COMPSCI.373 Tutorial 2

Primitives and Transforms

Primitives

(a) Convert the following homogeneous points into non-homogeneous points:

i. $[1, 2, 3, 1]^T$

ii. $[a, b, c, d]^T$

iii. $[3, 6, 9, 3]^T$

iv. $[1, 2, 3, 0]^T$

(b) Give the equation of the plane that goes through points $[1, 1, 1]^T$, $[2,1,3]^T$ and $[-1,-2,1]^T$.

(c) Give the equation of the line that goes through points $[1,0, 1]^T$ and $[2, 5, 1]^T$.

The Coordinate System

Assume that we have a new coordinate system defined in terms of an original Cartesian coordinate system as follows: Axis u = $[-1, 0, 0]^T$, Axis v = $[0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^T$, Axis w = $[0, -1, 0]^T$, and origin $[0, -1, 0]^T$.

a) What are the u, v, w coordinates of the point $[1, 2, 3]^T$ in the original system?

b) Are the axes u and w orthogonal to each other?

c) Are the axes v and w orthogonal to each other?

Distance from Point

(a) A plane as the equation $2x + 3y + z = 4$. Find the magnitude of the distance between the plane and point $[1, 1, 2]^T$.

(b) A line goes through the point $[2, 2, 1]^T$ at a gradient of $[1, 1, 0]^T$. Find the distance between the line and the point $[1, 1, 2]^T$.

Affine Transformations

An affine transformation preserves **collinearity** and **ratios of distance**. Which of the following homogeneous 3D transformations are not affine?

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 3 & 0 & 0 & 1 \\ 0 & \frac{3}{\sqrt{2}} & \frac{-3}{\sqrt{2}} & 1 \\ 0 & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Affine Transformations

(a) Write down the 3D homogeneous transformation matrix representing the operations in the following order: (1) A translation of 2 in the x direction and 1 in the y direction; (2) A rotation of 45 degrees around the x axis.

(b)Write down the 3D homogeneous transformation matrix representing the operations in the following order: (1) A rotation of 45 degrees around the x axis. (2) A translation of 2 in the x direction and 1 in the y direction.

(c) What do you notice about (a) and (b)?

(d) Find the inverse of the following matrix: $\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Operations

(a) Consider the point $p = [1, 1, 1]^T$ and point $q = [2, 3, 1]^T$. Find the location of q' which is a 30 degree rotation of q around p in the YZ plane.

(b) When plane $x+z=2$ and $x+y-z=3$ intersect, they form a line. What is the equation of this line?