THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, XXXX
Campus: City

COMPUTER SCIENCE

COMPSCI 773: Intelligent Vision Systems

(Time allowed: TWO hours)

NOTE: Attempt **all** questions!

Write the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

This is an open book exam. Candidates **may** bring calculators, notes, reference books, or other written material into the examination room.

Section:	**A**	**B**	**C**	Total
Possible marks:	40	20	40	100
Awarded marks:				

SURNAME:

FIRSTNAME:

ID:

QUESTION/ANSWER SHEETS FOLLOW

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Section A: Coordinates, affine transform, and scaling

Questions 1–8 involve the homogeneous coordinates notation.

Let a 3×3 matrix $\mathbf{P}$ transform a 2D point $\mathbf{M}$ with Cartesian coordinates (x, y) into the 2D point $\widetilde{\mathbf{M}}$ by 2D scaling, $\mathbf{S}$, with scaling factors (s_x, s_y) along the coordinate axes, 2D translation $\mathbf{T}$ with Cartesian coordinate increments (t_x, t_y), and 2D rotation $\mathbf{R}$ by angle θ.

1. Write down the 3×3 matrices $\mathbf{S}$, $\mathbf{T}$ and $\mathbf{R}$. [3 marks]

$$\mathbf{S} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}; \mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}; \mathbf{R} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

2. Specify the matrix $\mathbf{P}$ which transforms $\mathbf{M}$ into $\widetilde{\mathbf{M}}$ and represent its components in terms of the components of $\mathbf{S}$, $\mathbf{T}$ and $\mathbf{R}$. [2 marks]

$$\mathbf{P} = \mathbf{RTS}$$

$$\mathbf{P} = \begin{bmatrix} s_x\cos\theta & s_y\sin\theta & t_x\cos\theta + t_y\sin\theta \\ -s_x\sin\theta & s_y\cos\theta & -t_x\sin\theta + t_y\cos\theta \\ 0 & 0 & 1 \end{bmatrix};$$

3. Specify the matrix which transforms $\widetilde{\mathbf{M}}$ back into $\mathbf{M}$ (e.g. the inverse of the transformation in Question 2). [4 marks]

$$\mathbf{P^{-1}} = S^{-1}T^{-1}R^{-1}$$

$$\mathbf{P^{-1}} = \begin{bmatrix} \frac{\cos\theta}{s_x} & \frac{-\sin\theta}{s_y} & \frac{-t_x}{s_x} \\ \frac{\sin\theta}{s_y} & \frac{\cos\theta}{s_y} & \frac{-t_x}{s_y} \\ 0 & 0 & 1 \end{bmatrix}$$

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4. Compute $\mathbf{P}$ and find $\widetilde{\mathbf{M}}$ given $\mathbf{M}$ with Cartesian coordinates $(x = 3, y = 1)$ and transformation parameters $\theta = -45°$, $t_x = 1$, $t_y = 1$, $s_x = 1$, and $s_y = 2$. [4 marks]

$$\mathbf{P} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\sqrt{2} & 0 \\ \frac{\sqrt{2}}{2} & \sqrt{2} & \frac{3\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\widetilde{M} = PM$$

$$\widetilde{M} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{4\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

5. Compute $\mathbf{P}$ and find $\mathbf{M}$ given $\widetilde{\mathbf{M}}$ with Cartesian coordinates $(\widetilde{x} = 2, \widetilde{y} = 3)$ and parameters $\theta = 60°$, $t_x = 1$, $t_y = 2$, $s_x = 0.5$, and $s_y = 0.5$ of transforming $\mathbf{M}$ into $\widetilde{\mathbf{M}}$. [3 marks]

$$\mathbf{P} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{-\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \\ \frac{-3}{2} & \frac{3}{4} & 1 \end{bmatrix}$$

$$M = P^{-1}\widetilde{M}$$

$$M = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{4\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

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6. Extend the previous definitions for $\mathbf{S}$, $\mathbf{T}$ and $\mathbf{R}$ to specify the 4 by 4 matrix P which transforms a 3D point $\mathbf{M}$ with Cartesian coordinates (x, y, z) into the 3D point $\widetilde{\mathbf{M}}$. Assume $\mathbf{R} = \mathbf{R}_x\mathbf{R}_y\mathbf{R}_z$ with angle θ_x for rotation $\mathbf{R}_x$; $\mathbf{R}_y = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and $\mathbf{R}_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. [4 marks]

$$\mathbf{P} = \mathbf{R_x R_y R_z T S}$$

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x & 0 \\ 0 & -\sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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7. Find $\mathbf{M}$ given $\widetilde{\mathbf{M}}$ with Cartesian coordinates $(2, 1, 1)$ and transformation parameters $\theta_x = 45°$, $t_x = 1, t_y = 1, t_z = 0, s_x = 1, s_y = 2$, and $s_z = 2$. [4 marks]

8. Transform the following points from homogeneous to Cartesian coordinates or vice versa as indicated below: [3 marks]

Homogeneous coordinates		Cartesian 3D coordinates
$[6, 4, 2, 1]$	$\longleftrightarrow$	[6 , 4 , 2]
$[1, 0, 3, 2]$	$\longleftrightarrow$	[0.5 , 0 , 1.5]
$[3, -2, -4, 0]$	$\longleftrightarrow$	$[+\infty, -\infty, -\infty]$

Cartesian 3D coordinates		Homogeneous coordinates
$[15, 10, 5]$	$\longleftrightarrow$	[45 , 30 , 15 , 3]
$[-\infty, -\infty, +\infty]$	$\longleftrightarrow$	[-3 , -3 , 3 , 0]
$[0, 0, 0]$	$\longleftrightarrow$	[3 , 2 , 1 , ∞]

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9. Assuming the distortion free pinhole camera model (focal length f) for a single camera, specify the 2D coordinates (x, y) of the projection, $\mathbf{m}$, of a 3D point $\mathbf{M}$ with Cartesian coordinates (X, Y, Z) to the image plane. Both the 2D and 3D coordinates are in the camera coordinate system. [3 marks]

$x = \frac{fX}{Z}; y = \frac{fY}{Z}$

10. Assuming a pinhole camera model with first-order radial distortion (focal length f, distortion parameter κ_1), r distance between the image centre and distorted point M of coordinates (x_d, y_d), δ distance between distorted (x_d, y_d) and undistorted (x_u, y_u) 2D coordinates of a point in the image, Specify the relationship between κ_1, f,r and δ. [3 marks]

$x_u = x_d(1 + \kappa_1({x_d}^2 + {y_d}^2))$
$y_u = y_d(1 + \kappa_1({x_d}^2 + {y_d}^2))$
$(x_u - x_d)^2 = {x_d}^2{\kappa_1}^2({x_d}^2 + {y_d}^2)^2$
$(y_u - x_d)^2 = {y_d}^2{\kappa_1}^2({x_d}^2 + {y_d}^2)^2$
$\delta^2 = {\kappa_1}^2 r^6$
$\kappa_1 = \frac{\delta}{r^3}$

11. Compute the first order distortion parameter value (supposedly, positive) if distorted and undistorted coordinates differ by 0.08 *mm* (millimetre) at a radial distance equivalent to 400 pixels from the acquired image centre ($(c_x = 500, c_y = 500)$ pixels). The square pixel area is 5 microns $\times$ 5 microns and the focal length is 40 mm. [3 marks]

$\kappa_1 = 0.01mm^{-2}$

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12. Assuming a pinhole camera model with first and second-order radial distortion (focal length f, distortion parameter κ_1 and κ_2), specify the relationship between distorted (x_d, y_d) and undistorted (x_u, y_u) 2D coordinates of a point in the image. [2 marks]

$$x_u = x_d(1 + \kappa_1({x_d}^2 + {y_d}^2) + \kappa_2({x_d}^2 + {y_d}^2)^2)$$
$$y_u = y_d(1 + \kappa_1({x_d}^2 + {y_d}^2) + \kappa_2({x_d}^2 + {y_d}^2)^2)$$
$$\kappa_2 = \frac{\frac{\delta}{r^3} - \kappa_1}{r^2}$$

13. Using the first order radial distortion parameter $\kappa_1 = 0.01mm^{-2}$, Compute the second order radial distortion parameter value (supposedly, positive) if distorted and undistorted coordinates differ by 0.081 *mm* (millimetre) at a radial distance equivalent to 400 pixels from the acquired image centre. The square pixel area is 5 microns $\times$ 5 microns and the focal length is 40 mm. [3 marks]

$$\kappa_2 = 3.125 \times 10^{-5} mm^{-2}$$

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Section B: Fundamentals of applied early vision

Make sure to thoroughly detail your answers to the following questions. An answer with only a straight result is not enough.

Questions 14–19 assume the 2×2 matrix $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and the 2×1 vector $\mathbf{X} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

14. Compute the eigenvalues (λ_1, λ_2) of the matrix $\mathbf{A}$.
Hint: Eigenvalues of a matrix $\mathbf{U}$ are the roots of the polynomial defined by the determinant of the matrix $\mathbf{U} - \lambda\mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix. [4 marks]

$$\mathbf{P_A}(\lambda) = det \begin{pmatrix} 2-\lambda & 1 \\ 2 & 3-\lambda \end{pmatrix}$$
$$= (2-\lambda) * (3-\lambda) - 2 * 2 = 4 - \lambda + \lambda^2$$

The zeros of P_λ are the eigenvalues of A e.g. $\Lambda_2 = 4$ and $\Lambda_1 = 1$

15. Find the eigenvectors e_1, e_2 of $\mathbf{A}$ associated to the eigenvalues λ_1, λ_2.
Hint: By definition, an eigenvector b_i, of the matrix B, associated to an eigenvalue λ_i verifies: $Bb_i = \lambda b_i$. [5 marks]

$$\mathbf{Ae_1} = e_1$$

$$\begin{cases} 2x + y & = & x \\ 2x + 3y & = & y \end{cases}$$
$$x = -y$$
$$\mathbf{Ae_2} = 4e_2$$

$$\begin{cases} 2x + y & = & 4x \\ 2x + 3y & = & 4y \end{cases}$$

$$2x = y$$
$$e_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$e_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

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16. Compute the determinant of matrix $\mathbf{A}$. [2 marks]

$$\mathbf{det}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$
$$detA = 2 \times 3 - 2 \times 1$$

17. Compute the inverse matrix $\mathbf{A}^{-1}$. [3 marks]

$$\mathbf{A}^{-1} = \frac{1}{detA}{}^{t}comA = \frac{1}{4}\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}$$

18. Compute the vector $\mathbf{Y} = \mathbf{AX}$. [3 marks]

$$\mathbf{Y} = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

19. Compute the vector $\mathbf{Z} = \mathbf{A}^{-1}\mathbf{X}$: [3 marks]

$$\mathbf{Z} = \frac{1}{4}\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

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Section C: Epipolar Geometry and Matching of Stereo Images

20. How are two cameras placed one with respect to another if epipoles in both the images coincide with the principle points (traces of optical axes)? [4 marks]

The cameras view each other and are placed in such a way that their optical axes coincide.

21. How are two cameras placed one with respect to another if epipoles in both images seat infinitely far along the Y-axis of the world co-ordinate frame and have the same x-xoordinate? [5 marks]

The optical axes of the cameras are parallel and orthogonal to the XOZ plane of the world co-ordinate plane, and their baseline is parallel to the Y-axis.

22. Given the projection matrix $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & -0.5 & 0 & -1 \\ 0 & 0 & 2 & -1 \end{bmatrix}$, determine the optical centre of this camera? [5 marks]

The optical centre $\mathbf{O} = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0.5 \end{bmatrix}$.

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23. Let a given stereo pair have the epipoles with the Cartesian co-ordinates $\mathbf{e}_1 = (x_{e,1} = 2, y_{e,1} = 1)$ and $\mathbf{e}_2 = (x_{e,2} = 1, y_{e,2}) = 3$ in the left and right images of a stereo pair, respectively, and let

$$\mathbf{F} = \begin{bmatrix} 3 & 2 & -8 \\ 2 & 3 & -7 \\ -9 & -11 & 29 \end{bmatrix}$$

be the fundamental matrix for this stereo pair. Give the equations for the epipolar line in the left image that corresponds to the right image point $\mathbf{p}_2 = (x_2 = 0, y_2 = 0)$ and for the epipolar line in the right image that corresponds to the left image point $\mathbf{p}_1 = (x_1 = 0, y_1 = 0)$. [5 marks]

> The epipolar lines are $-9x - 11y + 29 = 0$ and $-8x - 7y + 29 = 0$ in the left and right images, respectively, as follows from the relationships $\widetilde{\mathbf{e}}_2^\top \mathbf{F}$ and $\mathbf{F}\widetilde{\mathbf{e}}_1$ where $\widetilde{\ }$ denotes the homogeneous co-ordinates for the points.

24. What relationship does exist between the fundamental matrix $\mathbf{F}$ of a pair of cameras and the corresponding points $\mathbf{x}_1$ and $\mathbf{x}_2$ in the images captured by the cameras? Does this relationship hold for the epipoles in Question 23? [3 marks]

> Let $\widetilde{\ }$ denotes the homogeneous co-ordinates for the points. Each point is sitting on the epipolar line for the other point, that is, $\widetilde{\mathbf{x}}_2^\top \mathbf{F} \widetilde{\mathbf{x}}_1 = 0$.

25. Describe, in brief, main reasons why stereo matching that searches for corresponding areas in a stereo pair of images is an ill-posed, in the math sense, problem. [3 marks]

> Multiplicity of 3D scenes forming the same stereo pair due to partial occlusions and uniform or repetitive colouring of surfaces.

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26. Describe which differences between the corresponding image signals are taken into account in the SSD (Sum of Squared Differences) based stereo matching and the correlation based matching. Your answer should include math models of corresponding signals $(g_1(x_1, y_1); g_2(x_2, y_2))$ and noise $(n_1(x_1, y_1); n_2(x_2, y_2)$ in the left (1) and right (2) images of a stereo pair leading to these matching scores. [6 marks]

> SSD assumes the signals are affected only by the centre-symmetric random noise: e.g. $g_1(x_1, y_1) = g_2(x_2, y_2) + n(x_1, y_1)$, while the correlation allows for additional contrast and offset deviations, e.g. $g_1(x_1, y_1) = \alpha g_2(x_2, y_2) + \beta + n(x_1, y_1)$

27. 3D stereo reconstruction of human heads / faces typically uses stereo pairs captured with cameras having a vertically oriented baseline. Explain why such pairs are more appropriate than the pairs with the conventional horizontal baseline. [4 marks]

> Due to approximate horizontal mirror symmetry of a head / face, stereo matching is less accurate for the conventional horizontal stereo pairs than for the vertical pairs.

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28. Consider the 5×5 image:

    Let a brightness difference threshold $t = 10$ and a 3 by 3 SUSAN mask be used. What is the USAN's area for pixel $(x = 3, y = 3)$? [2 marks]

29. Consider the image in Question 28, a brightness difference threshold $t = 10$, and a 3×3 SUSAN mask. What is the Cornerness R value for pixel $(x = 4, y = 4)$? [3 marks]

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