# Software Tools
# Type Systems

Part II - Lecture 9



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# Today's Outline

- Introduction to Type Systems
- Simplified Java Type Rules
- Type Derivation



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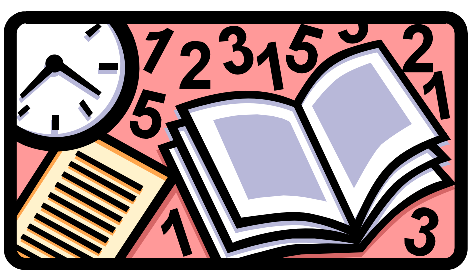
# Assignment 2





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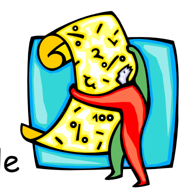
# Report Grading Schedule



Approx. **5 pages** (including figures) IEEE style

0. **IEEE style (5%), Abstract (5%)**
1. **Introduction (10%)**:
   Introduced & motivated the project and its aims?
2. **Related Work (20%)**:
   Cited & described academic related work (≥4)?
3. **Requirements (10%)**: What needed to be done & why?
4. **Design (20%)**: How did you design your solution?
   Why? Design alternatives? Strengths & weaknesses?
5. **Implementation (20%)**: How did you implement it?
   What did you contribute? The team work? Challenges?
6. **Conclusion (10%)**: Achievements? Conclusions?
   Lessons? Future/unfinished work?

# Introduction to Type Systems



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# Type Systems

- Detect potential runtime errors in source code
- Some errors cannot be detected in general, e.g. division by zero, array boundary violations etc.
- Idea: only detect some errors ("forbidden errors")
- General type-checker algorithm:
  - Use type rules that define how elementary parts of the source code should look like
  - Type rules give program parts a type
  - If a type can be derived for a program, then it does not contain any forbidden errors

```
int m(String s) {
   int y = s + 1;
   m(y,3);
   return s;
}
```



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# The Environment Gamma Γ

- Γ is the scope at a particular place in the program
- It contains the signatures of the variables and methods that can be accessed there

```
class MyClass {
   int x;
   String y;
   int m1(int z) {
      int a = 0;
      return a + z;
   }
   void m2() {
      String a = "hello";
      System.out
         .println(a);
   }
}
```

$\Gamma_2$ = { int x; String y; int m1(int z); void m2(); int z; int a; }

$\Gamma_3$ = { int x; String y; int m1(int z); void m2(); String a; }

$\Gamma_1$ = { int x; String y; int m1(int z); void m2(); }



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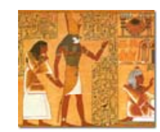
# Judgements



Statements about the correctness of program parts, e.g.

| Symbols | Meaning |
|---|---|
| { **int x;** } ├ ◊ | "{ **int x;** } is a correct environment" |
| { **int x;** } ├ **x+1**:**int** | "**x+1** is a correct expression of type **int** in environment { **int x;** }" |
| { **int x;** } ├ **x=x+1;** | "**x=x+1** is a correct statement in environment { **int x;** }" |
| { **int x;** } ├ **void m(){x=x+1;}** | "**void m(){x=x+1;}** is a correct method definition in env. { **int x;** }" |
| ∅ ├ **class A {int x; void m(){x=x+1;} }** | "**A** is a correct class in an empty environment" |

# Type Rules

Rule [*expr* +] can be used to derive/check additions of integer expressions (e.g. 1+1)

Rule Name — Precondition (everything above the line) — Judgements — "is correct in environment" — "has type"

$$[\texttt{int}\ \ +]\ \frac{\Gamma \vdash expr_1 : \texttt{int} \quad \Gamma \vdash expr_2 : \texttt{int}}{\Gamma \vdash expr_1 + expr_2 : \texttt{int}}$$

Postcondition (everything below the line) — Judgement

"**If** $expr_1$ is a correct **int** expression in environment $\Gamma$ and $expr_2$ is a correct **int** expression in environment $\Gamma$ **then** $expr_1$ + $expr_2$ is also a correct **int** expression in environment $\Gamma$"



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# Simplified Java Type Rules

$$[\texttt{int}\ \ +]\ \frac{\Gamma \vdash expr_1 : \texttt{int} \quad \Gamma \vdash expr_2 : \texttt{int}}{\Gamma \vdash expr_1 + expr_2 : \texttt{int}}$$



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# Type Derivation

**Idea**: derive smaller parts, combine them into big parts

From smallest to biggest:

1. Environments for methods (containing signatures for accessible methods and vars)
2. Expressions in methods
3. Statements in methods
4. The methods themselves
5. Member variables
6. The whole class

(4–6: Not covered in 732)

The start rule (for creating environments):

$$[env]\ \frac{sig_1, \ldots, sig_n \text{ are correct signatures}}{\{sig_1, \ldots, sig_n\} \vdash \diamond}$$



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# Expressions 1

**Literals**

$$[\texttt{boolean}\ lit]\ \frac{\Gamma \vdash \diamond \quad x \in \{\texttt{true}, \texttt{false}\}}{\Gamma \vdash x : \texttt{boolean}}$$

$$[\texttt{String}\ lit]\ \frac{\Gamma \vdash \diamond \quad x \in \texttt{String}}{\Gamma \vdash x : \texttt{String}} \qquad [\texttt{int}\ lit]\ \frac{\Gamma \vdash \diamond \quad x \in \texttt{int}}{\Gamma \vdash x : \texttt{int}}$$

**Operators** (e.g. + for **int**)

$$[\texttt{int}\ \ +]\ \frac{\Gamma \vdash expr_1 : \texttt{int} \quad \Gamma \vdash expr_2 : \texttt{int}}{\Gamma \vdash expr_1 + expr_2 : \texttt{int}}$$

$$[\texttt{String}\ \ +]\ \frac{\Gamma \vdash expr_1 : \texttt{String} \quad \Gamma \vdash expr_2 : \texttt{String}}{\Gamma \vdash expr_1 + expr_2 : \texttt{String}}$$

$$[\texttt{int}\ \ ==]\ \frac{\Gamma \vdash expr_1 : \texttt{int} \quad \Gamma \vdash expr_2 : \texttt{int}}{\Gamma \vdash expr_1 == expr_2 : \texttt{boolean}}$$

You can create analogous rules for other types, e.g. **double**

# Expressions 2

**Variable access** $[var]\ \dfrac{\Gamma \vdash \diamond \quad \{type\ id;\} \subseteq \Gamma}{\Gamma \vdash id\colon type}$

Pre: a correct environment with a variable signature

Post: an expression that accesses the variable

**Method calls**

$$[call]\ \frac{\Gamma \vdash expr_1\colon type_1 \quad \ldots \quad \Gamma \vdash expr_n\colon type_n \qquad \{type_{ret}\ id(type_1\ id_1, \ldots, type_n\ id_n);\} \subseteq \Gamma}{\Gamma \vdash id(expr_1, \ldots, expr_n)\colon type_{ret}}$$

Pre: n correct expressions in an environment with a method signature (has n parameters with same types)

Post: method call using the expressions as arguments



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# Statements

**Expressions** as statements $[stat\ expr]\ \dfrac{\Gamma \vdash expr\colon type}{\Gamma \vdash expr;}$

**Assignments** $[assign]\ \dfrac{\Gamma \vdash expr\colon type \quad \{type\ id;\} \subseteq \Gamma}{\Gamma \vdash id{=}expr;}$

**Blocks** of statements $[block]\ \dfrac{\Gamma \vdash stat_1 \quad \ldots \quad \Gamma \vdash stat_n}{\Gamma \vdash \{stat_1 \ldots stat_n\}}$

**If** statement $[if]\ \dfrac{\Gamma \vdash expr\colon \texttt{boolean} \quad \Gamma \vdash stat}{\Gamma \vdash \texttt{if}(expr)\ stat}$

**If-else** statement $[if\ else]\ \dfrac{\Gamma \vdash expr\colon \texttt{boolean} \quad \Gamma \vdash stat_1 \quad \Gamma \vdash stat_2}{\Gamma \vdash \texttt{if}(expr)\ stat_1\ \texttt{else}\ stat_2}$

You can create analogous rules for **for**, **while**, …



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# Type Derivation



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# Derivation Example 1

Given the environment $\Gamma = \{$ **boolean x; int y;** $\}$

derive the following code:

```
if (x)  y = y + 1;
```

$[var]\ \dfrac{\Gamma \vdash \diamond \quad \{type\ id;\} \subseteq \Gamma}{\Gamma \vdash id\colon type}$ $\quad [\texttt{int}\ lit]\ \dfrac{\Gamma \vdash \diamond \quad x \in \texttt{int}}{\Gamma \vdash x\colon \texttt{int}}$

1. Derive expression **x** $\quad [var]\ \dfrac{\Gamma \vdash \diamond \quad \{\texttt{boolean x;}\} \subseteq \Gamma}{\Gamma \vdash \texttt{x}\colon \texttt{boolean}}$
2. Derive expression **y** $\quad [var]\ \dfrac{\Gamma \vdash \diamond \quad \{\texttt{int y;}\} \subseteq \Gamma}{\Gamma \vdash \texttt{y}\colon \texttt{int}}$
3. Derive expression **1** $\quad [\texttt{int}\ lit]\ \dfrac{\Gamma \vdash \diamond \quad 1 \in \texttt{int}}{\Gamma \vdash 1\colon \texttt{int}}$

## Derivation Example 1 Cont.

Given the environment $\Gamma = \{$ **`boolean x; int y;`** $\}$
derive the following code:

```
if (x)  y = y + 1;
```

$$[assign]\ \frac{\Gamma \vdash expr : type \quad \{type\ id;\} \subseteq \Gamma}{\Gamma \vdash id = expr;}$$

$$[if]\ \frac{\Gamma \vdash expr : \texttt{boolean} \quad \Gamma \vdash stat}{\Gamma \vdash \texttt{if}(expr)\ stat}$$

4. Derive **y+1** $\quad [\texttt{int}\ +]\ \dfrac{\Gamma \vdash \texttt{y} : \texttt{int} \quad \Gamma \vdash \texttt{1} : \texttt{int}}{\Gamma \vdash \texttt{y+1} : \texttt{int}}$
5. Derive **y=y+1** $\quad [assign]\ \dfrac{\Gamma \vdash \texttt{y+1} : \texttt{int} \quad \{\texttt{int y;}\} \subseteq \Gamma}{\Gamma \vdash \texttt{y=y+1;}}$
6. Derive **if** $\quad [if]\ \dfrac{\Gamma \vdash \texttt{x} : \texttt{boolean} \quad \Gamma \vdash \texttt{y=y+1;}}{\Gamma \vdash \texttt{if(x)}\ \ \texttt{y=y+1;}}$





## Derivation Example 2

Given the environment $\Gamma = \{$ **`int x; int m(String s);`** $\}$
derive the following code:

```
x = m("hello") + 7;
```

$$[call]\ \frac{\Gamma \vdash expr_1 : type_1 \quad \ldots \quad \Gamma \vdash expr_n : type_n \qquad \{type_{ret}\ id(type_1\ id_1, \ldots, type_n\ id_n);\} \subseteq \Gamma}{\Gamma \vdash id(expr_1, \ldots, expr_n) : type_{ret}}$$

1. Derive **"hello"** $\quad [\texttt{String}\ lit]\ \dfrac{\Gamma \vdash \diamond \quad \texttt{"hello"} \in \texttt{String}}{\Gamma \vdash \texttt{"hello"} : \texttt{String}}$
2. Derive **m("hello")**

$$[call]\ \frac{\Gamma \vdash \texttt{"hello"} : \texttt{String} \quad \{\texttt{int m(String s);}\} \subseteq \Gamma}{\Gamma \vdash \texttt{m("hello")} : \texttt{int}}$$





## Derivation Example 2 Cont.

Given the environment $\Gamma = \{$**`int x; int m(String s);`**$\}$
derive the following code:

```
x = m("hello") + 7;
```

$$[assign]\ \frac{\Gamma \vdash expr : type \quad \{type\ id;\} \subseteq \Gamma}{\Gamma \vdash id = expr;}$$

4. Derive **7** $\quad [\texttt{int}\ lit]\ \dfrac{\Gamma \vdash \diamond \quad \texttt{7} \in \texttt{int}}{\Gamma \vdash \texttt{7} : \texttt{int}}$
5. Derive addition $\quad [\texttt{int}\ +]\ \dfrac{\Gamma \vdash \texttt{m("hello")} : \texttt{int} \quad \Gamma \vdash \texttt{7} : \texttt{int}}{\Gamma \vdash \texttt{m("hello")+7} : \texttt{int}}$
6. Derive assignment

$$[assign]\ \frac{\Gamma \vdash \texttt{m("hello")+7} : \texttt{int} \quad \{\texttt{int x;}\} \subseteq \Gamma}{\Gamma \vdash \texttt{x=m("hello")+7;}}$$





## Summary

# Today's Summary

- **Type systems** detect potential runtime errors in code
- **Environment** $\Gamma$ contains the signatures of the accessible variables and methods in a method
- **Type rules** with pre- and postcondition & judgements, e.g.

$$[\texttt{int +}]\ \frac{\Gamma \vdash expr_1 : \texttt{int} \quad \Gamma \vdash expr_2 : \texttt{int}}{\Gamma \vdash expr_1 + expr_2 : \texttt{int}}$$

- **Type derivation**: using the type rules, first derive smallest parts, then combine them into larger parts

**Reference**:

Luca Cardelli. Type Systems.
http://www.eecs.umich.edu/~bchandra/courses/papers/Cardelli_Types.pdf





# Quiz

1. What is a type system?
2. What is an environment $\Gamma$ ? Why do we need it?
3. What is a judgement? Give examples.
4. Given the environment
   $\Gamma = \{$ **`String s; String m(int a, int b);`** $\}$
   Derive the following program:

```
if(s=="hello") s = m(1,2); else s = "abc";
```