THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2014
Campus: City

COMPUTER SCIENCE

COMPSCI 773: Intelligent Vision Systems

(Time allowed: TWO hours)

NOTE: Attempt **all** questions!

Write the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

This is an open book exam. Candidates **may** bring calculators, notes, reference books, or other written material into the examination room.

Section:	**A**	**B**	Total
Possible marks:	50	50	100
Awarded marks:			

SURNAME:

FORENAME(S):

STUDENT ID:

QUESTION/ANSWER SHEETS FOLLOW

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Section A: Camera Calibration: 50 marks

1. Consider a 3 by 3 matrix, $R = [\mathbf{r}_1\ \mathbf{r}_2\ \mathbf{r}_3]$, describing a 3D rotation in the right-handed Cartesian coordinate system. What are the relationships between the three column vectors, $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ of this matrix R?

   Hint: Consider dot and cross products of the vectors. [4 marks]

2. Given the vector $\mathbf{r}_1 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, produce vectors $\mathbf{r}_2$ and $\mathbf{r}_3$ such that all three vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ are the column vectors of the 3D rotation matrix R considered in Question 1. [2 marks]

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3. Consider a camera with the pixel width p, the width W_{chip} of the sensor, and the fixed focal length f. Let the origin, O, of the camera reference frame be at its centre. Assuming the first-order Tsai model of radial distortion with parameter κ_1, and an image point with undistorted, (X_u, Y_u), and distorted, (X_d, Y_d), Cartesian coordinates, demonstrate that $\kappa_1 = \frac{\delta}{r_d^3}$ where δ is the distance between the distorted and undistorted image points and r_d is the distance between the distorted point and the image centre. [4 marks]

4. Considering a 4000×3000 pixels camera with the width $W_{chip} = 6.4$ mm $\times\, 4.8$ mm of the sensor in Question 3, compute the maximum first-order radial distortion parameter $\kappa_{1:\max}$ such that the maximal distance between the distorted and undistorted coordinates of any point in the image is kept below the pixel width. Given your experimental experience with camera calibration, is this a realistic requirement? [6 marks]

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5. Consider the extended first- and second-order Tsai model of radial distortion with parameters κ_1 and κ_2. Relate κ_2 to κ_1 and the distances δ and r_d specified in Question 3. [4 marks]

6. Considering the camera parameters in Questions 3 and 4, the second-order radial distortion model in Question 5, and the given parameter $\kappa_1 = 0.0001mm^{-2}$, compute the maximum second-order radial distortion $\kappa_{2:\max}$, such that the maximal distance between the distorted and undistorted coordinates of any point in the image is kept below the equivalent width of **two** pixels. Given your experimental experience with camera calibration, is this a realistic requirement? [6 marks]

7. Considering the Tsai's calibration and your work on Assignments 2 and 3, specify three major causes of calibration errors and realistic strategies to decrease such errors. [3 marks]

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8. Consider a camera calibration experiment involving an off-the-shelf camera and a calibration cube made of three orthogonal planes with round calibration patches. The world reference frame (WRF) is located on the cube and follows the clock-wise orientation rule (the rightward horizontal x-axis and the upward vertical y-axis). The circular planar target patches P_1, P_2, and P_3 with centres at Cartesian coordinates, $[100, 100, 0]$, $[104, 100, 0]$, $[100, 104, 0]$, respectively, are used to calibrate a camera. After calibration, the rotation matrix R linking the camera frame to the WRF is the identity matrix with the unit columns $[1, 0, 0]^\mathsf{T}$; $[0, 1, 0]^\mathsf{T}$; and $[0, 0, 1]^\mathsf{T}$ where T denotes the transposition; and the translation vector $T = [0, 0, -1000]^\mathsf{T}$ links the WRF centre to the camera optical centre. The calculated focal length f is equal to 5 mm, and the camera sensor width $\times$ height specification is 4.8 mm (480 elements) $\times$ 3.6 mm (360 elements). A distortion-free lens is assumed.

   Given the above calibration parameters, an image of the cube is acquired. After some processing steps on the image of the calibration plane acquired for the calibration purpose, the centre of the target P_1 is found at pixel coordinates $[289, 231]$ in the image.

   Draw the reference frames (World, Camera, Image sensor), the patches centres, and the above image point with due regard for the usual convention and aforementioned requirements. [2 marks]

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9. Relate the position of a point with homogeneous coordinates $\widetilde{\mathbf{P}} = [X\ Y\ Z\ 1]^\mathsf{T}$ on the cube to its position (in pixels) on the image $\widetilde{\mathbf{p}} = [x\ y\ 1]^\mathsf{T}$ where T denotes the transposition. [3 marks]

10. Given the framework in Question 8, compute the error in pixels between the actual position, $[289, 231]$, in the image plane and projection of the centre of the target patch P_1 to this plane. [4 marks]

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11. One tenth of a pixel is often cited as the smallest attainable distance in an imaging system. How is this translated in terms of the calibration error in the image plane and on the calibration object framework in Question 8? Is such an attainable measurement requirement realistic given your assignment's calibration setup? [6 marks]

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12. Current off-the-shelf printers generate maximum errors of up to 0.5 mm (both in x- and y- directions) when printing grid patterns. Calibration objects printed out of such printers carry an intrinsic measurement error of 0.5 mm on each cube target points. Considering the framework in Question 8, how is such intrinsic error translated in the image plane? Is this an acceptable error given your camera calibration experience? [6 marks]

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Section B: Epipolar Geometry and Matching: 50 marks

13. Determine optical centres, $\mathbf{O}_1$ and $\mathbf{O}_2$, of two cameras with the 3×4 projection matrices P_1 and P_2, respectively:

$$P_1 = [Q_1 \, \mathbf{q}_1] = \begin{bmatrix} 1.0 & 0.5 & 0.0 & -2.0 \\ 0.5 & 1.0 & 1.0 & -2.0 \\ 0.0 & 1.0 & 2.0 & -2.0 \end{bmatrix} \text{ and } P_2 = [Q_2 \, \mathbf{q}_2] = \begin{bmatrix} 1.0 & 0.5 & 0.0 & -1.0 \\ 0.5 & 1.0 & 1.0 & -1.0 \\ 0.0 & 1.0 & 2.0 & -2.0 \end{bmatrix}$$

Hint: The 3×3 matrix $Q = \begin{bmatrix} 1.0 & 0.5 & 0.0 \\ 0.5 & 1.0 & 1.0 \\ 0.0 & 1.0 & 2.0 \end{bmatrix}$ has the inverse matrix $Q^{-1} = \begin{bmatrix} 2.0 & -2.0 & 1.0 \\ -2.0 & 4.0 & -2.0 \\ 1.0 & -2.0 & 1.5 \end{bmatrix}$

[4 marks]

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14. Given the same cameras as in Question 13 above, determine first the point $\widetilde{\mathbf{D}}_1 = \begin{bmatrix} \mathbf{D}_1 \\ 0 \end{bmatrix}$ at the infinity of the optical ray, projecting the 3D point $\mathbf{S}$ with homogeneous coordinates $\widetilde{\mathbf{S}} = \begin{bmatrix} 0.0 \\ 0.0 \\ 2.0 \\ 1.0 \end{bmatrix}$ to the image plane of the first camera, and then project the computed infinitely far point $\widetilde{\mathbf{D}}_1$ to the image plane of the second camera. [6 marks]

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15. For the same cameras as in Question 13 above, find the homogeneous coordinates of the epipole $\widetilde{\mathbf{e}}_1$, i.e. of the projection of the optical centre $\mathbf{O}_2$ of the second camera to the image plane of the first camera, and determine its Cartesian coordinates. [4 marks]

16. Write down the basic relationship between homogeneous coordinates $\widetilde{\mathbf{s}}_1$ and $\widetilde{\mathbf{s}}_2$ of two corresponding points with Cartesian coordinates $\mathbf{s}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ and $\mathbf{s}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$, respectively, in the left and right images of a stereo pair captured by the cameras, described by the fundamental matrix $\mathbf{F} = [F_{i,j}]_{i,j=1}^{3}$. Which properties of the corresponding points and conjugate epipolar lines does this basic relationship describe? [3 marks]

17. In terms of the relationship in Question 16, specify the epipolar line $\mathbf{a}_1^\mathsf{T}\widetilde{\mathbf{s}}_1 = a_1x_1 + b_1y_1 + c_1 = 0$ in the left image that corresponds to the point with the Cartesian coordinates $\mathbf{s}_2 = (x_2, y_2)$ in the right image, that is, specify the individual coefficients $\mathbf{a}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ in terms of the components of the fundamental matrix and the corresponding point coordinates. [3 marks]

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18. In which point do all the epipolar lines of the right or left image of a stereo pair intersect? [2 marks]

19. Describe, in brief, the 8-point algorithm for estimating the fundamental matrix of rank 2, in particular, which system of equations is solved by this algorithm, why it is called ”the 8-point algorithm”, and which data points are to be used? [5 marks]

20. Describe, in brief, to what extent an observed optical 3D surface can be reconstructed from a given stereo pair if both the intrinsic and extrinsic parameters of stereo cameras are unknown. [3 marks]

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21. Explain, in brief, basic reasons why the same stereo pair of images might depict multiple different 3D scenes. [2 marks]

22. Specify which local stereo matching method allows for uniform contrast and offset deviations between the corresponding areas in stereo images and describe in brief the underlying math model of image signals corrupted with the centre-symmetric Gaussian noise in addition to the contrast/offset deviations. [3 marks]

23. Compare, in brief, the accuracy and computational complexity of stereo matching methods based on the local and global optimisation. [3 marks]

24. Semi-global stereo matching (SGM) builds an output depth map of a goal 3D surface by averaging up to 8-16 intermediate depth maps. Each intermediate map is a collection of parallel planar 2D profiles, having the same, but different for the different maps spatial orientation. Each profile is reconstructed by the one-dimensional dynamic programming (DP) algorithm independently of all other profiles. Experiments have shown that by accuracy the SGM is close to the known global optimisation algorithms, such as loopy 2D belief propagation or iterative graph-cut matching. What, in your opinion, might be an advantage of the SGM over the latter two algorithms? [2 marks]

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25. Describe the dynamic programming solution of a global minimisation problem $\mathbf{v}^* = \arg\min_{\mathbf{v}} D(\mathbf{v})$ where the n-variate functional $D(\mathbf{v}) = \sum_{i=1}^{n} \varphi_i(v_i) + \sum_{i=2}^{n} f_i(v_{i-1}, v_i)$, which depends on a goal sequence of n scalar variables, $\mathbf{v} = (v_1, \ldots, v_n)$, is the sum of arbitrary non-negative univariate, $\phi_i(v_i)$, and bivariate, $f_i(v_{i-1}, v_i)$, of the variables, v_i, and their successive pairs, v_{i-1} and v_i, respectively. [6 marks]

26. Describe in brief differences between the dynamic programming and belief propagation solutions of the global minimisation problem in Question 25. [4 marks]

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