



# Part 1: 2D/3D Geometry, Colour, Illumination

## Colours

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COMPSCI 373 Computer Graphics and Image Processing



<http://socks-studio.com/2013/...>



<http://www.mutluduvar.com/...>

- ① Colours: Real and Graphical World
- ② Spectral density function (SDF)
- ③ Interaction of Light with Materials
- ④ Spectral response function (SRF)
- ⑤ Human perception of light
- ⑥ Colour Spaces
- ⑦ RGB and CMYK colour spaces
- ⑧ HSL and HSV colour spaces

# Colours in Real and Graphical World



# Colours in Real and Graphical World

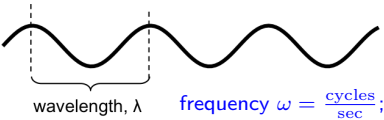


# Electromagnetic Radiation: Waves

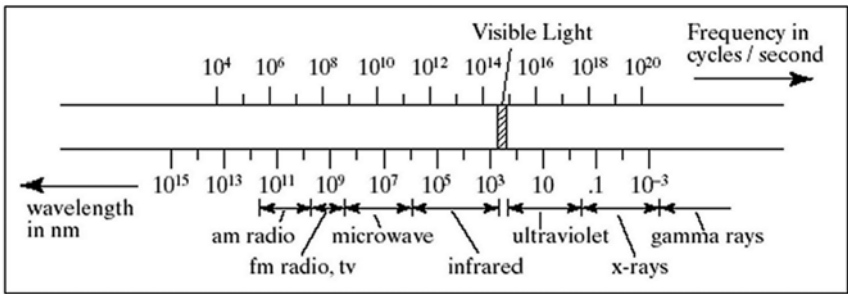
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Theories of light:

- ① Waves (classical physics)
- ② Particles (quantum physics)



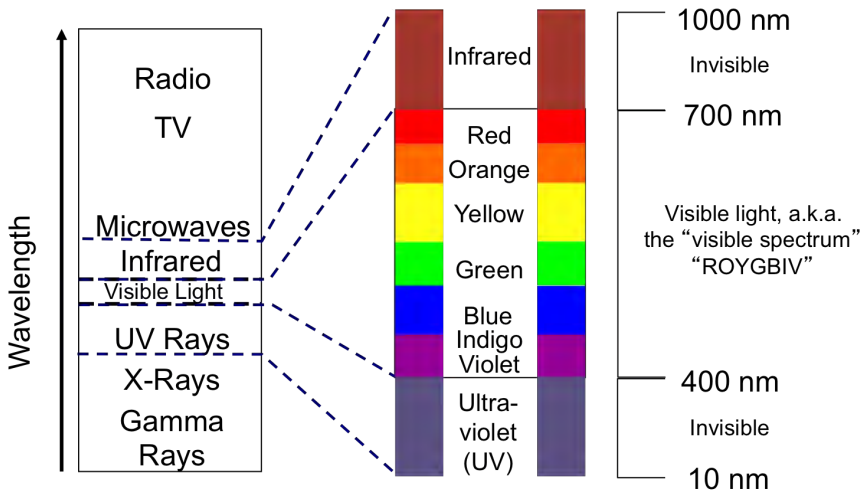
frequency  $\omega = \frac{\text{cycles}}{\text{sec}}$ ;  $\lambda \cdot \omega = c$ ;  
 $c = 3 \cdot 10^8 \frac{m}{\text{sec}}$  – speed of light.



**FIGURE 12.1** Electromagnetic spectrum.

# Neighbours of Visible Light

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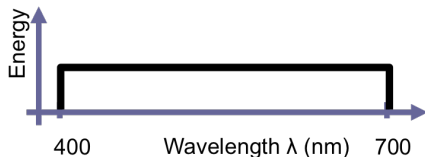
# Spectral Density Function (SDF): $S(\lambda)$

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$$S(\lambda) = \text{power} / \text{unit wavelength} = \text{energy}$$



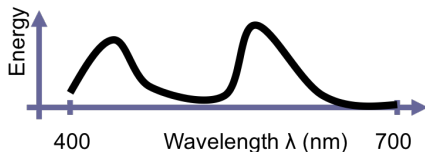
spike or “single” wavelength  
= “spectral colour”



uniform white/gray light



white light plus a dominant wavelength

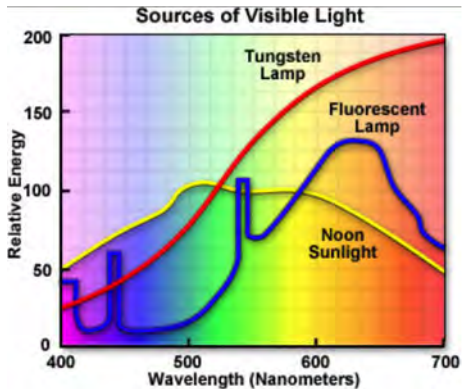
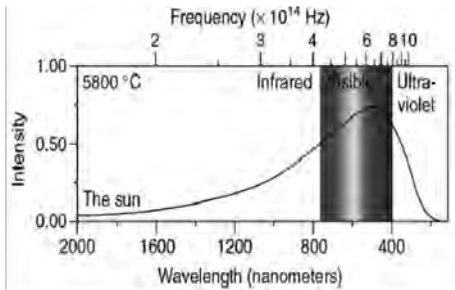


arbitrary SDF: blue plus orange/yellow

# SDFs for Different Light Sources

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Some other light spectra: sunlight, tungsten lamp, fluorescent lamp



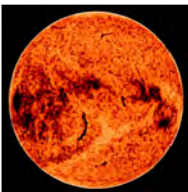
<http://www.micro.magnet.fsu.edu/optics/lightandcolor/sources.html>



# The Sun (across the electromagnetic spectrum)



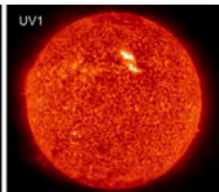
Low frequency



IR (infrared)



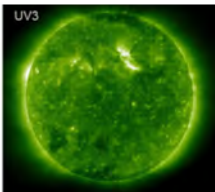
Visible light



UV<sub>1</sub> (ultraviolet)



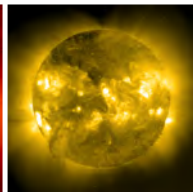
UV<sub>2</sub>



UV<sub>3</sub>

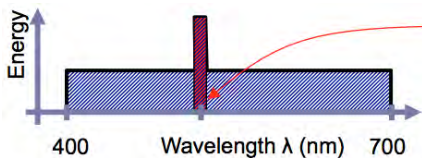





X-ray

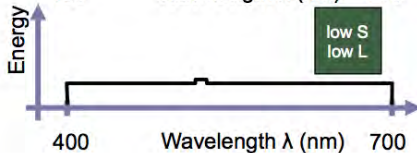
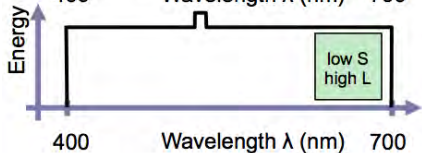
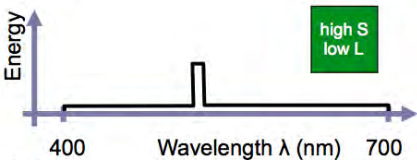


Gamma-ray

# Describing Colors using the SDF



- **Hue**: dominant wavelength
- **Luminance** (brightness): total power (integral of SDF) 
- **Saturation** (also purity): % of luminance residing in dominant component  



# Interaction of Light with Materials



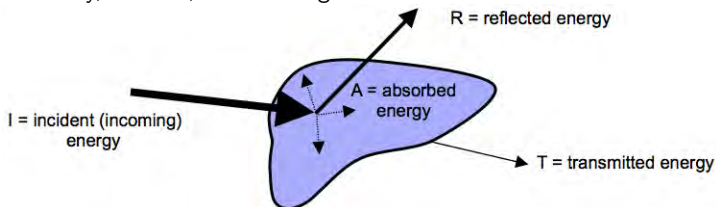
# Interaction of Light with Materials

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Light  $\Rightarrow$  surface of some “body”: 3 possible results:

- 1 **Absorption** – energy of selected wavelengths retains in the body.
- 2 **Transmission** – energy of selected wavelengths travels through and exits the body; *refraction* of light occurs at boundaries.
- 3 **Reflection** – an energy of selected wavelengths bounces off the surface: angle of reflection = angle of incidence.

+ combinations of 1–3, such as “internal reflection” when light enters a semi-translucent body, scatters, and some light reflects back out: human or Shreks skin. . .



# Interaction of Light with Materials

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Incident (incoming) light energy:

= **A**bsorbed energy + **T**ransmitted energy + **R**eflected energy

= retained + passed through + bounced off

Chemical properties of the body determine the % of each.



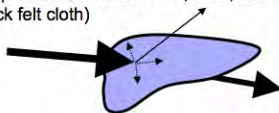
opaque coloured surface:  $A > 0$ ,  $T = 0$ ,  $R > 0$



opaque black surface:  $A = 1$ ,  $T = 0$ ,  $R = 0$   
(black felt cloth)



perfect mirror surface:  $A = 0$ ,  $T = 0$ ,  $R = 1$



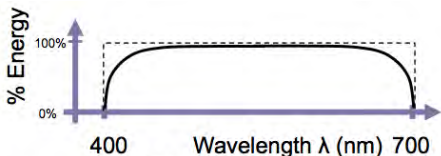
transparent surface:  $A$ ,  $R$  small,  $T \sim 1$   
(clear glass)

Computer graphics: **reflected** (and refracted), **transmitted light**.

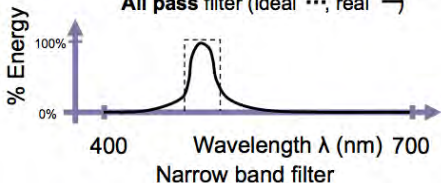
# Spectral Response Function (SRF)

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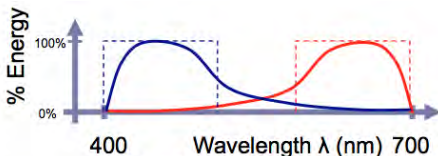
- Molecular structure of a body determines which wavelengths of light and what amount are absorbed, transmitted, or reflected.
- Can be measured with a **spectral response function (SRF)**, or *filter function*.



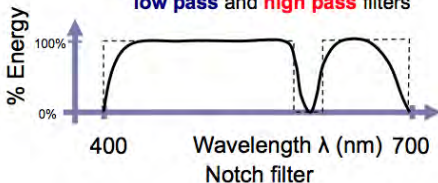
All pass filter (ideal  $\cdots$ ; real  $\rightarrow$ )



Narrow band filter



low pass and high pass filters



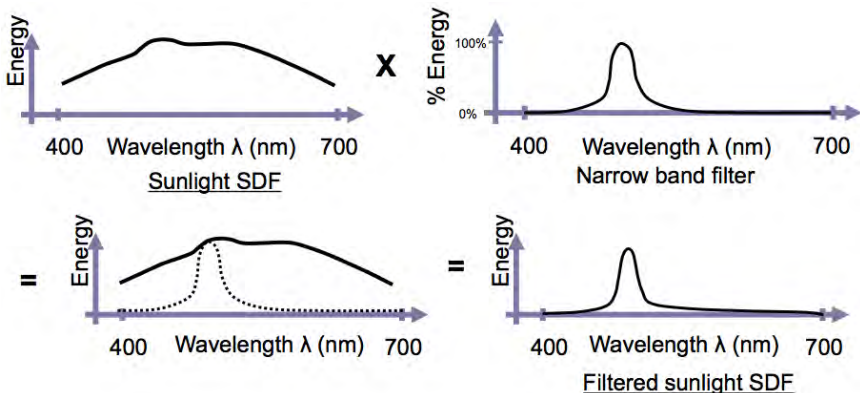
Notch filter

$$\text{SDF}_{\text{light source}} \times \text{SRF} = \text{SDF}_{\text{result}}$$

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SDF of result = product of the SRF and the light-source's SDF:

Multiply at each wavelength  $\lambda$  – the SRF % times the source energy.



$$\text{SDF} \times \text{SRF} = \text{SDF}_{\text{result}}$$

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Why is this relevant for computer graphics?

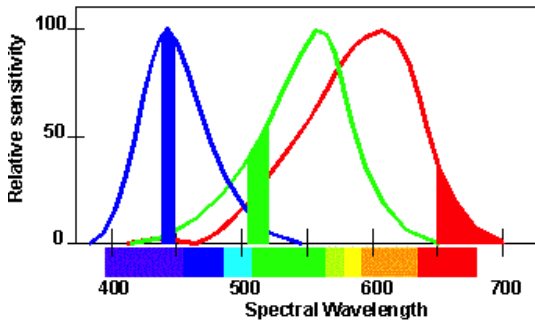
- 1 All light sources can be defined by their SDF.
  - Natural light source: sun, fire.
  - Artificial light source: light bulb, laser, LED, computer display.
- 2 All light absorbers, transmitters, or reflectors can be defined by their SRF.
  - Sensing devices: an **absorbed light SRF**.
  - Camera (digital photocell, film).
  - Human eye (retina).
    - Definition of “colour” = integral of  $(\text{SDF}_{\text{light source}} \times \text{SRF}_{\text{sensor}})$ .
  - Glass, still water, cellophane: a **transmitted light SRF**.
  - Surface material of an object: a **reflected light SRF**.



# Human Perception of Light



## How the Eye Perceives Color

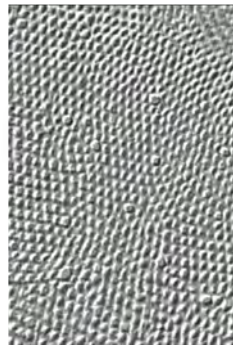
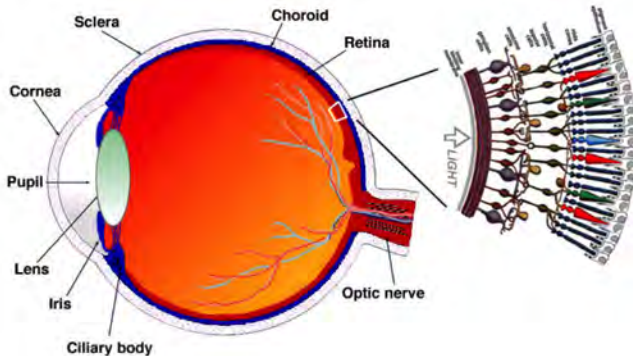


[pixshark.com](http://pixshark.com)

[www.colorcube.com](http://www.colorcube.com)

# The Eye

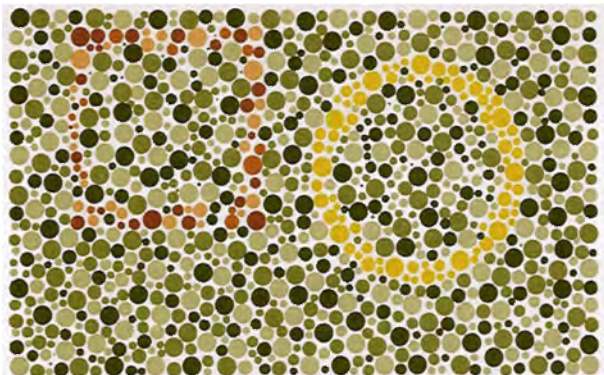
Four types of receptors (sensors):  
R/G/B cones + rods, each having the unique SRF.



<http://webvision.med.utah.edu/imageswv/fovmoswv.jpeg>  
<http://webvision.med.utah.edu/imageswv/Sagschem.jpeg>

# Colour “Blindness”

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<http://members.aol.com/protanope/card2.html>

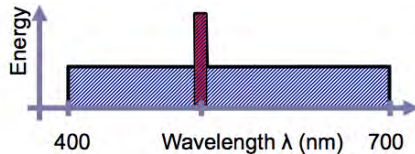
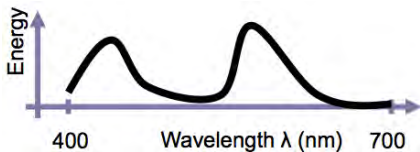
If you didn't see both a yellow circle and a faint brown square, you are somewhat colour blind (in the USA 5.0% of males, 0.5% of females). To find out more, visit:

<http://www.kcl.ac.uk/teares/gktvc/vc/lt/colourblindness/cblind.htm>

# Colours and the SDF

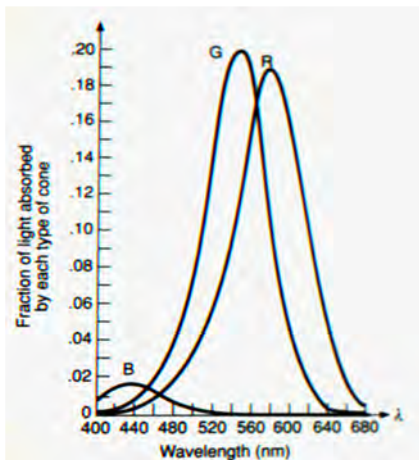
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- Many different SDFs are perceived by us as the same color!
- When describing a colour (as seen by the eye) exactly, we do not need to know the full SDF.
- Three parameters are enough.
- For example: just use hue, luminance and saturation.

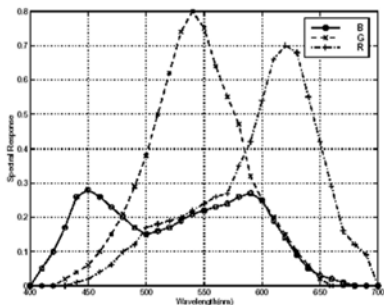


# SRFs for the Eye and a Camera

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RGB spectral responses for a Kodak digital camera



<http://www.ecs.csun.edu/~dsalomon/DC2advertis/AppendH.pdf> <http://www.stanford.edu/class/ee392b/handouts/color.pdf>

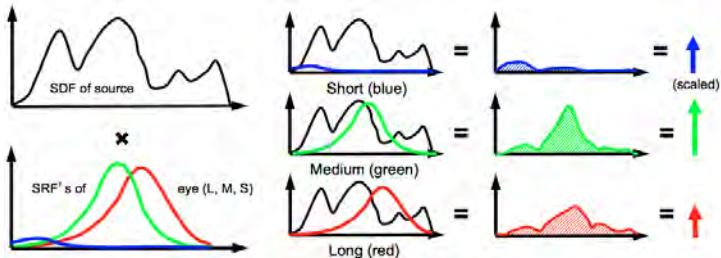
# Seeing Red, Green and Blue

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- A cone cell in the retina measures amount of red, green, or blue wavelength energy (3 SRF's). Responds only in bright light.
- The SRF of a **rod** cell covers all wavelengths (measures grey level, or intensity). Responds in low light, but not in bright light.
- Integral of R, G, or B cone response produces a single value.

**Note:** SRFs are really L, M, S (long, medium, short) wave responses, not R, G, B.

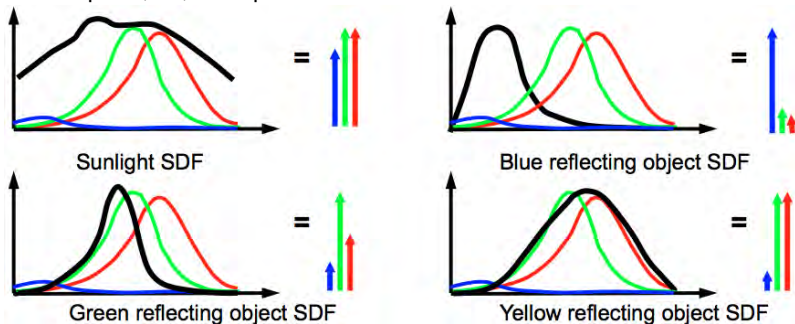
**Note:** Low short (blue) response is scaled up by vision system (after retina).



# Seeing Red, Green and Blue

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- Example L, M, S responses for various SDFs.



- Resulting L, M, and S SRF responses are independent values.
- The three SRF response values are interpreted as hues by our brain, e.g.,

**red + green = yellow** , **red + green + blue = white**

# Summary 1

- 1 **Spectral Density Function (SDF)**: describes the wave composition of light with power for each wave length segment.
- 2 **Spectral Response Function (SRF)**: can be used to specify how much % of each wave length are absorbed or reflected or transmitted.
- 3 Light with the different SDFs can have the same colour for our eye.

## References:

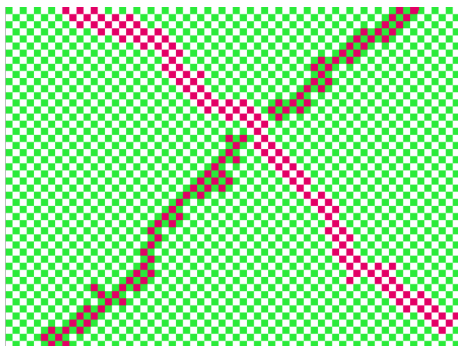
- Light and colours: Hill, Chapter 11.1
- Dominant wave length: Hill, Chapter 11.2.1



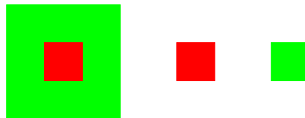


# Quiz

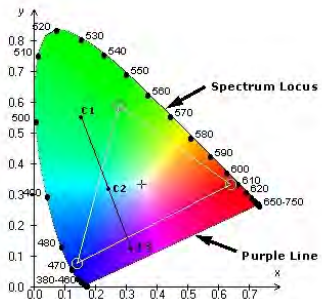
- 1 What is a spectral density function (SDF)?
- 2 In what ways can light interact with a material?
- 3 How can we describe this interaction?
- 4 What are hue, luminance and saturation?



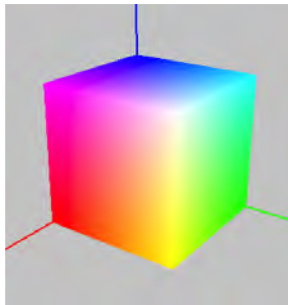
⇐ How many different colors?



# Colour Spaces



CIE XYZ



RGB



HLS

<http://www.hf.faa.gov/webtraining/visualdisplays/HumanVisSys2c5.htm>

# Colour Coordinate Space

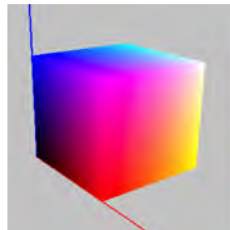
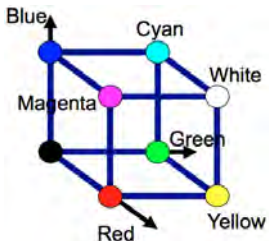
- Defines 3 SRFs (**colour matching functions**) for some sensor.
- One dimension for each SRF ( $\rightarrow$  **tristimulus colour space**).
  - Each dimension represents a **primary color P**.
  - Coordinate value = resulting SDF integral normalised to (0, 1).
- Colour triple: a 3D point defined by **chromaticity values** ( $c_0, c_1, c_2$ )
- Example: the RGB colour space (Red, Green, Blue primaries).

Basis vectors:

$$\mathbf{R} = [1, 0, 0]^T$$

$$\mathbf{G} = [0, 1, 0]^T$$

$$\mathbf{B} = [0, 0, 1]^T$$

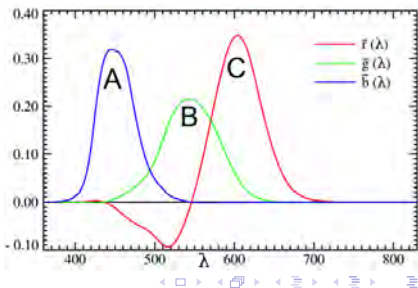


Chromaticity values:  $[r, g, b]^T = r\mathbf{R} + g\mathbf{G} + b\mathbf{B}$

# Finding Color Matching Functions

- **Given** 3 primaries A, B, C, **find** 3 SRFs, one for each primary.
- **An idea:**
  - 1 Show light  $L$  with pure color of wavelength  $\lambda$  and brightness 1 to test persons.
  - 2 Let them adjust another light  $P$  using chromaticity values  $a, b, c$  until  $L$  and  $P$  match.
  - 3 Do this with the whole range of wavelengths  $\lambda$  and note down the  $a, b, c$  values for each  $\lambda$ .

**Problem:** when using normal, visible colours as primaries, some wavelengths  $\lambda$  need negative chromaticity values because adding colours decreases saturation.



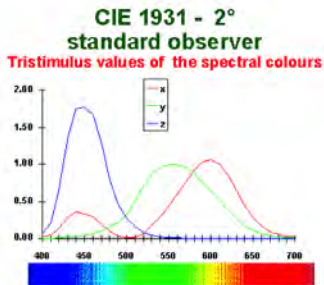
# CIE XYZ Colour Space (1931)

International Commission on Illumination (Commission internationale de l'éclairage)  
Normalised standard space designed according to requirements:

- Standard primaries "R", "G", "B".
- Only positive chromaticities.
- Equal chromaticities are greys.
- Easy conversion to brightness.

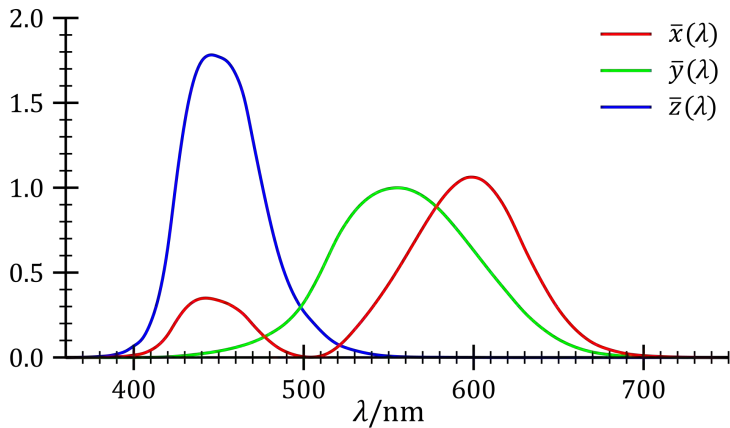
Three primaries: X, Y, Z

- All are imaginary (not real colours).
- The SRFs were designed by engineers to meet above requirements.
- Y corresponds to brightness
- Conversion to RGB by matrix multiplication (linear combination of X,Y,Z = R,G,B and vice versa).

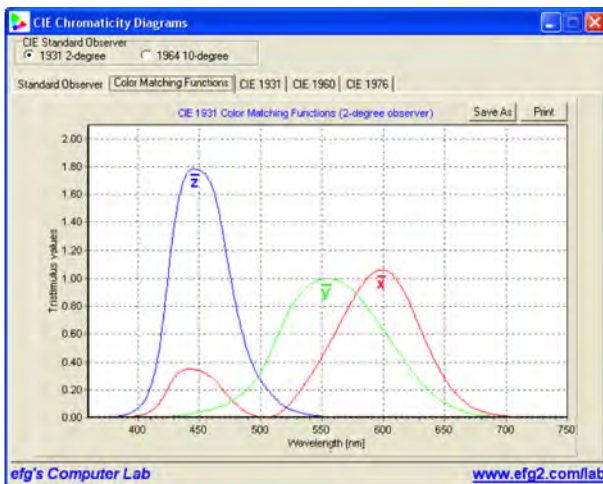


# CIE 1931 - 2° Standard Observer Colour Matching Functions

Tristimulus values of the spectral colours:



# CIE XYZ Colour Matching Functions



$(X, Y, Z)$ -coordinates of any input SDF are found by multiplying and integrating, thus defining the  $(X, Y, Z)$  colour of the SDF.

# CIE Chromaticity Diagram: 2D Chromaticity Space

Projection of the 3D  $XYZ$  space onto the 2D plane

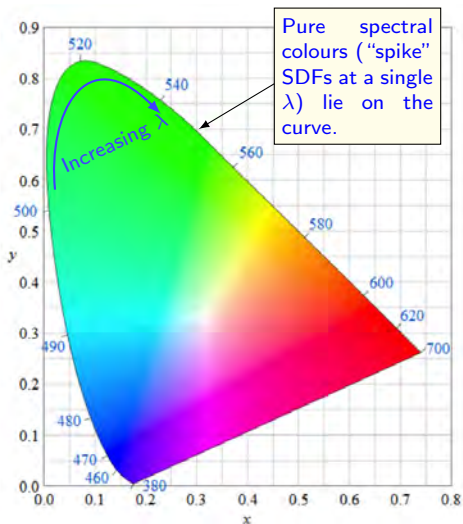
$$x + y + Z = 1:$$

- Looking only at colours with brightness 1.
- 2D coordinates  $(x, y)$ :

$$x = \frac{X}{X+Y+Z}$$

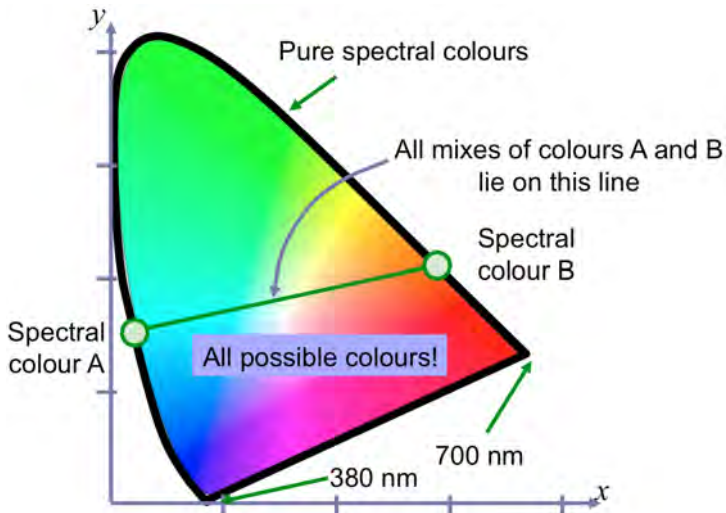
$$y = \frac{Y}{X+Y+Z}$$

$(x, y)$  is the chromaticity of the colour.

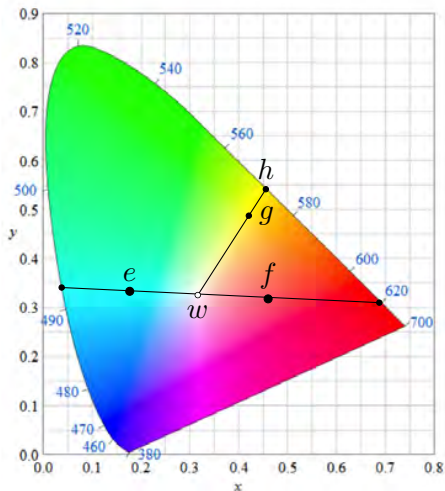




# CIE Chromaticity Diagram



# Using the CIE Chromaticity Diagram

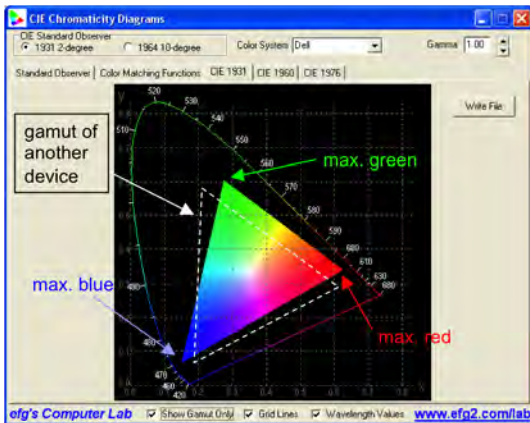


- $w$  – white.
- $e$  and  $f$  – complementary colours  
 $\left(\frac{x_e+x_f}{2} = x_w; \frac{y_e+y_f}{2} = y_w\right)$ .
  - Can be combined to white.
- $h$  – the dominant wavelength of chromaticity  $g$ .
- $\frac{(\overline{wg})}{(\overline{wh})}$  – the saturation of  $g$ .
  - How close (in %) the chromaticity  $g$  is to its pure colour  $h$ .

# Colour Gamut: A Subset of Colours A Device Can Represent

The CIE colour space can be used to describe the colour gamut.

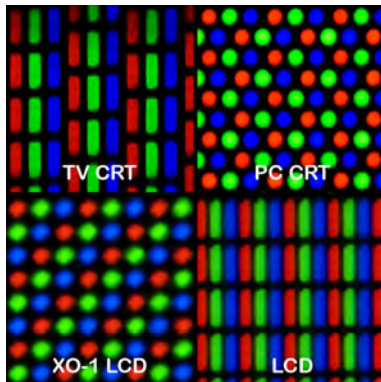
- 1 Measure maximum intensity of each **device primary** in CIE (use filters with SRFs = CIE SRFs).
- 2 Convert to the  $(x, y)$  chromaticity.
- 3 The 2D triangle defines possible device colours (i.e., the colour gamut).



Different devices have different gamuts (colour conversion problems).

# Additive Colour Systems

- Colours are mixed by adding up appropriate amounts of primaries (adding SDF spikes to black).
- Widely used in screens with subpixels emitting R,G,B.
- Cones in retina respond to light emitted by each subpixel.
- Brain adds the individual cone responses to produce perception of hue, luminance, and saturation.



<http://www.csfieldguide.org.nz/DataRepresentation.html>

⇒ **Demo program:** ColorMix.exe

<http://www.efg2.com/Lab/Graphics/Colors/ColorMix.htm>



# Subtractive Colour Systems

- Colours are mixed by subtracting appropriate amounts of colours from white (like using notch SRFs on white).
- White light is reflected or transmitted, and some wavelengths are absorbed (subtracted), e.g., coloured glass, printed images.



The colours to subtract are the complements of the primaries, e.g., **cyan magenta yellow** (CMY).

- Cyan absorbs red.
- Magenta absorbs green.
- Yellow absorbs blue.

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} c \\ m \\ y \end{bmatrix}$$

CMYK (K = black) is often used for four-colour printers.

# Troubles with RGB

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Difficult to use for colour design as selecting a hue is sometimes non-intuitive.

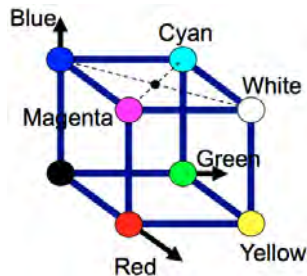
- E.g., what RGB combination would you use to make brown?
- $[128, 80, 50]$  is a good choice. Could you figure that out?

Not a good colour space for interpolating between colours. For example:

$$\frac{1}{2} \text{ blue } \blacksquare + \frac{1}{2} \text{ white } \square = \blacksquare$$

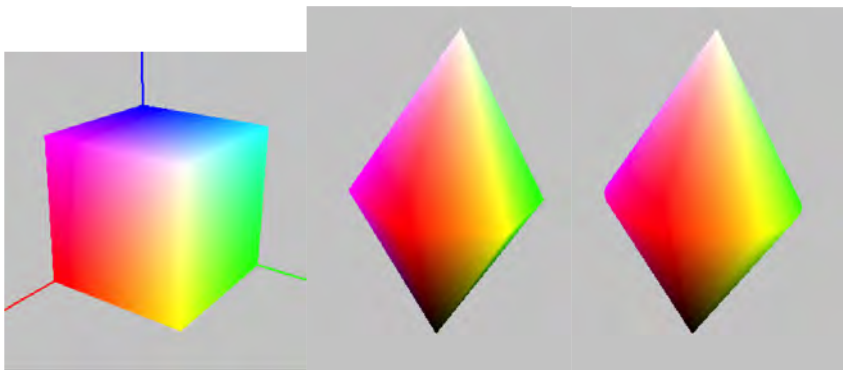
$$\frac{1}{2} \text{ magenta } \blacksquare + \frac{1}{2} \text{ cyan } \blacksquare = \blacksquare$$

Linear interpolation between  $(r, g, b)$  chromaticities does not linearly interpolate the saturation or the luminance.

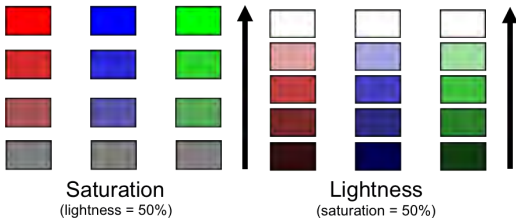
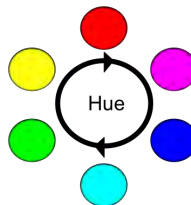
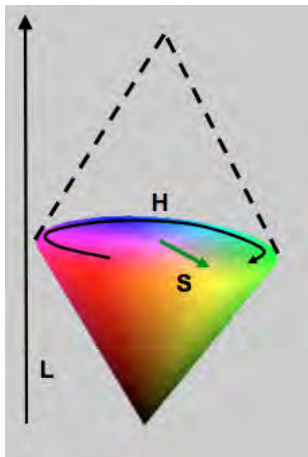


# HSL Colour Space

- **Hue, Saturation, Lightness.**
- Based on transforming the RGB cube  $\rightarrow$  double hexcone  $\rightarrow$  double cone.



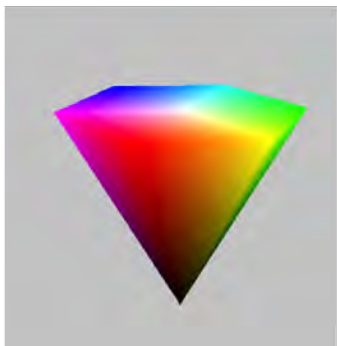
# HSL Color Space





# HSV Color Space

- **Hue, Saturation, Value** (similar to Lightness).
- Only a single cone: at the top all colours are the brightest.



**V**  
(value)



# Colour interpolation: RGB vs. HSL

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Linear interpolation between two RGB colours

$C_0 = [r_0, g_0, b_0]$  and  $C_1 = [r_1, g_1, b_1]$  in the RGB space:

$$\begin{aligned} r(t) &= r_0 + t \cdot (r_1 - r_0) \\ g(t) &= g_0 + t \cdot (g_1 - g_0); \\ b(t) &= b_0 + t \cdot (b_1 - b_0); \quad 0 \leq t \leq 1 \end{aligned}$$

## Problem:

- Saturation and luminance are **not** linearly interpolated.
- Interpolation may correctly vary from one hue (H) to another, but the saturation (S) and luminance (L) may vary in strange ways!

$C_0 = [1.0, 0.8, 0.2]$



$C_1 = [0.2, 0.0, 1.0]$

# Colour interpolation: RGB vs. HSL

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Linear interpolation between two HSL colours

$C_0 = [h_0, s_0, l_0]$  and  $C_1 = [h_1, s_1, l_1]$  in the HSL space:

$$h(t) = h_0 + t(h_1 - h_0)$$

$$s(t) = s_0 + t(s_1 - s_0);$$

$$l(t) = l_0 + t(l_1 - l_0); \quad 0 \leq t \leq 1$$

All three components (HSL) are linearly interpolated.

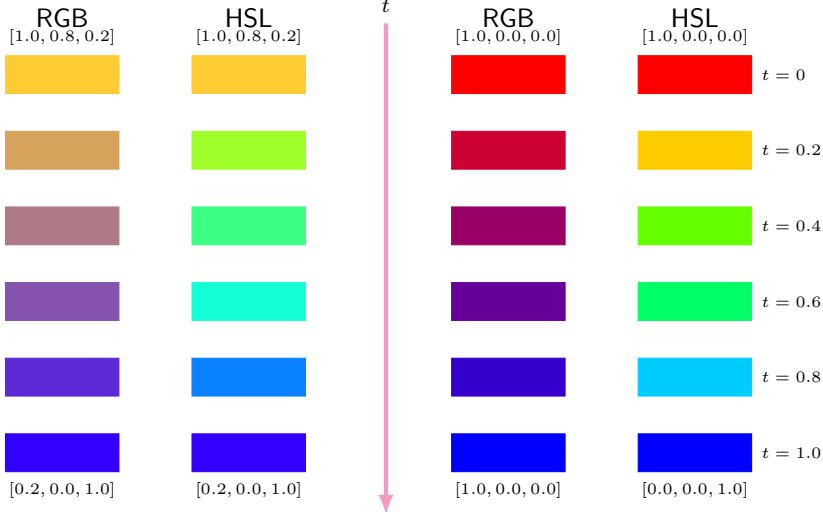
- **Solution:** Convert  $C_0$  and  $C_1$  to HSL; interpolate in HSL, convert results back to RGB.

$$C_0 = [1.0, 0.8, 0.2]$$



$$C_1 = [0.2, 0.0, 1.0]$$

# Colour interpolation: RGB vs. HSL



# Summary

- 1 Colors can be represented using a 3D colour space.
- 2 RGB: is easy to use for additive colour mixing, but has limited gamut.
- 3 CIE XYZ: can represent all visible colours.
- 4 HSL: makes proper linear interpolation between hue, saturation and lightness.

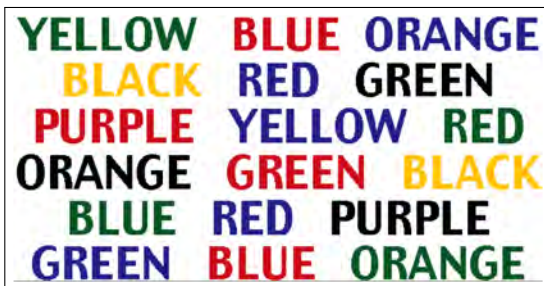
## References:

- Colour description: Hill, Chapter 11.2
- CIE colour Model: Hill, Chapter 11.3
- Other colour spaces: Hill, Chapter 11.4



# Quiz

- 1 What is a colour coordinate space?
- 2 Name an advantage of the CIE XYZ colour model.
- 3 What is the disadvantage of the RGB colour model?



Say the colour of the word (not the word itself)