THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2009
Campus: Tamaki

COMPUTER SCIENCE

COMPSCI 773: Vision Guided Control

(Time allowed: TWO hours)

NOTE: Attempt **all** questions!

Write the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

This is an open book exam. Candidates **may** bring calculators, notes, reference books, or other written material into the examination room.

Section:	**A**	**B**	**C**	Total	
Possible marks:	30	36	33	100	
Awarded marks:					

SURNAME:

FORENAME(S):

STUDENT ID:

QUESTION/ANSWER SHEETS FOLLOW

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Section A: Homogeneous coordinates, affine transform and scaling

Let's define a function T which transforms a 2D point M (Cartesian coordinates: (x, y)) into the point $\tilde{M}$ after performing a rotation (angle θ), a 2D translation (Cartesian coordinates (t_x, t_y) then a scaling (2D scaling factor s_x and s_y).

1. Using homogeneous coordinates notation, write the matrix which characterise T. [4 marks]

2. Write the matrix which transforms $\tilde{M}$ back into M (e.g. the inverse of the transformation in Question 1). [4 marks]

3. Find M given $M(2, 3)$, $\theta = 60$ degrees, $T(1, 2)$ and scalings $s_x = 0.5$ and $s_y = 0.5$. [2 marks]

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4. Find M given $M^{'}(2,3)$, $\theta = 60$ degrees, $T(1,2)$ and scalings $s_x = 0.5$ and $s_y = 0.5$. [2 marks]

5. Transform the following points from homogeneous to Cartesian coordinates or vice versa as indicated below: [5 marks]

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6. Assuming the distortion free pinhole camera model (focal length f) for a single camera, compute the 2D coordinates of point m, projection in the image I of 3D point M $((X, Y, Z))$. [3 marks]

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7. Assuming first order radial distortion pinhole camera model (focal length f, distortion parameter κ_1) for a single camera, compute the 2D coordinates of point m, projection in the image I of 3D point M $((X, Y, Z))$. [2 marks]

8. The coordinates of the image centre are given as $(500, 500)^T$ in pixels. What is the first order distortion parameter value (supposed positive) if distorted and undistorted coordinates differ by 0.01mm (millimetre) at a radial distance from the image centre, equivalent to 500 pixels?
The squared pixel width is 5 microns (micrometres). [5 marks]

9. Using the first order radial distortion parameter calculated in Question 8, compute the undistorted metric coordinates for a point with pixel coordinates $(1000, 1000)^T$. [3 marks]

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Section B: PCA-LDA

Section B:.1 PCA

10. The database A contains six $2D$ points:

$$x_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, x_2 \begin{pmatrix} 0 \\ 2 \end{pmatrix}, x_3 \begin{pmatrix} 0 \\ 3 \end{pmatrix}, x_4 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, x_5 \begin{pmatrix} 2 \\ 0 \end{pmatrix}, x_6 \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

Points x_1, x_2, x_3 belong to class 1, x_4, x_5, x_6 belong to class 2.

Compute the database mean and the centred coordinates y_i for each element x_i [2 marks]

11. Compute the covariance matrix of the centred database given by: $C = \sum_{i=1}^{i=6} y_i y_i^T$

[5 marks]

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12. Compute the eigenvalues (λ_1, λ_2) of the matrix C. [2 marks]

    Hint: The eigenvalues of a matrix B are the zeros of the $\lambda-$second order polynomial form defined by the determinant of the matrix $B - \lambda I_2$, where I_2 is the 2×2 identity matrix.

    The determinant of a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is equal to $a \cdot d - b \cdot c$

13. Find the eigenvectors e_1, e_2 associated to the eigenvalues λ_1, λ_2.

    Hint: By definition, an eigenvector b_i, of the matrix B, associated to an eigenvalue λ_i verifies: $Bb_i = \lambda b_i$. [3 marks]

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14. Express each vector of the database x_i as a weighted linear combination of eigenvectors e_1 and e_2. [6 marks]

    Hint: e_1 and e_2 are orthogonal.

15. Using only the coordinates of the database points x_i long the main PCA direction (e.g. along the eigenvector with the largest eigenvalues), compute the Euclidean distances $d(x_1, x_4)$ and $d(x_1, x_2)$. Compare to the distance sin the original orthogonal basis (e.g. pre-PCA). Any comments? [4 marks]

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Section B:.2 LDA

16. Use the same database as in question 10. Compute the between-class scatter matrix S_B and the within-class scatter matrix S_W for the database A. [5 marks]

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17. Find the direction (described by the vector e) which maximises the distance between the projected mean values of the classes of the database A while keeping the within class variances low. In other terms, do the LDA ! [3 marks]

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18. Draw the database set and the directions of projections computed via PCA and LDA. [2 marks]

19. Compare the distances obtained in question 15 with their values when the database points are projected along the direction obtained via LDA analysis. Comments? [3 marks]

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20. Compute for class 1 and 2 the mean and standard deviation before and after projection on the main PCA and LDA axis. What do you think (a few lines for each answer) of the following three assertions:

    (a) PCA maximises the variance of the overall dataset

    (b) LDA maximises the between-class variance while minimizing the within-class variance

    (c) LDA works better than PCA [3 marks]

Section C: Epipolar Geometry and Matching of Stereo Images

21. How are two cameras placed one with respect to another if epipoles in both the images coincide with the principle points (traces of optical axes)? [3 marks]

22. How are two cameras placed one with respect to another if epipoles in both images seat infinitely far along the Y-axis of the world co-ordinate frame and have the same x-xoordinate? [5 marks]

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23. Given a camera with the projection matrix $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & -0.5 & 0 & -1 \\ 0 & 0 & 2 & -1 \end{bmatrix}$, determine the optical centre of this camera? [4 marks]

24. Let a given stereo pair have the epipoles with the Cartesian co-ordinates $\mathbf{e}_1 = (x_{e,1} = 2, y_{e,1} = 1)$ and $\mathbf{e}_2 = (x_{e,2} = 1, y_{e,2}) = 3$ in the left and right images of a stereo pair, respectively, and let

$$\mathbf{F} = \begin{bmatrix} 3 & 2 & -8 \\ 2 & 3 & -7 \\ -9 & -11 & 29 \end{bmatrix}$$

be the fundamental matrix for this stereo pair. Give the equation for the epipolar line in the left image that corresponds to the point $(x_2 = 0, y_2 = 0)$ in the right image and the equation for the epipolar line in the right image that corresponds to the point $(0, 0)$ in the left image. [5 marks]

25. What relationship does exist between the fundamental matrix $\mathbf{F}$ of a pair of cameras and the corresponding points $\mathbf{x}_1$ and $\mathbf{x}_2$ in the images captured by the cameras? Does this relationship hold for the epipoles in Question 24? [3 marks]

26. Describe, in brief, main reasons why stereo matching that searches for corresponding areas in a stereo pair of images is an ill-posed, in the math sense, problem. [3 marks]

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27. Describe which differences between the corresponding image signals are taken into account in the SSD (Sum of Squared Differences) based stereo matching and the correlation based matching. Your answer should include math models of signals and noise leading to these matching scores. [6 marks]

28. 3D stereo reconstruction of human heads / faces typically uses stereo pairs captured with cameras having a vertically oriented baseline. Explain why such pairs are more appropriate than the pairs with the conventional horizontal baseline. [4 marks]

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