THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2013
Campus: City

COMPUTER SCIENCE

COMPSCI 773: Intelligent Vision Systems

(Time allowed: TWO hours)

NOTE: Attempt **all** questions!

Write the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

This is an open book exam. Candidates **may** bring calculators, notes, reference books, or other written material into the examination room.

Section:	**A**	**B**	**C**	Total
Possible marks:	30	50	20	100
Awarded marks:				

SURNAME:

FORENAME(S):

STUDENT ID:

QUESTION/ANSWER SHEETS FOLLOW

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Section A: Camera calibration and stereo vision

1. Briefly explain, in your own words, the goal of a single camera calibration (no equation allowed). [3 marks]

2. Given (*i*) a camera with a fixed focal length f, an $n \times n$ pixels resolution, and a square pixel width w, (*ii*) a 3D calibration object with the world coordinates (X_w, Y_w, Z_w), (*iii*) 3D coordinates (X, Y, Z) of the optical centre of a pin-hole camera, (*iv*) undistorted image coordinates (x_u, y_u), and (*v*) the distorted image coordinates (x_d, y_d), provide a set of equations which relate the distorted image coordinates to the world coordinates following the Tsai's camera calibration implementation. [4 marks]

3. Application: Given understanding of radial distortion in the Tsai's camera calibration, provide the equation which relates the first order radial distortion coefficient κ_1 to the distance, r, between the image center and a point P in the distorted image, and the distance, d, between the distorted and undistorted positions of the point P. Compute κ_1 observing that the distances of a given point P to the image centre in the distorted and undistorted images differ by 0.1 millimetre (*mm*). The point P is supposed to be at the radial distance (from the image center) of 100 *mm*. The square pixel width is 1 μm (micron, or 10^{-3} *mm*), and the focal length is 50 *mm*. [3 marks]

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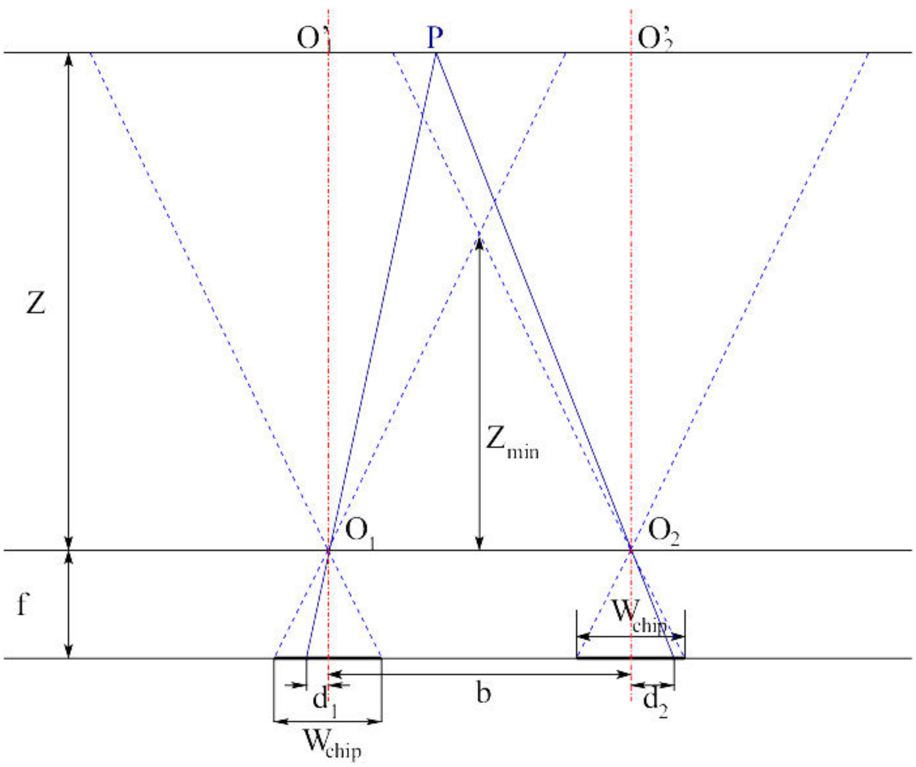
4. Given your experience with the camera calibration in this course, describe, in a few words, TWO factors, which may impact the camera calibration accuracy, and provide possible solutions to correct each of the described factors. [4 marks]

Stereo set-up



Let two identical cameras be placed in canonical geometric position with optical centres O_1 and O_2 and parallel optical axes (O_1O_1' and O_2O_2') as displayed in the above figure. The distance between the optical centres is the baseline length b and the focal length for both the cameras is f. Let p, n, and W_{chip} denote the physical width of one pixel on the camera's sensor, the number of pixels along one scanline, and the width of the camera's sensor, or the scanline width, respectively. A point P at depth Z appears in each image at different positions, d_1 and d_2, along a scanline. The disparity, $D_P = d_1 - d_2$, for the point P is linked to the depth Z of the point P, the focal length f, and the baseline length, b, as $D_P = f\frac{b}{Z}$.

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5. Given the discrete nature of a camera sensor, the depth Z is calculated in discrete increments. Given a point P at the depth Z corresponding to a disparity D in the above stereo system, calculate the upper ΔZ^{+} and lower ΔZ^{-} depth increment to the previous and next depth values when the disparity decreases, or respectively, increases, by 1. [5 marks]

6. Calculate the upper and lower depth increments for both the minimum ($D_{\min} = 1$) and the maximum disparity values ($D_{\max}$) for the above set-up when $p = 1\mu m$ (micron), $f = 5$ *mm* (millimeters), and $n = 1000$ pixels. [4 marks]

ΔZ_1^- is not defined

$\Delta Z_1^+ =$

$\Delta Z_{d_{max}}^- =$

$\Delta Z_{d_{max}}^+ =$

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7. Consider the values in Question 6 and further assume that you may use the above stereo set-up to capture human faces. Let the subjects be placed at $1m$ from the stereo set-up with the face distances from the set-up ranging between 0.90 *m* (the tip of the nose) to 1.1 *m* (thye ear). Calculate the number of disparity levels, which will code the face surface. Calculate the minimum and maximum depth increments for a human face. [5 marks]

Section B: Epipolar Geometry and Stereo Matching: 50 marks

8. Given two cameras with the 3×4 projection matrices $P_1 = [Q_1\ \mathbf{q}_1] = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & -2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$, respectively, determine the optical centres $\mathbf{O}_1$ and $\mathbf{O}_2$ of the cameras.

Hint: The 3×3 matrix $Q = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ has the inverse matrix $Q^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ -1 & 2 & -3 \end{bmatrix}$ [4 marks]

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9. Given the same cameras as in Question 8 above, determine the point $\widetilde{\mathbf{D}}_1 = \begin{bmatrix} \mathbf{D}_1 \\ 0 \end{bmatrix}$ at the infinity of the projection ray, projecting the 3D point $\mathbf{S}$ with homogeneous coordinates $\widetilde{\mathbf{S}} = [0, 0, 2, 1]^{\mathsf{T}}$ to the image plane of the first camera, and project the point $\widetilde{\mathbf{D}}_1$ to the image plane of the second camera. [6 marks]

10. For the same cameras as in Question 8 above, find the homogeneous coordinates of the epipole $\widetilde{\mathrm{e}}_1$, i.e. the projection of the optical centre $\mathbf{O}_2$ of the second camera to the image plane of the first camera. [4 marks]

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11. Write down the basic relationship between homogeneous coordinates $\widetilde{\mathbf{p}}_1$ and $\widetilde{\mathbf{p}}_2$ of two corresponding points with Cartesian coordinates $\mathbf{p}_1$ and $\mathbf{p}_2$ in the left and right images of a stereo pair captured by the cameras, being described by the fundamental matrix $\mathbf{F} = [F_{i,j}]_{i,j=1}^{3}$. [3 marks]

12. In terms of the relationship in Question 11, specify the epipolar line $\mathbf{a}^\mathsf{T}\widetilde{\mathbf{p}}_1 = 0$ in the left image that corresponds to the point with Cartesian coordinates $\mathbf{p}_2 = (x_2, y_2)$ in the right image. [5 marks]

13. In which point(s) do all the epipolar lines of the right or left image of a stereo pair intersect? [3 marks]

14. Describe, in brief, the 8-point algorithm for estimating the fundamental matrix of rank 2. [5 marks]

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15. Describe, in brief, to what extent an observed optical 3D surface can be reconstructed in the cases when both intrinsic and extrinsic parameters of stereo cameras are unknown, or only intrinsic parameters are known, or both intrinsic and extrinsic parameters are known. [5 marks]

16. Describe, in brief, the main reasons why stereo matching that searches for corresponding areas in a stereo pair of images is an ill-posed, in the math sense, problem. [5 marks]

17. Describe in brief which differences between the magnitudes of the corresponding image signals are taken into account in the correlation based stereo matching.

    Hint: Consider math models of signals and noise that lead to this matching score. [5 marks]

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18. Compare, in brief, methods of stereo matching based on local and global maximisation of similarity or minimisation of dissimilarity between signals in corresponding points of a stereo pair. [5 marks]

    Hint: Consider pros and cons with respect to accuracy, computational complexity, and simplifying constraints.

Section C: Principal Component Analysis (PCA): 20 marks

Consider a database made of 2D face images of 100 different subjects with each subject described with 10 images thus giving a N=1000 images database. Each image has a resolution of 1000 by 1000 pixels. Consider that the face images have been registered one to another, that is, face features for different faces are placed as close as possible one to another.

19. In the context of human subject recognition, (following the PCA implementation as seen in class), describe steps, which lead to constructing the correlation matrix of the above database. Explain in own words, how you may circumvent the high dimensionality posed by this database? [5 marks]

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20. Consider that you have succesfully extracted 100 eigenvectors and eigenvalues from the above face database correlation matrix. Explain the necessary steps to create a classifier which may assess whether any given unknown face image input belongs to a subject being already in the face database? [5 marks]

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21. The database A contains 6 $2D$ points:

$$x_1 \begin{pmatrix} 3 \\ 3 \end{pmatrix}, x_2 \begin{pmatrix} 2 \\ 2 \end{pmatrix}, x_3 \begin{pmatrix} 5 \\ 5 \end{pmatrix}, x_4 \begin{pmatrix} 1 \\ 1 \end{pmatrix}, x_5 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \text{ and } x_6 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Vectors x_1, x_2, x_3 are supposed to belong to class 1, while vectors x_4, x_5, x_6 belongs to class 2. Compute the covariance matrix C of the centred database set and the associated eigenvalues (λ_1, λ_2) and eigenvectors (λ_1, λ_2). [7 marks]

Hint: The eigenvalues of a matrix B are the zeros of the $\lambda-$second order polynomial form defined by the determinant of the matrix $B - \lambda I_2$, where I_2 is the 2×2 identity matrix.
The determinant of a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is equal to $a \cdot d - b \cdot c$
The eigenvector e_λ of eigenvalue λ for Matrix M verifies: $Me_\lambda = \lambda e_\lambda$

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22. Find the set of parameters α_1 and α_2 which best (in the average sense) represent the elements of class c_1 and c_2 in the eigenvectors base. [3 marks]

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