Monads – Part III

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4 June 2019
7 June 2019

1. Revisions
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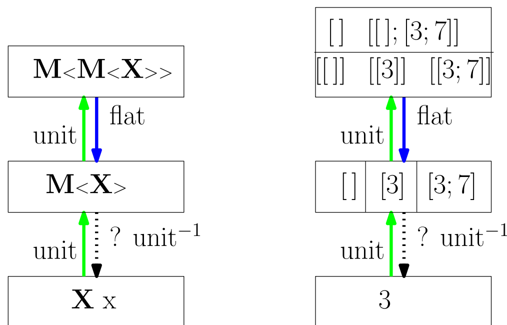
3. List Monad
4. Option aka Maybe Monad
5. Homework Suggestions

Revision: what is a monad

- In category theory: an endo-functor in the category of types, that formalizes the concept of natural transformations or lifting – NOT in this course
- In practical programming: a systematic uniform way to create and manipulate container-like types
  - Passive containers – just data, no code: lists, arrays, other non-virtual sequences, options (nullables), references, ...
  - Active containers – run code that creates data: virtual sequences, traces, tasks, asyncs, actors, jobs, cloudlets, ...

Revision: unit and flat

- In programming, unit creates singleton containers
- unit cannot create containers that are empty or contain more then one item – thus unit^{-1} is not a function (generally)
- flat breaks the encapsulation of the double container and removes all inner borders (if any)

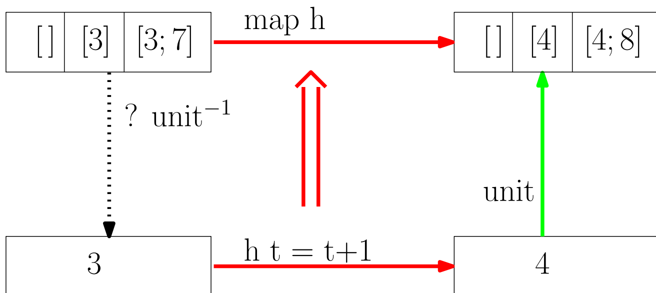


Revision: map horizontal – typical natural lifting

- In programming, map h breaks the encapsulation of the container ("unit^{-1}"), applies h to each item, and finally recreates a container (unit):

```
1 | [3;7] =map h=> [h 3; h 7]
```

- In the special case when the container is empty, the result is directly empty, bypassing function h.

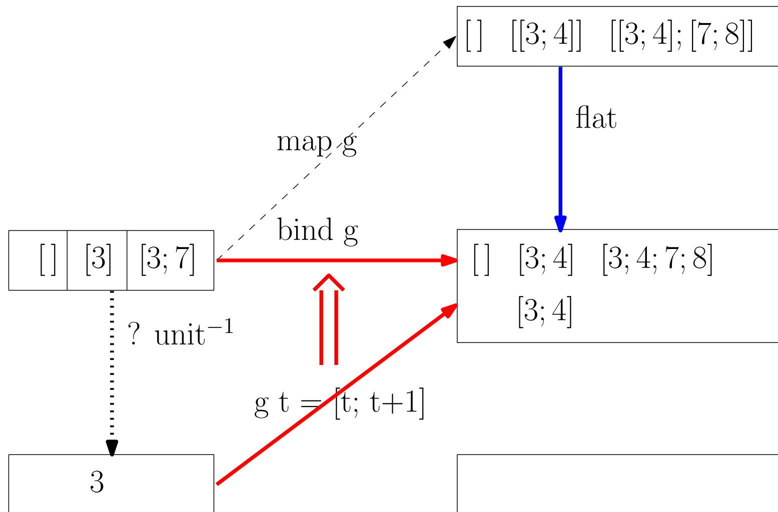


Revision: map diagonal

- map g, when g is "diagonal"
- map g >> flat is bind!

Revision: bind (diagonal)

- bind g is map g >> flat ... so it's aka flatMap



Revision: homework

- If needed, create such explanatory diagrams for other interesting scenarios, e.g.

```
1 let flat m2 = bind id m2
2
3 let map h = bind (h >> unit)
```

Monad summary

- Fundamental functions: unit, flat, bind, map
- In programming
  - Base functions: unit, bind
  - Derived functions: flat, map ...
    but directly implemented (for efficiency)
  - Three laws for unit and bind
- In category theory
  - Base functions: unit, flat, map
  - Derived function: bind
  - Four laws for unit, flat, map
- In programming we prefer bind, because its relation to
  - sugared monads (**let**!, **do**!)
  - Kleisli operators (>>=, >=>)

List Monad: unit, and sample functions

- Fundamental functions: unit, flat, bind, map

```
1 let unit t = [t]
```

- Sample functions:

```
1 let h t = t+1
2 let f t = [t]        // f = id >> unit = unit
3 let f' t = [t+1]     // f = h >> unit
4 let g t = [t; t+1]   // more complex
```

- Samples:

```
1 h 3 ⇒ 4
2 f 3 ⇒ [3]
3 f' 3 ⇒ [4]
4 g 3 ⇒ [3; 4]
```

List Monad: flat

- flat (requires at least a double nested list):

```
1 let flat' m2 = List.concat m2  // F# core lib
2
3 let flat m2 =
4     let mutable r = []
5     for m in m2 do
6         for t in m do
7             r <- r @ [t]
8     r
```

- Samples:

```
1 flat [[3]; []; [5;7]] ⇒ [3; 5; 7]
2 flat [[]] ⇒ []
3 flat [] ⇒ []
```

List Monad: map

- map (fun h is arbitrary):

```
1  let map' h m = List.map h m  // F# core lib
2
3  let map h m =
4      let mutable r = []
5      for t in m do
6          r <- r @ [h t]
7      r
```

- Samples:

```
1  map h [3; 5; 7] ⇒ [4; 6; 8]
2  map h [] ⇒ []
3
4  map g [3; 5; 7] ⇒ [[3; 4]; [5; 6]; [7; 8]]
5  map g [] ⇒ []
```

List Monad: bind

- bind (function g returns a list, possibly empty):

```
1 let bind' g m = List.collect g m  // F# core lib
2
3 let bind g m =
4     let mutable r = []
5     for t in m do
6         for u in g t do
7             r <- r @ [u]
8     r
```

- Samples:

```
1 bind g [3; 5; 7] ⇒ [3; 4; 5; 6; 7; 8]
2 bind g [] ⇒ []
```

List Monad: Kleisli

- Kleisli operators

```
1 let (>>=) m g = m |> bind g
2
3 let (>=>) f g = f >> bind g
```

- Samples:

```
1 (f >=> g) [3; 5; 7] => [3; 4; 5; 6; 7; 8]
2
3 (f >=> g) [] => []
```

List Monad: Builder, Sugared

- List monad builder

```
1 type ListBuilder () =
2   member this.Bind (m, g) = bind g m // let! do!
3   member this.ReturnFrom (m) = m     // return!
4   member this.Return (t) = unit t    // return
5
6 let mu = ListBuilder ()  // new
```

- Sugared

```
1 let x m = mu {
2     let! t = m
3     let! u = f t
4     return! g u
5 }
```

List Monad: Builder, Desugared

- Desugared

```
1 let y m =
2     mu.Bind (m,
3         fun t ->
4             mu.Bind (f t,
5                 fun u -> mu.ReturnFrom (g u)))
```

- Samples:

```
1 x [3; 5; 7] ≡ y [3; 5; 7] ⇒ [3; 4; 5; 6; 7; 8]
2
3 x [] ≡ y [] ⇒ []
```

Option Type

- Intuitively, corresponds to a singleton or empty list

```
1 type option<'a> =    // 'a option
2       | Some of 'a   // singleton container
3       | None         // empty or null container
```

- It is a tagged container, with zero or at most one item
- Similar to C#.NET nullable types, e.g. **int**?
  - Tag Some : it contains a value (≈ is NOT "null")
  - Tag None : no value (≈ is "null")
- Reading: the billion dollar mistake = null

Option Type

- Safe model for functions that return null or raise exceptions
- This function seems **int** −> **int**, but ...

```
1 let divby t = 100 / t
2 divby 5 ⇒ 20
3 divby 0 ⇒ // ? exception
```

- Better function **int** −> **option<int>**

```
1 let divby t = if t <> 0 then Some (100/t) else None
2 divby 5 ⇒ Some 20
3 divby 0 ⇒ None
```

Option Monad: unit, and sample functions

- Fundamental functions: unit, flat, bind, map

```
1 let unit t = Some t
```

- Sample functions:

```
1 let h t = t+1
2 let f t = Some t        // f = id >> unit = unit
3 let f' t = Some (t+1)   // f' = h >> unit
4 let g t = Some (t+1)    // g = h >> unit
```

- Samples:

```
1 h 3 ⇒ 4
2 f 3 ⇒ Some 3
3 f' 3 ⇒ Some 4
4 g 3 ⇒ Some 4
```

Option Monad: flat

- flat (requires at least a double nested option):

```
1 let flat' m2 =
2     if Option.isSome m2 then Option.get m2
3     else None
4
5 let flat m2 =
6     match m2 with
7     | Some m -> m
8     | None -> None
```

- Samples:

```
1 flat (Some (Some 3)) => Some 3
2 flat (Some None) => None
3 flat None => None
```

Option Monad: map

- map (fun h is arbitrary):

```
1 let map' h m = Option.map h m  // F# core lib
2
3 let map h m =
4     match m with
5     | Some t -> h t |> unit
6     | None -> None
```

- Samples:

```
1 map h (Some 3) ⇒ Some 4
2 map h None ⇒ None
3
4 map g (Some 3) ⇒ Some (Some 4)
5 map g None ⇒ None
```

Option Monad: bind

- bind (function g returns an option, possibly None):

```
let bind' g m = Option.bind g m

let bind g m =
    match m with
    | Some t -> g t
    | None -> None
```

- Samples:

```
bind g (Some 3) ⇒ Some 4
bind g None ⇒ None
```

Option Monad: Kleisli

- Kleisli operators

```
1 let (>>=) m g = m |> bind g
2
3 let (>=>) f g = f >> bind g
```

- Samples:

```
1 (f >=> g) (Some 3) => Some 4
2
3 (f >=> g) None => None
```

Option Monad: Builder, Sugared

- Option monad builder

```
1 type OptionBuilder () =
2   member this.Bind (m, g) = bind g m // let! do!
3   member this.ReturnFrom (m) = m     // return!
4   member this.Return (t) = unit t    // return
5
6 let mu = OptionBuilder ()  // new
```

- Sugared

```
1 let x m = mu {
2       let! t = m
3       let! u = f t
4       return! g u
5 }
```

Option Monad: Builder, Desugared

- Desugared

```
1 let y m =
2     mu.Bind (m,
3         fun t ->
4             mu.Bind (f t,
5                 fun u -> mu.ReturnFrom (g u)))
```

- Samples:

```
1 x (Some 3) ≡ y (Some 3) ⇒ Some 4
2
3 x None ≡ y None ⇒ None
```

Homework Suggestions

- Consolidate your understanding by repeating similar exercises with other monads
- Arrays
- Sequences
- Task – cf. MyTask
- Async – hard!
- Trace
- References