THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2006
Campus: Tamaki

COMPUTER SCIENCE

COMPSCI 773: Vision Guided Control

(Time allowed: TWO hours)

NOTE: Attempt **all** questions!

Write the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

This is an open book exam. Candidates **may** bring calculators, notes, reference books, or other written material into the examination room.

Section:	**A**	**B**	**C**	**D**	Total
Possible marks:	20	40	25	15	100
Awarded marks:					

SURNAME: __________

FORENAME(S): __________

STUDENT ID: __________

QUESTION/ANSWER SHEETS FOLLOW

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Section A: Homogeneous coordinates and calibration

1. Write the formulae relating homogeneous coordinates of a 2D point to its Cartesian (X,Y) coordinates. [3 marks]

$x/t = X$
y/t=Y
With x,y,t homogeneous coordinates of a 2D (cartesian) point

2. Transform the following points from homogeneous to Cartesian coordinates or vice versa as indicated below: [7 marks]

Homogeneous coordinates		Cartesian 3D coordinates
$[9,6,3,3]$	$\longleftrightarrow$	[3 , 2 , 1]
$[6,2,2,0]$	$\longleftrightarrow$	[∞ , ∞ , ∞]
$[10,2,2,2]$	$\longleftrightarrow$	[5 , 1 , 1]
$[0,0,0,1]$	$\longleftrightarrow$	[0 , 0 , 0]

Cartesian 3D coordinates		Homogeneous coordinates
$[9,6,3]$	$\longleftrightarrow$	[27 , 18 , 9 , 3]
$[0,0,0]$	$\longleftrightarrow$	[anything except ∞,same,same,∞]
$[\infty,\infty,\infty]$	$\longleftrightarrow$	[anything,anything,anything, 0]

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3. Explain meaning of the term Radius of Ambiguity and whether it is used for one camera or two camera calibration error assessment? [3 marks]

The radius of ambiguity corresponds to the distance between a calibration target centre and its backprojected position (for perfect calibration it is the same point), in the calibration target plane, using the ray passing through the camera optical centre and the image point which represents the target centre. The radius of ambiguity reflects radial distorsion, error in optical centre position as well as target centre image position errors. It relates to single camera calibration error assessment.

4. Point $[100, 100, 0]$ is the centre of a calibration patch in the plane XY of a target object which was used to calibrate a camera. This point is imaged by the camera sensor with coordinates $[X_t, Y_t, Z_t]$=$[5.05, 5.1, -1000]$. The lens of the camera has a fixed focal length of 50 millimetres. Compute the coordinates of the point where the back-projected ray intersects the calibration plane. Compute the corresponding Radius of Ambiguity. All coordinates are given in millimetres in the world reference frame (or basis) whose origin is attached to the calibration cube. [7 marks]

Lets call X_b, Y_b, 0 the coordinates of the intersection between the back-projected ray (passing through the image plane point $[x_t, y_t, z_t]$=$[5.05, 5.1, -1000]$ which represents the calibration target in the image) and the calibration plane.
$\frac{X_b}{x_t}$=$\frac{Z}{f}$ where Z represents the distance between the optical centre and the calibration plane origin.
We have as well:
$\frac{Y_b}{y_t}$=$\frac{Z}{f}$
X_b=$\frac{5.05\times1000}{50}$=$101$
Y_b=$\frac{5.1\times1000}{50}$=102
The radius of ambiguity is given as:
$r_{ambiguity}$=$d([x_t, y_t, z_t]$,$[X_t, Y_t, Z_t])$
I would accept both euclidean distance and manhattan distance or any other distance as long as the students is clear about it. So the answer is, 3 or $\sqrt{5}$

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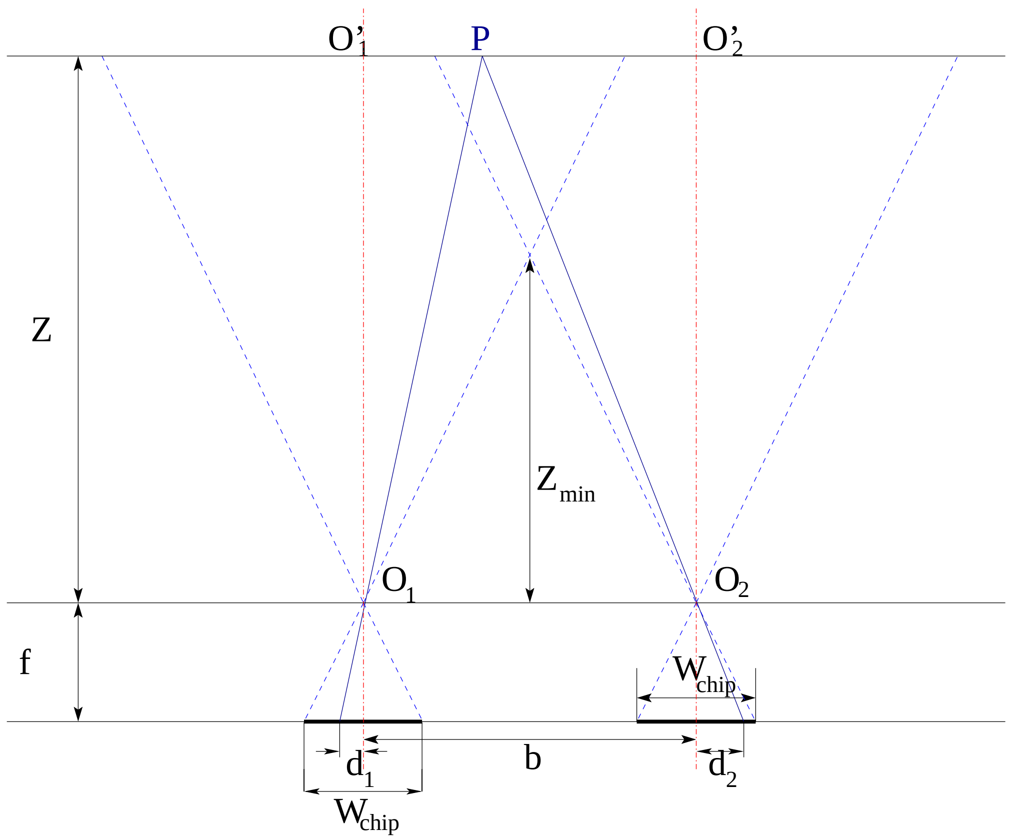
Section B: Stereo vision and binary machine vision



Two identical cameras, with optical centres O_1 and O_2, are placed with parallel optical axes (O_1O_1' and O_2O_2') as displayed in the figure. The distance between the optical centres (the baseline distance) is b and the focal length for both cameras is f. p is the physical width of one pixel on the camera's sensor, n is the number of pixels on one scanline, and W_{chip} is the camera's sensor width, or scanline width. A point P at depth Z appears in each image at a different position on a scanline. The disparity, d_P, of point, P, is given by $d_P = d_1 - d_2$.

5. What is the disparity when point P is on the horizon? [2 marks]

the disparity is equal to zero, a point on the horizon is imaged at the same place in both cameras

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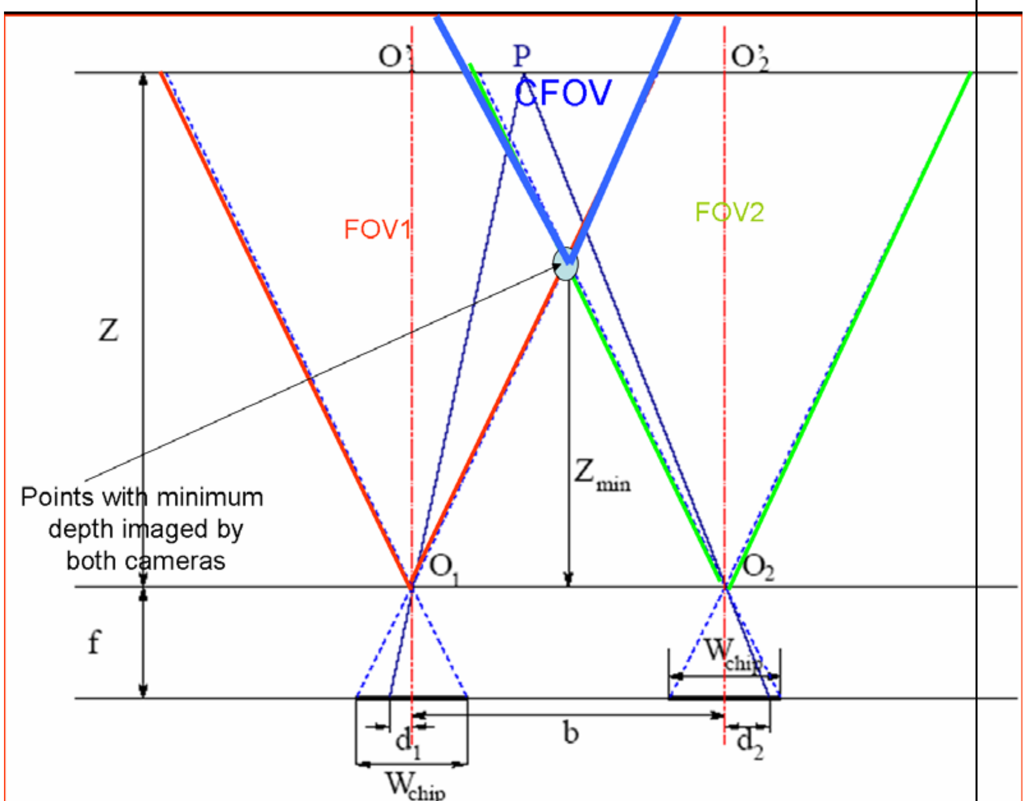
6. Draw the fields of view for each camera and the common field of view (e.g. points imaged by both cameras). Show where the minimum depth, $Z_{\min}$, is located. [6 marks]



7. What is the disparity when point P is at the minimum depth Z_{min}? [2 marks]

the disparity at minimum depth Z_{min} is equal to the width of the ccd sensor (given in mm or in pixels).
$d = n$
or $d = W_{chip}$

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8. Practically, the camera resolution (that is, its pixel width, p, and sensor width, W_{chip}) will determine the minimum depth, Z_{min}. Give the formula for Z_{min} as a function of the sensor's width, the baseline distance, b, and the focal length, f. Compute Z_{min} when the cameras are 100 millimetres apart, the focal length is 25 mm, the number of pixels per line is 500, and the pixel width is 10 μm. [11 marks]

Using similar triangles one can reach the disparity as:
$d = \frac{fb}{Z}$ where b is the baseline and d the disparity given in mm.
AS seen earlier we reach Z_{min} for d, disparity equal to the ccd sensor width $d = W_{chip} = p \times n$ where p is the pixel width and n the number of pixels per scanline
This gives $Z_{min} = \frac{fb}{pn}$
$= \frac{25 \times 100}{500 \times 0.01} = 500mm$

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9. The camera image plane resolution is the pixel. Pixel size influences the depth resolution of a stereo system. What is the formula for the depth resolution along the cyclopean axis (i.e. the line parallel to and midway between the camera axes). What is the depth resolution along one camera axis (you can chose the left or right camera). Any comments? [9 marks]

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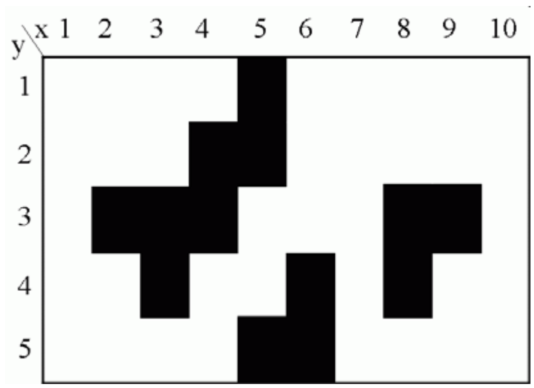
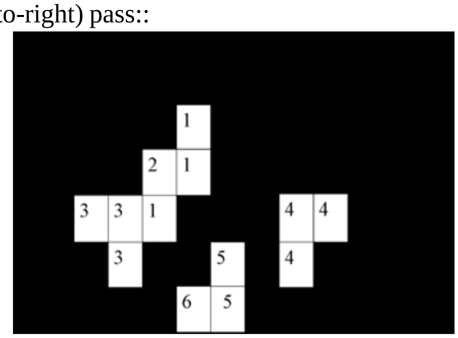
10. Label the different blobs (connected regions with an 8-neighbourhood) in the binary image using the two-pass labelling technique. [10 marks]

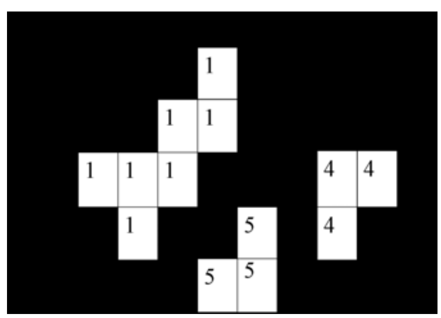
Binary image:



First (top-down left-to-right) pass::



Second pass:



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Section C: Principal Component Analysis

The database A contains six $2D$ points:

$$x_1 \begin{pmatrix} 3 \\ 3 \end{pmatrix}, x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}, x_3 \begin{pmatrix} 2 \\ 3 \end{pmatrix}, x_4 \begin{pmatrix} 2 \\ 1 \end{pmatrix}, x_5 \begin{pmatrix} 4 \\ 5 \end{pmatrix}, x_6 \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

The points x_1, x_2, x_3 belong to class 1, and the points x_4, x_5, x_6 belong to class 2. Let y_i; $i = 1, \ldots, 6$, denote the centred database vectors.

11. Compute the covariance matrix of the centred database given by: $C = \frac{1}{6} \sum_{i=1}^{6} y_i y_i^T$ [5 marks]

$x_m ean \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

The centered points are:

$$\bar{x_1} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \bar{x_2} \begin{pmatrix} -2 \\ -2 \end{pmatrix}, \bar{x_3} \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \bar{x_4} \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \bar{x_5} \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \bar{x_6} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$C = \frac{1}{6} \begin{pmatrix} 16 & 14 \\ 14 & 16 \end{pmatrix}$

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12. Compute the eigenvalues, (λ_1, λ_2), of the matrix C. Note: **eigenvalues can be approximated to their first decimal digit.** [4 marks]

$P_\lambda = det(C - \lambda I_2)$
The eigenvalues of C are the zeros of P_λ
We have: $det \begin{pmatrix} 8/3 - \lambda & 7/3 \\ 7/3 & 8/3 - \lambda \end{pmatrix} = 0$
$\lambda^2 - 16/3\lambda + 5/3 = 0$
$\lambda_1 = 5$
$\lambda_2 = 1/3$

13. Find the eigenvectors, e_1 and e_2, associated with the eigenvalues λ_1 and λ_2, respectively. [4 marks]

the eigenvectores are given by:
$Ce_i = \lambda_i e_i$
For $\lambda_1 = 5$:
$\begin{pmatrix} -7/3 & 7/3 \\ 7/3 & -7/3 \end{pmatrix} = 0$
$e_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
For $\lambda_1 = 1/3$:
$\begin{pmatrix} 7/3 & 7/3 \\ 7/3 & 7/3 \end{pmatrix} = 0$
$e_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

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14. Express each vector, x_i, of the database as a weighted linear combination of eigenvectors, e_1 and e_2. [6 marks]

$x_i = \frac{x_i.e_1}{e_1.e_1}e_1 + \frac{x_i.e_2}{e_2.e_2}e_2$
In the orthonormal basis e_1, e_2:
The centered points are:

$$\bar{x_1}\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \bar{x_2}\begin{pmatrix} -2\sqrt{2} \\ 0 \end{pmatrix}, \bar{x_3}\begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}, \bar{x_4}\begin{pmatrix} -3/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}, \bar{x_5}\begin{pmatrix} 3/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}, \bar{x_6}\begin{pmatrix} 5/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

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15. Compute the Euclidean distances between points x_1 and x_4 (d_{14}), x_2 and x_5 (d_{25}). Compute the same distances using coordinates projected in the direction of the largest variance. Any comments? [6 marks]

whether the data is centered or not does not affect the distances between the points. Choosing the original coordinates:
$d_{14} = \sqrt{5}$
$d_{25} = 5$
Along the direction of the largest variance, that is along e_1:
$d_{14} = 3/\sqrt{2}$
$d_{25} = 3/\sqrt{2} + 2\sqrt{2} = 7/\sqrt{2}$
PCA tends to spread best the data along the direction of largest variance. It does not look like in this example the between classes distance has been increased. This is not anyway the goal of PCA.

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Section D: Kalman Filtering

16. A linear system has a single scalar state, x_k, and a scalar output, y_k, depending on discrete time, k. The system is described by the scalar factors, a_k and c_k, scalar state uncertainty, u_k, and scalar observation error, v_k:

$$\begin{aligned} x_{k+1} &= a_k x_k + u_k; \\ y_k &= c_k x_k + v_k \end{aligned}$$

The error correlation matrix $\mathbf{P}_k$ is replaced in this case by the scalar error variance, p_k.
Derive the Kalman filter assuming that the state uncertainty has variance σ_k^2 and the measurement error has variance θ_k^2. [15 marks]

Hint: In this case, matrix-vector operations of the conventional Kalman filtering are simplified: the matrix-vector product is replaced by the usual scalar product, the transposition does not effect the scalar value, and the inversion has its usual scalar meaning: $a^{-1} = \frac{1}{a}$.

In this case the Kalman filter equations are as follows:

- the intermediate state estimate and the intermediate error variance at time k:

- the gain factor at time k:

- the state estimate and the error variance at time k:

You may reduce these equations but it is not required that you do so.

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