THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2008
Campus: Tamaki

COMPUTER SCIENCE

COMPSCI 773: Vision Guided Control

(Time allowed: TWO hours)

NOTE: Attempt **all** questions!

Write the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

This is an open book exam. Candidates **may** bring calculators, notes, reference books, or other written material into the examination room.

Section:	**A**	**B**	**C**	**D**	Total
Possible marks:	25	17	25	33	100
Awarded marks:					

SURNAME:

FORENAME(S):

STUDENT ID:

QUESTION/ANSWER SHEETS FOLLOW

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Section A: Homogeneous coordinates and calibration

1. Write the matrix relating the homogeneous coordinates of a 2D point after effectuating first a +60 degrees rotation then a translation along the direction supported by the Cartesian vector $(2, 1)$. [3 marks]

2. Write the matrix relating the inverse of the transformation described above. [2 marks]

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3. Transform the following points from homogeneous to Cartesian coordinates or vice versa as indicated below: [5 marks]

Homogeneous coordinates		Cartesian 3D coordinates
$[-6, 2, 2, 0]$	$\longleftrightarrow$	[, ,]
$[3, 1, 1, 1]$	$\longleftrightarrow$	[, ,]
$[0, 1, 2, 1]$	$\longleftrightarrow$	[, ,]

Cartesian 3D coordinates		Homogeneous coordinates
$[0, 1, 0]$	$\longleftrightarrow$	[, , ,]
$[\infty, \infty, \infty]$	$\longleftrightarrow$	[, , ,]

4. Explain why, using the pinhole camera model for a single camera, you cannot compute the 3D coordinates of a point, knowing its (x, y) position in the image plane. [2 marks]

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5. Write the equations relating the distorted and undistorted image coordinates in presence of first order radial distortion. [3 marks]

6. The coordinates of the image centre are given as $(500, 500)^T$ in pixels. What is the first order distortion parameter value (supposed positive) if distorted and undistorted coordinates differ by 0.01mm (e.g. 0.01 millimetre) at a distance equivalent to 500 pixels of the centre?. The squared pixel width is 5 microns. [5 marks]

7. Using the first order radial distortion parameter calculated above, compute the undistorted metric coordinates for point M of pixel coordinates $(1000, 1000)^T$. [5 marks]

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Section B: Line fitting

8. Specify formulae and compute the mean co-ordinates $\overline{x}$, $\overline{y}$, variances μ_{xx}, μ_{yy}, and covariance μ_{xy} of the following six 2D points: [5 marks]

Point n:	1	2	3	4	5	6
Co-ordinate x:	0	2	1	3	4	5
Co-ordinate y:	2	0	1	3	4	5

Mean $\overline{x} =$

Mean $\overline{y} =$

Variance $\mu_{xx} =$

Variance $\mu_{yy} =$

Covariance $\mu_{xy} =$

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9. Using the least-square method, find parameters (α, β, γ) of the best-fitting line $\alpha x + \beta y + \gamma = 0$ for an array of 2D points $\{(x_n, y_n) : n = 1, ..., N\}$ in function of the mean coordinates $\overline{x}$, $\overline{y}$, variances μ_{xx}, μ_{yy} and the covariance μ_{xy} of the points. [7 marks]

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10. Compute the best-fitting line parameters for the six points in Question 8. [3 marks]

11. Represent the points and the associated best-fitting line. Give for each point, its distance to the best fitting-line. [2 marks]

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Section C: Principal Component Analysis

12. The database A contains six $2D$ points:

$$x_1\begin{pmatrix}3\\3\end{pmatrix}, x_2\begin{pmatrix}5\\5\end{pmatrix}, x_3\begin{pmatrix}4\\4\end{pmatrix}, x_4\begin{pmatrix}1\\1\end{pmatrix}, x_5\begin{pmatrix}0\\2\end{pmatrix}, x_6\begin{pmatrix}2\\0\end{pmatrix}$$

Points x_1, x_2, x_3 belong to class 1, x_4, x_5, x_6 belong to class 2.

Section C:.1 PCA

(a) Compute the database mean and the centred coordinates y_i for each element x_i [3 marks]

(b) Compute the covariance matrix of the centred database given by: $C = \sum_{i=1}^{i=6} y_i y_i^T$ [7 marks]

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(c) Compute the eigenvalues (λ_1, λ_2) of the matrix C. [4 marks]

(d) Find the eigenvectors e_1, e_2 associated to the eigenvalues λ_1, λ_2.

[3 marks]

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(e) Express each vector of the database x_i as a weighted linear combination of eigenvectors e_1 and e_2. [6 marks]

(f) What would you call the transformation between the original reference frame and the new reference frame defined by the PCA eigenvectors? [2 marks]

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Section D: Epipolar Geometry and Matching of Stereo Images

13. How are two cameras placed one with respect to another if epipoles in both the images coincide with the principle points (traces of optical axes)? [3 marks]

14. How are two cameras placed one with respect to another if epipoles in both images seat infinitely far along the Y-axis of the world co-ordinate frame and have the same x-xoordinate? [5 marks]

15. Given a camera with the projection matrix $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & -0.5 & 0 & -1 \\ 0 & 0 & 2 & -1 \end{bmatrix}$, determine the optical centre of this camera? [4 marks]

16. Let a given stereo pair have the epipoles with the Cartesian co-ordinates $\mathbf{e}_1 = (x_{e,1} = 2, y_{e,1} = 1)$ and $\mathbf{e}_2 = (x_{e,2} = 1, y_{e,2}) = 3$ in the left and right images of a stereo pair, respectively, and let

$$\mathbf{F} = \begin{bmatrix} 3 & 2 & -8 \\ 2 & 3 & -7 \\ -9 & -11 & 29 \end{bmatrix}$$

be the fundamental matrix for this stereo pair. Give the equation for the epipolar line in the left image that corresponds to the point $(x_2 = 0, y_2 = 0)$ in the right image and the equation for the epipolar line in the right image that corresponds to the point $(0, 0)$ in the left image. [5 marks]

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17. What relationship does exist between the fundamental matrix $\mathbf{F}$ of a pair of cameras and the corresponding points $\mathbf{x}_1$ and $\mathbf{x}_2$ in the images captured by the cameras? Does it hold for the epipoles in Question 16? [3 marks]

18. Describe, in brief, main reasons why stereo matching that searches for corresponding areas in a stereo pair of images is an ill-posed, in the math sense, problem. [3 marks]

19. Describe which differences between the corresponding image signals are taken into account in the SSD (Sum of Squared Differences) based stereo matching and the correlation based matching. Your answer should include math models of signals and noise leading to these matching scores. [6 marks]

20. 3D stereo reconstruction of human heads / faces typically uses stereo pairs captured with cameras having a vertically oriented baseline. Explain why such pairs might or might not be more efficient than the images for the conventional horizontal baseline. [4 marks]

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