# Tutorial Answers

260-SL



## 1. Primitives

(a)(i) $[1,2,3]^T$

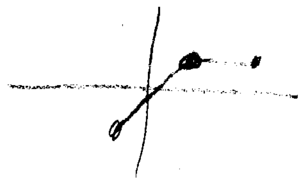
(ii) $[a/d, b/d, c/d]^T$

(iii) $[1,2,3]^T$

(iv) $[\infty,\infty,\infty]^T$

(4)

(b) vector 1: $\begin{pmatrix}2\\1\\3\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\0\\2\end{pmatrix}$



vector 2: $\begin{pmatrix}-1\\-2\\1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-2\\-3\\0\end{pmatrix}$

(5)

$\begin{pmatrix}1\\0\\2\end{pmatrix} \times \begin{pmatrix}-2\\-3\\0\end{pmatrix} = \begin{pmatrix}6\\-4\\-3\end{pmatrix} = 6x-4y-3z = -\underline{1}$

$\therefore 6x-4y-3z+1=0$

(c) $\begin{pmatrix}1\\0\\1\end{pmatrix} - \begin{pmatrix}2\\5\\1\end{pmatrix} = \begin{pmatrix}-1\\-5\\0\end{pmatrix}$ $\therefore$ line $\begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}-1\\-5\\0\end{pmatrix}$

(1)

(12:10)

(10)

2(a) 1. translate centre

$$\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}0\\-1\\0\end{pmatrix} = \begin{pmatrix}1\\3\\3\end{pmatrix}$$

2. Project onto vectors

$$u = \begin{pmatrix}1\\3\\3\end{pmatrix} \cdot \begin{pmatrix}-1\\0\\0\end{pmatrix} = -1$$

$$v = \begin{pmatrix}1\\3\\3\end{pmatrix} \cdot \begin{pmatrix}0\\1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} = 3/\sqrt{2} + 3/\sqrt{2} = 6/\sqrt{2}$$

$$w = \begin{pmatrix}1\\3\\3\end{pmatrix} \cdot \begin{pmatrix}0\\-1\\0\end{pmatrix} = -3 \qquad (4)$$

(b) $\begin{pmatrix}-1\\0\\0\end{pmatrix} \cdot \begin{pmatrix}0\\-1\\0\end{pmatrix} = 0 \therefore$ yes (1)

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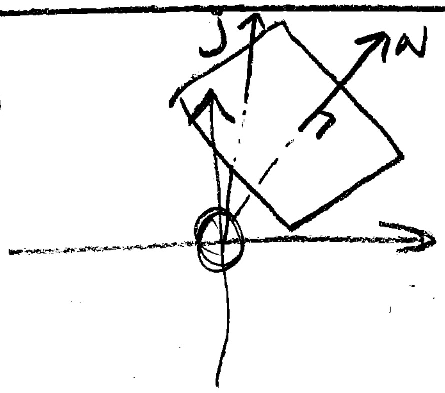
(c) $\begin{pmatrix}0\\1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} \cdot \begin{pmatrix}0\\-1\\0\end{pmatrix} = -1/\sqrt{2} \therefore$ no

12:15 (15)

tutorial answers (2)

3(a)




1. normal of plane $= \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

2. project point $\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ onto $N$

$$= \frac{j \cdot N}{|N|} = \frac{ax_0 + by_0 + cz_0}{\sqrt{a^2+b^2+c^2}}$$

3. Subtract d $= \frac{d}{|N|}$

$\therefore$ final formula

$$\frac{2.1+3.1+1.2-4}{\sqrt{2^2+3^2+1^2}} = \frac{3}{\sqrt{14}} \longleftarrow = \frac{ax_0+by_0+cz_0+d}{\sqrt{a^2+b^2+c^2}}$$

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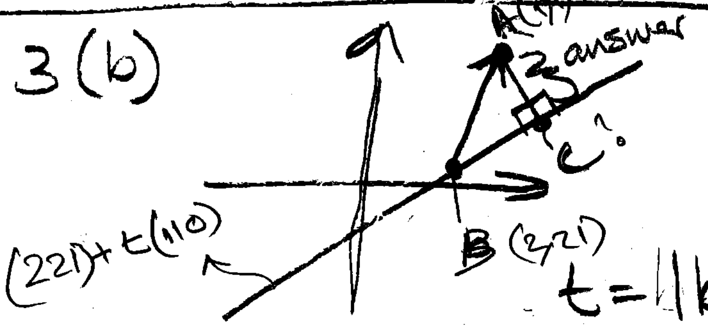
3(b)




1. Vector kb $= \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

2. project onto d

$$t = \|bc\| = \frac{kb \cdot d}{|d|^2} = \frac{[-1,-1,1]\cdot[1\ 1\ 0]}{2} = -1$$

3. Find $\perp$ point

$$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

4. Find the difference

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$ $\therefore$ mag $= 1 =$ the distance between the two

$$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \frac{BA \cdot G}{|G|^2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

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4 (a) translation $\Rightarrow$ affine

(b) rotation + translation $\Rightarrow$ rigid transform (Euclidean) = affine

(c) rotation + translation + scale $\Rightarrow$ scale transform (Similarity) = affine

(d) projection = <u>not affine</u>

(5) 12:35

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5(a)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5(b)

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 1 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5(c) no so order matters

5(d)

$$\begin{bmatrix} \sqrt{3}/2 & 0.5 & 0 & 0 \\ -0.5 & \sqrt{3}/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(5)
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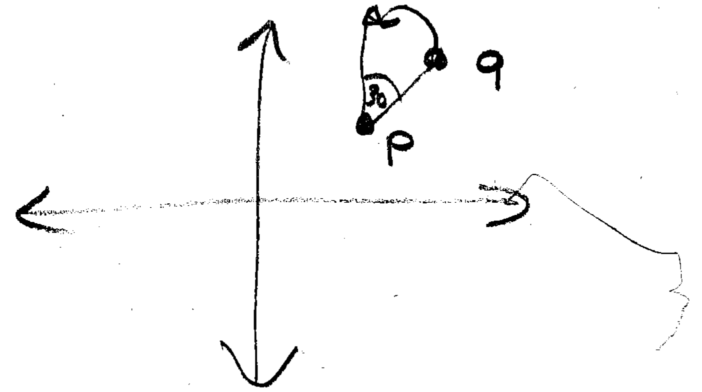
6 (a)




1. translate $\longrightarrow$ $\begin{pmatrix}2\\3\\1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\2\\0\end{pmatrix}$

2. rotate

3. translate back

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \sqrt{3}/2 & -0.5 & 0\\ 0 & 0.5 & \sqrt{3}/2 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\2\\0\\1\end{bmatrix} = \begin{pmatrix}1\\ \sqrt{3}\\ 1\\ 1\end{pmatrix}$$

$$\begin{pmatrix}1\\ \sqrt{3}\\ 1\end{pmatrix} + \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\ \sqrt{3}+1\\ 1\end{pmatrix}$$

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6 (b)

$$x + 0y + z = 2$$
$$x + y - z = 3$$

$$\therefore \begin{pmatrix}1\\0\\1\end{pmatrix} \times \begin{pmatrix}1\\1\\-1\end{pmatrix} = \begin{pmatrix}-1\\2\\1\end{pmatrix}$$

$$\therefore \begin{pmatrix}2\\1\\0\end{pmatrix} + t\begin{pmatrix}-1\\2\\1\end{pmatrix}$$

let $z=0$

$\therefore x = 2$

$y = 1$

$z = 0$

(50)