COMPSCI.373 TUTORIAL 3 SOLUTIONS

COLOR AND ILLUMINATION

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1 Vector Representation

Exercise 1.a.

$$\mathbf{AB} \bullet \mathbf{AC} = |\mathbf{AB}||\mathbf{AC}|cos(\theta)$$

$$cos(\theta) = \frac{\mathbf{AB} \bullet \mathbf{AC}}{|\mathbf{AB}||\mathbf{AC}|}$$

$$\boldsymbol{AB} = \begin{pmatrix} \sqrt{3}+1 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ 1 \\ 0 \end{pmatrix}$$

$$\boldsymbol{AC} = \begin{pmatrix} \sqrt{3}+1 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{AB} \bullet \mathbf{AC} = \sqrt(3) \times \sqrt(3) = 3$$

$$|\mathbf{AB}||\mathbf{AC}| = \sqrt(4) \times \sqrt(3) = 2\sqrt(3)$$

$$cos^{-1}(\frac{3}{2\sqrt(3)}) = 30 degrees$$

Exercise 1.b.

$$volume = (u \times v) \bullet w$$

$$= \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} = 47$$

Exercise 1.c.

$$(i) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} (ii) \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} (iii) \begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix}$$

$$(iv) \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Exercise 2.a.

$$I(x, y) = \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

Exercise 2.b. There are 3 types of receptors (cones) in our eyes that are roughly sensitive to Red, Green and Blue.

Exercise 2.c. There are 3 types of percepts in our eyes that are sensitive to high, mid and low range frequencies bands respectively. Any frequency within the sensitive range (band) is perceived to be the same. Thus colors with different wavelengths within the same bands appear the same.

Exercise 2.d. Not all visible colors have positive Red, Green and Blue components. RGB has no obvious interpolation scheme with respect to saturation or brightness.

Exercise 2.e. Easy for hardware to understand how to generate the colour.

Exercise 3.a. Describes the total proportion of light given off (emitted, transmitted or reflected) by a color sample at each visible wavelength.

Exercise 3.b. (i) Absorption + Reflection (ii) Absorption (iii) Reflection (iv) Transmission

Exercise 3.c. Reflection "bounces" off an object at an angle determined by the angle of incidence. Refraction "bends" at boundaries of different (semi)transparent mediums and is associated with transmittion.

Exercise 4.a. Using a transformation matrix.

Exercise 4.b. All visible light can be represented as positive components of XYZ. The Y component represents luminance (brightness).

Exercise 4.c. Convert to HSL, interpolate, convert back.

Exercise 5.a. The Phong reflection model determines the color of a pixel to be based on a combination of diffuse reflection (rough surfaces), specular reflection (shiny surfaces) and ambient reflection (scattered light). Phong shading propagates color across the faces of a 3D model using the interpolation of surface normals.

Exercise 5.b. Specular reflection and Diffuse reflection

Exercise 5.c. GPUs allow for massively parallel processing.

Exercise 5.d. Specular reflection

Exercise 5.e. Diffuse reflection