}{\sqrt{\pi}[(1+\gamma)^l - 1]} \, \frac{\int_0^\infty e^{-r^2}\, r^{2m-1}\, dr}{\int_0^\infty e^{-r^2}\, r^{2m-2}\, dr},$$

which can be calculated via introduction an auxiliary parameter

$$\int_0^\infty e^{-r^2}\, r^{2m-1}\, dr = \frac{d^{m-1}}{d\alpha^{m-1}} \int_0^\infty e^{-r^2}\, r\, dr\Big|_{\alpha=1} = 1/2 \;\; (m-1)!,$$

$$\int_0^\infty e^{-r^2}\, r^{2m-2}\, dr = \frac{d^{m-1}}{d\alpha^{m-1}} \int_0^\infty e^{-r^2}\, dr\Big|_{\alpha=1} = \sqrt{\pi}(m-3/2)(m-5/2)(m-7/2)\dots(3/2)1/2.$$

With regard of

$$\frac{(m-1)(m-2)(m-3)\dots 1}{(m-3/2)(m-5/2)(m-7/2)\dots(3/2)1/2} = \left(1+\frac{1}{2m-3}\right)\left(1+\frac{1}{2m-5}\right)\left(1+\frac{1}{2m-7}\right)\dots\left(1+\frac{1}{1}\right).$$

For large m the asymptotic of the product is represented as $\exp\,[\text{Const} + \sum_{s=1}^{m} \frac{1}{2m-1}] = \text{Const} + 2m \to \infty$ when $m \to \infty$. Hence the non-click probability $G(m,l) \approx \frac{\varepsilon(\;C+m)}{\sqrt{\pi}[(1+\gamma)^l-1]}$ has a natural behavior $G(m,l) \to 0$ if $l \to \infty$, but does not tend to zero, if the dimension m of the quantum space is changing and $m[(1+\gamma)^l - 1]^{-1}$ does not have zero limit.

Dear Cris,

This naive calculation shows, that the gaussian probability can't be extended to the Infinitely-dimensional case in most explicit way. I regret that we missed this elementary criticism of the gaussian probability when discussing the staff in out Turing-barrier text. Of course I do not have time to tell the story in my talk, but probably I just mention it as a most obvious reason to choose another probabilistic structure of the quantum space. Then consider please this my letter as a chance to get another comment from you concerning the deep subject of Turing barrier. Really, I enjoyed working with you a couple of years in this fascinating direction. Wish you strong health, great success in your research, and more recognition- not only in the World scene, which you already enjoy, but also locally. I am sure it will come soon.

Warm regards and best wishes for Elena and Andrea.

Boris.