Part 3: Image Processing

Basics of Mathematical Morphology

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COMPSCI 373 Computer Graphics and Image Processing

1. Main concepts
2. Structuring element
3. Erosion
4. Dilation
5. Opening
6. Closing
7. Hit-and-Miss transform
8. Greyscale Morphology

Basic Concepts

Morphology: a study of structure or form.

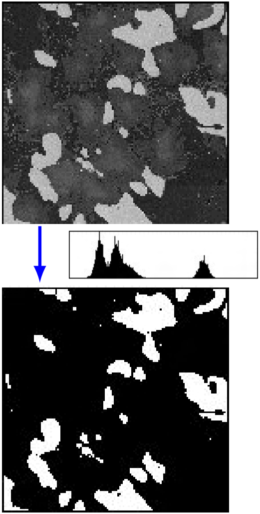
(http://www.merriam-webster.com/dictionary/morphology)

Morphological image processing may remove imperfections of a binary image.

- Regions in binary images produced by simple thresholding are typically distorted by noise.

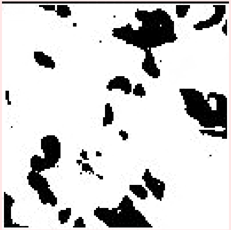
Morphological operations are non-linear and account for structure and forms of regions (objects) to improve an image.

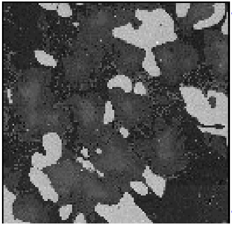
- These operations can be extended to greyscale images.



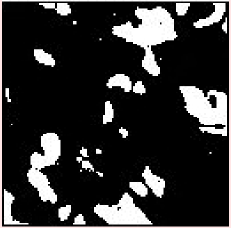
Binary Images: Object/Background

White $\underbrace{\text{background}}_{0}$ – black foreground $\underbrace{\text{objects}}_{1}$ $\Rightarrow$





For morphological processing, the foreground (i.e. object) and background pixels are treated as 1s and 0s, respectively, independently of their visual black-white coding.



Black $\overbrace{\text{background}}^{0}$ – white foreground $\overbrace{\text{objects}}^{1}$ $\Rightarrow$

Morphological Operations: Several Examples

White objects on black background: "**1**" / "**0**" as white / black

Image | Erosion | Dilation | Closing | Opening | Gradient

Black objects on white background: "**1**" / "**0**" as black / white

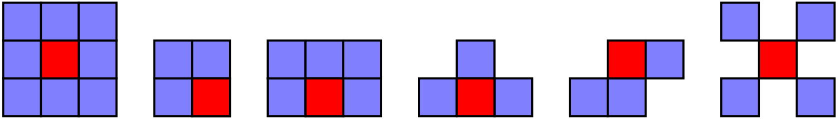
Morphological Operations: Basic Properties

Morphological operations rely on relative ordering of pixel values, not on their numerical values.

- Thus the operations are especially suited to binary image processing.
- These operations can be applied also to greyscale images such that their absolute pixel values are of no or minor interest.
  - E.g. images with unknown light transfer functions.

Morphological operations probe an image with a small shape or template called a **structuring**, or structure **element** (SE).

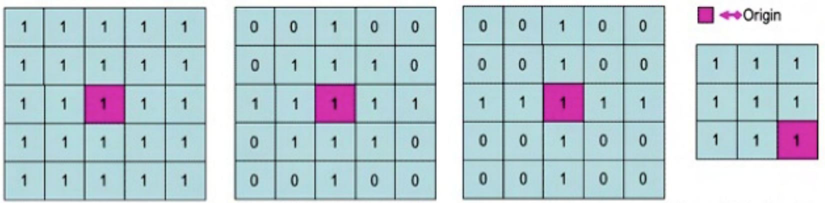
- SE resembles a convolution kernel in linear filtering.



Structuring Element

A small binary image, i.e. a small matrix of pixels, each with a value of zero (0) or one (1).

- Zero-valued pixels of the SE are ignored.
- **Size** of the SE: the matrix dimensions.
- **Shape** of the SE: the pattern of ones and zeros.
- **Origin** of the SE: usually, one of its pixels.
    - Generally, the origin can be also outside the matrix.

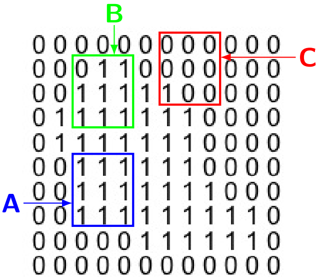


Square 5x5 element | Diamond-shaped 5x5 element | Cross-shaped 5x5 element | Square 3x3 element

Structuring Element

When an SE is placed in a binary image, each its pixel is associated with the pixel of the area under the SE:

- The SE **fits** the image if **for each** of its pixels set to 1 the corresponding image pixel is also 1.
- The SE **hits** (intersects) the image if **at least for one** of its pixels set to 1 the corresponding image pixel is also 1.



s_1

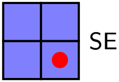
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

s_2

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

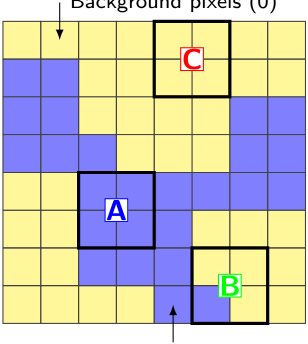
		A	**B**	**C**
fit	s_1	yes	no	no
	s_2	yes	yes	no
hit	s_1	yes	yes	yes
	s_2	yes	yes	no

Probing with a Structuring Element (SE)



A: SE **fits** the foreground (object);
B: SE **hits** (intersects with) the object;
C: SE neither fits, nor hits the object.

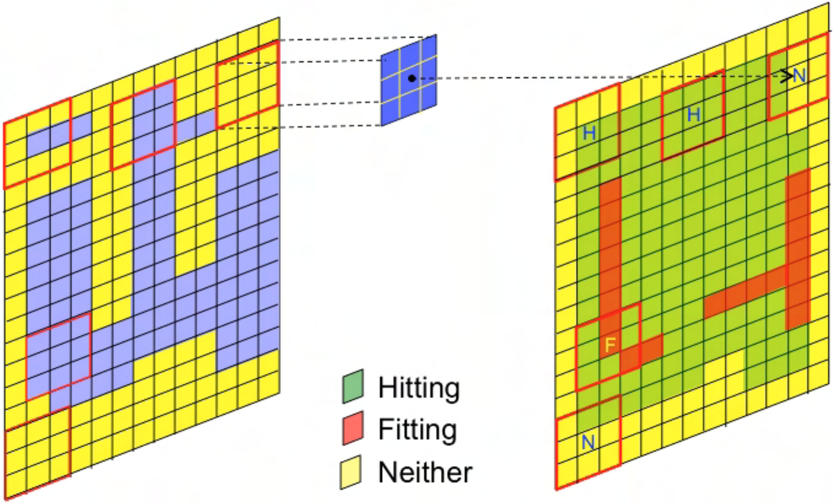
Background pixels (0)



Foreground pixels, or objects of interest (1 or any non-zero value)

- SE is positioned at all possible locations in an image and compared with the pixel neighbourhood at each location.
- Operations test whether the SE “fits” within the neighbourhood or “hits” the neighbourhood.
- An operation on a binary image creates a new binary image in which the pixel has a foreground value only if the test at that place in the input image is successful.

Probing with a Structuring Element (SE)

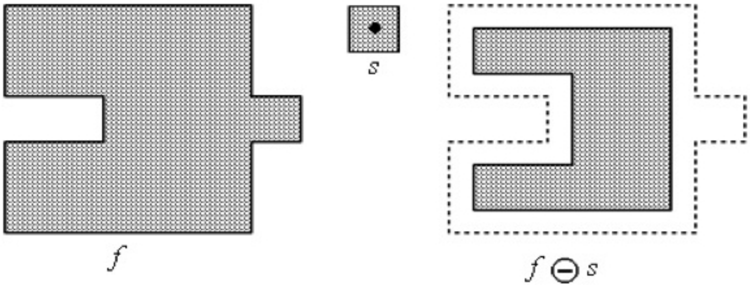


Fundamental Operation: Erosion

Erosion $f \ominus s$ of a binary image f by an SE s produces a new binary image $g = f \ominus s$.

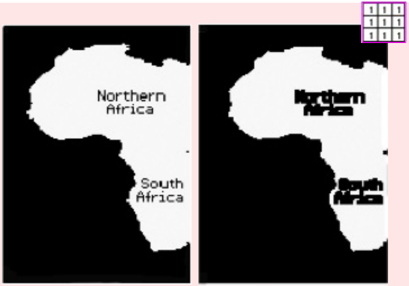
Eroded image g has ones in all locations (x, y) of an origin of the SE s at which s fits the input image f.

For all pixel coordinates (x, y), $g(x, y) = 1$ if s fits f and 0 otherwise.

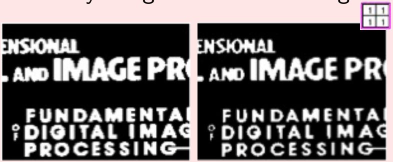


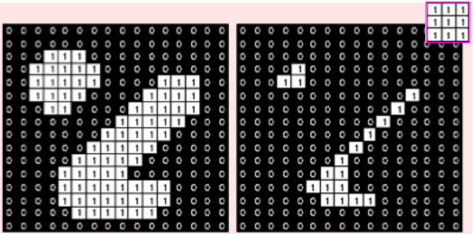
http://www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Erosion of a Binary Image with 2×2 and 3×3 SE



Binary image Eroded image





- Erosion with small (e.g. $2 \times 2 - 5 \times 5$) square SEs shrinks an image by stripping away a layer of pixels from both inner and outer region boundaries.
- Holes in and gaps between foreground objects become larger, and small details are eliminated.

www.cs.princeton.edu/ pshilane/mosaic/
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html
//documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Erosion of a Binary Image

Larger SEs have a more pronounced effect.

- Erosion with a large SE is similar to an iterated erosion with a smaller SE of the same shape. I
- If a pair of SEs s_1 and s_2 are identical in shape, with s_2 twice the size of s_1, then $f \ominus s2 \approx (f \ominus s_1) \ominus s_1$.

Erosion removes fine (small-scale) details, as well as noise from a binary image.

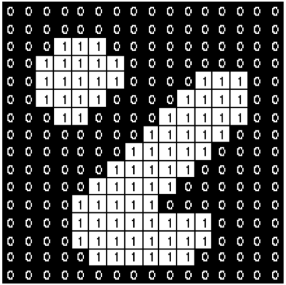
- Simultaneously, erosion reduces the size of regions of interest (objects), too.
- Background areas are growing, i.e. an image with the black or white background becomes blacker or whiter, respectively.

Boundary Detection by Erosion

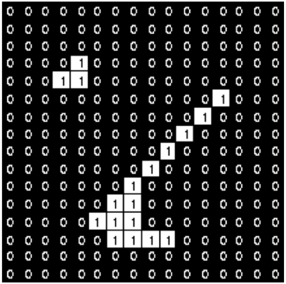
Internal gradient $b = f - (f \ominus s)$ of each region:

by subtracting the eroded image from an original image.

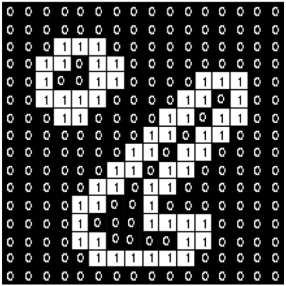
Gradient is computed with the 3×3 square SE: $\boxed{\begin{array}{|c|c|c|}\hline 1&1&1\\\hline 1&1&1\\\hline 1&1&1\\\hline\end{array}}$



Binary image f



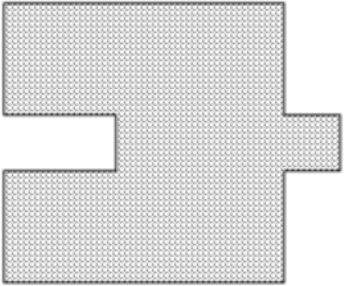
Eroded image $f \ominus s$



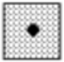
Boundary $b = f - (f \ominus s)$

Boundary Detection by Erosion

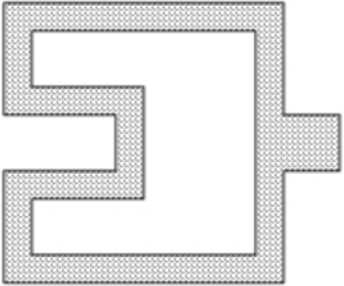
Internal gradient of each region:



f



s



$f - (f \ominus s)$

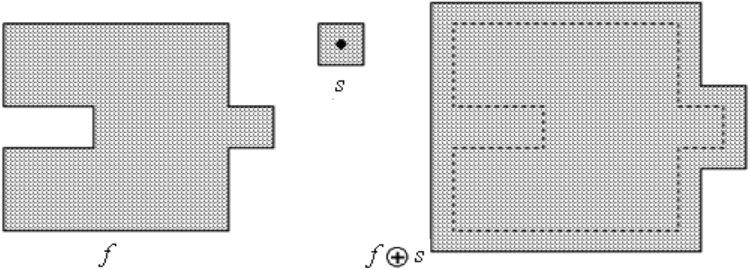
http://www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Fundamental Operation: Dilation

Dilation $f \oplus s$ of a binary image f by an SE s produces a new binary image $g = f \oplus s$.

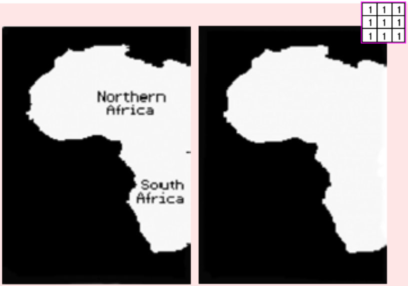
Dilated image g has ones in all locations (x, y) of an origin of the SE s at which s hits the input image f.

For all pixel coordinates (x, y), $g(x, y) = 1$ if s hits f and 0 otherwise.

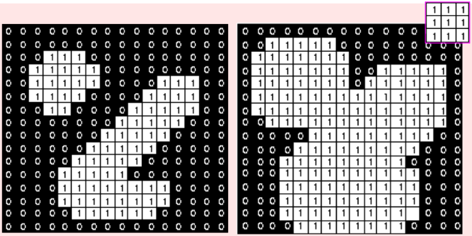


http://www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Dilation of a Binary Image with 2×2 and 3×3 SE



Binary image Dilated image



- Dilation is opposite to erosion: it adds a layer of pixels to the inner and outer boundaries of regions.
- Holes enclosed by a single region and gaps between different regions become smaller; small intrusions into boundaries of a region are filled in.

www.cs.princeton.edu/ pshilane/mosaic/
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html
//documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Dilation and Erosion: Duality

Dilation and erosion are **dual operations** in that they have opposite effects: $f \oplus s = f^c \ominus s_{\mathrm{rot}}$.

- f^c – the complement of f produced by replacing "1"s / "0"s with "0"s / "1"s.
- s_{rot} – the SE s rotated by $180°$.
    - If a SE is symmetric with respect to rotation, then s_{rot} does not differ from s.

If a binary image is considered as a collection of connected regions of "1"s on background of "0"s:

- **Erosion** is the fitting of the SE to these regions.
- **Dilation** is the fitting of the SE, rotated if necessary, into the background, followed by inversion of the result.

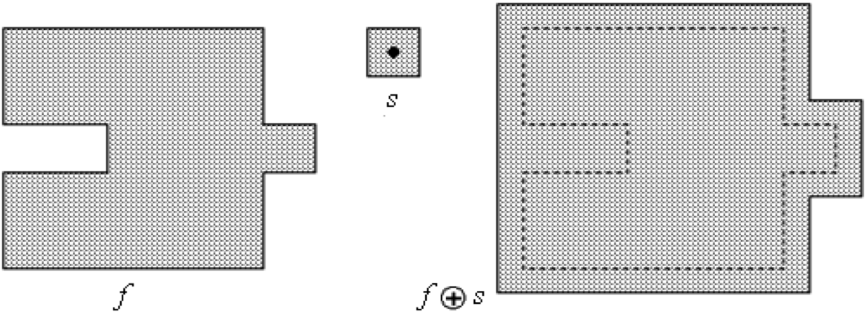
Boundary Detection by Dilation

External gradient $b = (f \oplus s) - f$ of each region:

by subtracting an original image from the dilated image.

Gradient is computed with the 3×3 square SE:

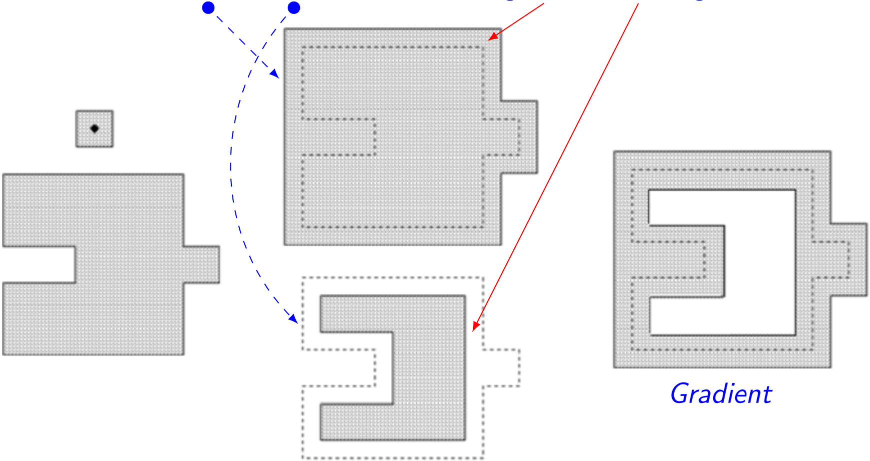
1	1	1
1	1	1
1	1	1



http://www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Morphological Gradient = Dilation − Erosion

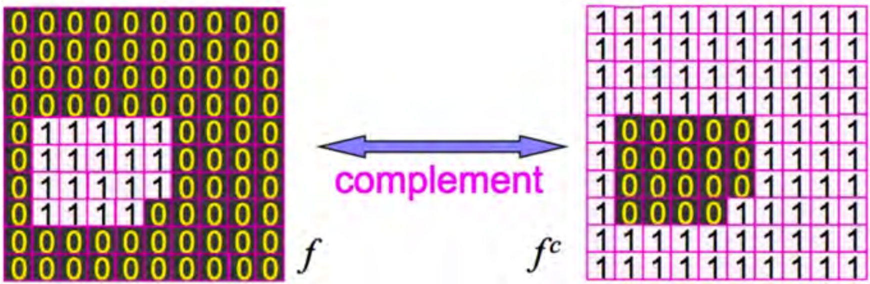
$$\nabla f = \underbrace{(f \oplus s)}_{\text{Dilation}} - \underbrace{(f \ominus s)}_{\text{Erosion}} \equiv \underbrace{[(f \oplus s) - f]}_{\text{External gradient}} + \underbrace{[f - (f \ominus s)]}_{\text{Internal gradient}}$$



Set-theoretic Binary Operations

- Many morphological operations are combinations of erosion, dilation, and simple set-theoretic operations.
- Set-theoretic **complement** of a binary image:

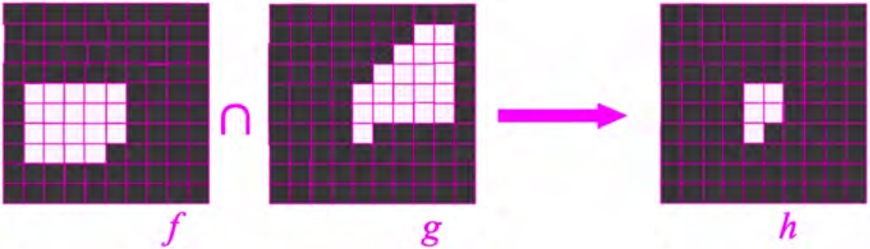
$$f^{c}(x,y) = \begin{cases} 1 & \text{if} \quad f(x,y) = 0 \\ 0 & \text{if} \quad f(x,y) = 1 \end{cases}$$



Set-theoretic Binary Operations: Intersection

Intersection $h = f \bigcap g$ of two binary images f and g:

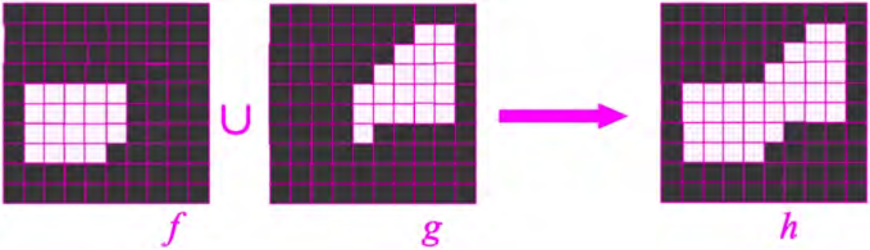
$$h(x,y) = \begin{cases} 1 & \text{if} \quad f(x,y) = 1 \ \ \mathbf{AND} \ \ g(x,y) = 1 \\ 0 & \text{otherwise} \end{cases}$$



Set-theoretic Binary Operations: Union

Union $h = f \bigcup g$ of two binary images f and g:

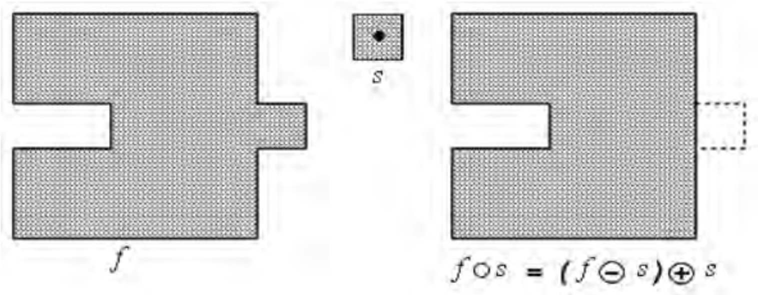
$$h(x,y) = \begin{cases} 1 & \text{if} \quad f(x,y) = 1 \ \ \mathbf{OR} \ \ g(x,y) = 1 \\ 0 & \text{otherwise} \end{cases}$$



Opening

Opening $f \circ s$ of an image f by a structuring element s is an **erosion** followed by a **dilation**:

$$f \circ s = (f \ominus s) \oplus s$$

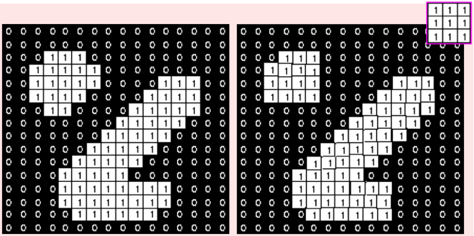


http://www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Opening of a Binary Image with Square 2×2 and 3×3 SE

Binary image

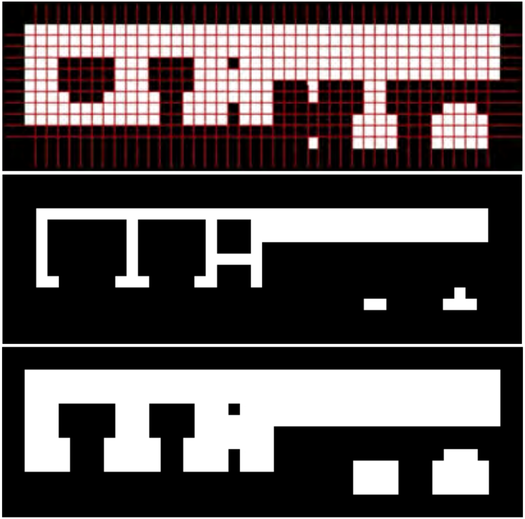
Opened image



Opening is so called because it can open up a gap between objects connected by a thin bridge of pixels.

www.cs.princeton.edu/ pshilane/mosaic/
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html
//documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Opening a Binary Image with a Square 3×3 SE



Initial image f:
white "1"s and black "0"s

SE s 

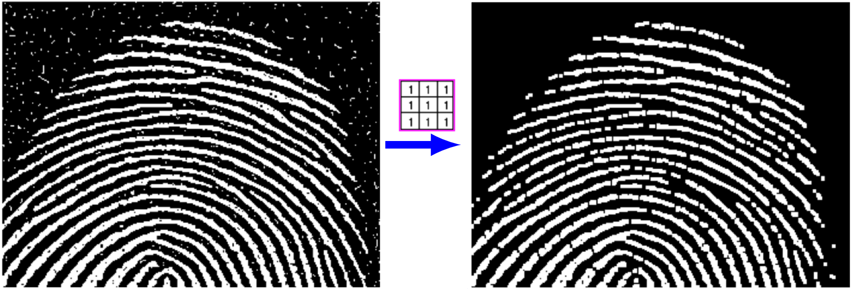
Erosion $f \ominus s$

Opening $f \circ s = (f \ominus s) \oplus s$

www.cs.ru.nl/ ths/rt2/col/h11/11morphENG.html

Opening a Binary Image

Any regions survived the erosion are restored to their original size by the dilation:

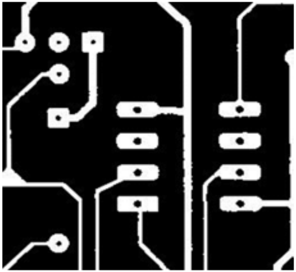


http://www.cs.ru.nl/∼ths/rt2/col/h11/11morphENG.html

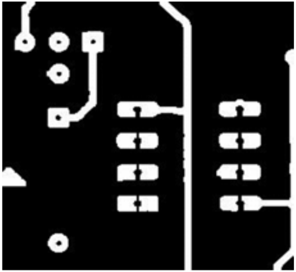
Idempotent operation: $(f \circ s) \circ s = f \circ s$

- Once an image is opened, next openings with the same structuring element have no further effect.

Opening a Binary Image with Square 5×5 and 9×9 SE



Binary image



Opening with a 5×5 SE

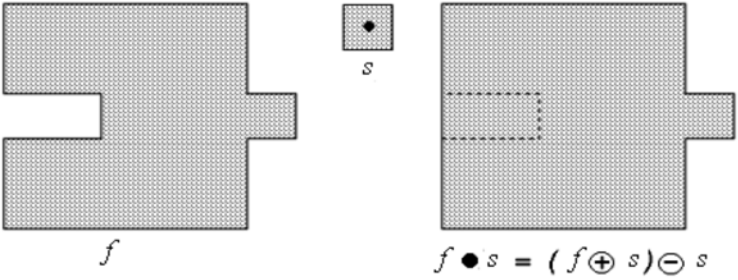
Opening with a 9×9 SE

www.mmorph.com/html/morph/mmopen.html/

Closing

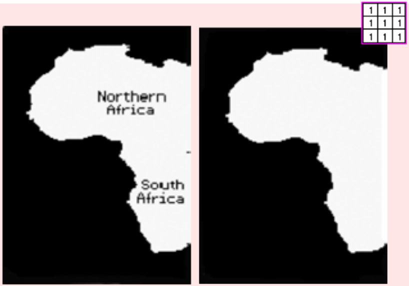
Closing $f \bullet s$ of an image f by a structuring element s is a **dilation** followed by an **erosion**:

$$f \bullet s = (f \oplus s) \ominus s$$

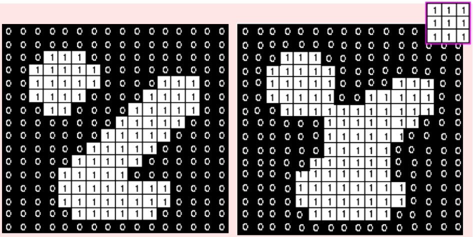


http://www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Closing of a Binary Image with Square 2×2 and 3×3 SE



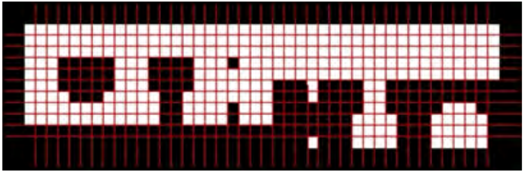
Binary image Closed image



Closing is so called because it can fill holes in the regions while keeping the initial region sizes.

www.cs.princeton.edu/ pshilane/mosaic/
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html
//documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Closing a Binary Image with a Square 3×3 SE



Initial image f:
white “1”s and black “0”s

SE s 

Dilation $f \oplus s$

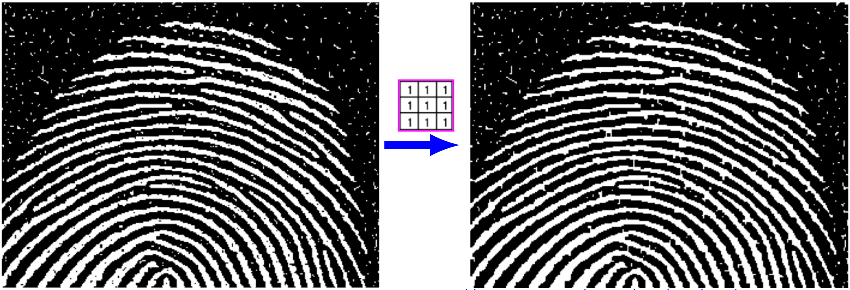
Closing $f \circ s = (f \oplus s) \ominus s$

www.cs.ru.nl/ ths/rt2/col/h11/11morphENG.html

Closing a Binary Image

Dilation and erosion with a rotated by 180° SE.

- Typical symmetric SE: the rotated and initial SE do not differ.



http://www.cs.ru.nl/∼ths/rt2/col/h11/11morphENG.html

Idempotent operation: $(f \bullet s) \bullet s = f \bullet s$

- Once an image is closed, next closings with the same structuring element have no further effect.

Closing Vs. Opening

Closing is the **dual operation** of opening.

- Just as opening is the dual operation of closing.

Closing of a binary image (*dual implementation*):

- Take the **complement** of that image ("1 / 0" $\Rightarrow$ "0 / 1").
- Perform **opening** with the structuring element.
- Take the **complement** of the result.

Opening of a binary image (*dual implementation*):

- Take the **complement** of that image ("1 / 0" $\Rightarrow$ "0 / 1").
- Perform **closing** with the structuring element.
- Take the **complement** of the result.

Closing Vs. Opening

Closing with a square or disk SE:

- Fills thin connections within an object.
- Eliminates small holes and fills dents in contours.
- Fills small gaps in parts of an object.

Opening with a square or disk SE:

- Breaks thin connections within an object.
- Eliminates small islands and sharp protrusions.

Binary Image: Closing + Opening

Initial image f

Closed, then opened image

$$g = (f \bullet s) \circ s$$

http://www.cs.ru.nl/∼ths/rt2/col/h11/11morphENG.html

Binary Image: Opening + Closing

Initial image f

Opened, then closed image
$g = (f \circ s) \bullet s$

http://www.cs.ru.nl/~ths/rt2/col/h11/11morphENG.html

Opening+Closing vs. Closing+Opening

$$\underbrace{\text{erode} - \text{dilate}}_{\text{open}} - \underbrace{\text{dilate} - \text{erode}}_{\text{close}}$$

$$\overbrace{\text{dilate} - \text{erode}}^{\text{close}} - \overbrace{\text{erode} - \text{dilate}}^{\text{open}}$$

Hit-and-Miss Transform

The hit-and-miss transform tests how objects in a binary image relate to their surroundings.

Matched pair of SEs, $\{s_1, s_2\}$, probes the inside and outside, respectively, of objects in the image:

$$f \otimes \{s_1, s_2\} = (f \ominus s_1) \bigcap (f^c \oplus s_2)$$

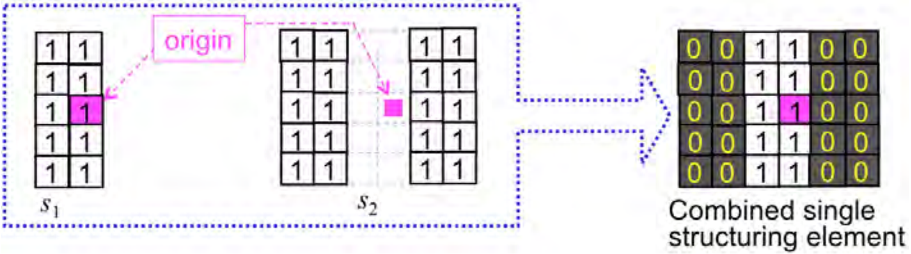
Hit and miss transform with an elongated 2×5 prototype formed by the two SEs.

http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Hit-and-Miss Transform

Preserves a pixel of a region if and only if s_1 for that pixel fits inside the region AND s_2 for that pixel fits outside the region.

- Structuring elements s_1 and s_2 do not intersect, otherwise it would be impossible for both fits to occur simultaneously.



Hit-and-Miss Transform

Easier description: by considering s_1 and s_2 as a single element with "1"s for pixels of s_1 and "0"s for pixels of s_2:

0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0

- An output pixel is set to 1 only if the 1s and 0s in the SE exactly match the 1s and 0s, respectively, in the input image.
- Otherwise, that pixel is set to 0.

If the two SEs together present a specific shape (spatial pattern of foreground 1s and background 0s), the transform can detects the desired shapes.

- The transform can be used for thinning or thickening of linear elements of objects.

Morphological Filtering

Compound operations (e.g. opening and closing) act as non-linear filters of shape in a binary image.

- Opening and closing with a disc SE smooth corners from the inside and the outside, respectively.
- Details smaller in size than the disc are also filtered out.
  - Opening is filtering at a scale of the size of the SE.
  - Only those portions of the image that fit the SE are passed by the filter.
  - Smaller structures are blocked and excluded.

The size of the SE is most important in order to eliminate noisy details but not to damage objects.

- If the SE is too large, the object could be degraded by the operation.

Greyscale Morphology

Definitions of operations:

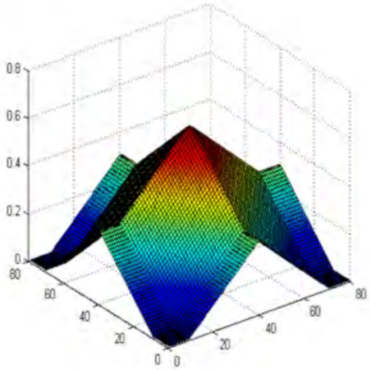
Similar to the binary case but with the additional dimension of pixel grey level.

An image is considered as a 3D surface $f(x, y)$.

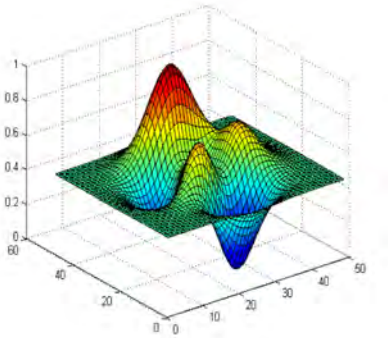
- The height at any point represents the non-negative integer grey level f at that point.
- The structuring element $s(\xi, \eta)$ is also 3D surface, but its pixels take any integer value s, including zero and negative values.

Greyscale Morphology

$$f(x, y)$$



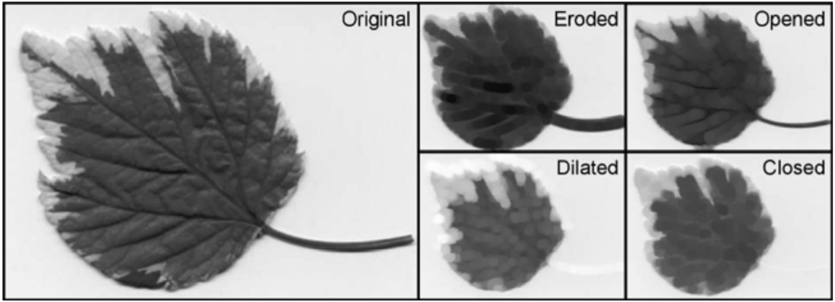
$$s(\xi, \eta)$$



http://www.cs.wits.ac.za/ michael/Lab5.html

Greyscale Morphology

- The SE is sometimes referred in the greyscale morphology to as a **structuring function**.
- Zero value is now significant: pixels that do not participate in morphological operations have to be indicated by some other means.



http://rsb.info.nih.gov/ij/plugins/images/gray-morphology.jpg

Greyscale Erosion

Greyscale erosion $f \ominus s$ of an image f by the SE s:

Replacing the grey level in the pixel aligned with the origin of s with the minimum difference between each pixel grey level in f and the corresponding value in s over the domain $\mathbb{S}_s$ of the SE:

$$f_{\mathrm{ero}}(x,y) = \min_{(\xi,\eta)\in\mathbb{S}_s} \{ f(x+\xi, y+\eta) - s(\xi,\eta) \}$$

220	210	120	45	50
225	200	130	67	53
202	199	100	73	45
189	190	110	68	49
190	200	134	71	57

$\ominus$

−1	−9	−1
11	11	11
−1	−9	−1

$\Rightarrow$

	101	56	42	
	89	62	34	
	101	57	38	

Greyscale Erosion

Greyscale erosion $f_{\text{ero}} = f \ominus s$ of an image f by the SE s:

$$f_{\text{ero}}(x,y) = \min_{(\xi,\eta)\in\mathbb{S}_s} \{f(x+\xi, y+\eta) - s(\xi,\eta)\}$$

where (ξ,η) — 2D indices of a pixel in the SE s, i.e. pixel coordinates with respect to the origin of the SE s.

- For a $(2k+1)\times(2k+1)$ square SE s, the indices $\xi,\eta \in [-k,-k+1,\ldots,0,1,\ldots,k]$.

220	210	120	45	50
225	200	130	67	53
202	199	100	73	45
189	190	110	68	49
190	200	134	71	57

−1	−9	−1
11	11	11
−1	−9	−1

$$\min\left\{\begin{matrix} 220+\ \ 1, 210+\ \ 9, 120+\ \ 1, \\ 225-11, 200-11, 130-11, \\ 202+\ \ 1, 199+\ \ 9, 100+\ \ 1 \end{matrix}\right\}$$

$$= \min\{221, 219, 121, 214, 189, 119, 203, 208, 101\} = \mathbf{101}$$

Greyscale Erosion

Flat structuring element: $s(\xi,\eta) = 0$ for all $(\xi,\eta) \in \mathbb{S}_s$.

- Erosion with the flat SE: $f_{\text{ero}}(x,y) = \min\limits_{(\xi,\eta)\in\mathbb{S}_s} \{f(x+\xi, y+\eta)\}$
- The same effect as the **minimum rank**, or **minimum filter**:

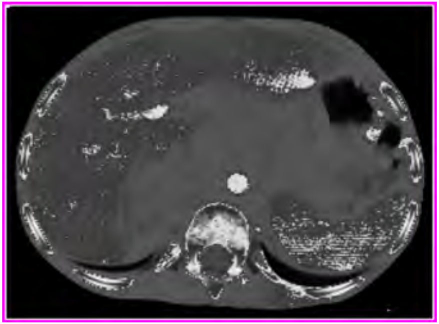
Greyscale image → Eroded image (3×3 minimum filter)

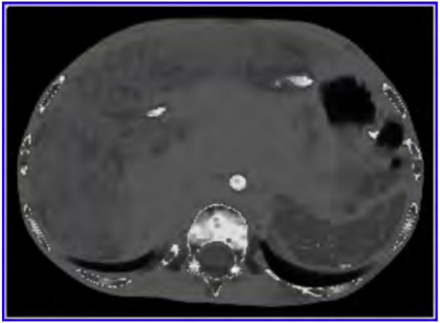
http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Flat SE: The Minimum Filter

Selects the minimum (bottom-ranked) grey level from the neighbourhood as the output value.

Greyscale image → Eroded image (3×3 minimum filter)





http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Greyscale Dilation

Greyscale dilation $f \oplus s$ of an image f by the SE s:

Replacing the grey level in the pixel aligned with the origin of s with the maximum sum of each pair of the pixel grey level in f and the corresponding value in s over the domain $\mathbb{S}_s$ of the SE:

$$f_{\text{dil}}(x,y) = \max_{(\xi,\eta)\in\mathbb{S}_s} \{f(x-\xi, y-\eta) + s(\xi,\eta)\}$$

220	210	120	45	50
225	200	130	67	53
202	199	100	73	45
189	190	110	68	49
190	200	134	71	57

$\oplus$

−1	−9	−1
11	11	11
−1	−9	−1

$\Rightarrow$

	236	211	141	
	224	210	111	
	201	201	133	

Greyscale Dilation

Greyscale dilation $f_{\text{dil}} = f \oplus s$ of an image f by the SE s:

$$f_{\text{dil}}(x,y) = \max_{(\xi,\eta)\in\mathbb{S}_s} \{f(x-\xi, y-\eta) + s(\xi,\eta)\}$$

where (ξ, η) — 2D indices of a pixel in the SE s.

- Greyscale dilation is a dual operation with respect to erosion.

220	210	120	45	50
225	200	130	67	53
202	199	100	73	45
189	190	110	68	49
190	200	134	71	57

−1	−9	−1
11	11	11
−1	−9	−1

$$\max\left\{\begin{matrix} 220-\ 1, 210-\ 9, 120-\ 1, \\ 225+11, 200+11, 130+11, \\ 202-\ 1, 199-\ 9, 100-\ 1 \end{matrix}\right\}$$

$$= \max\{219, 201, 119, 236, 211, 141, 201, 190, 99\} = \mathbf{236}$$

Greyscale Dilation

Flat structuring element: $s(\xi, \eta) = 0$ for all $(\xi, \eta) \in \mathbb{S}_s$.

- Dilation with the flat SE: $f_{\text{dil}}(x, y) = \max\limits_{(\xi,\eta)\in\mathbb{S}_s} \{f(x - \xi, y - \eta)\}$
- The same effect as the **maximum rank**, or **maximum filter**:

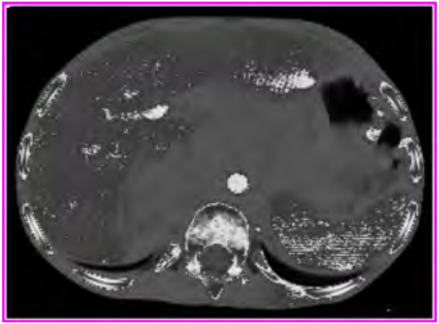
Greyscale image → Dilated image (3×3 maximum filter)

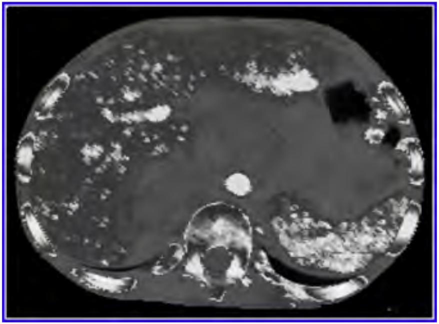
http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Flat SE: The Maximum Filter

Selects the maximum (top-ranked) grey level from the neighbourhood as the output value.

Greyscale image → Dilated image (3×3 maximum filter)





http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Greyscale Opening / Closing

Opening and **closing** for greyscale images are defined just as those for binary images:

Opening $f \circ s = (f \ominus s) \oplus s$

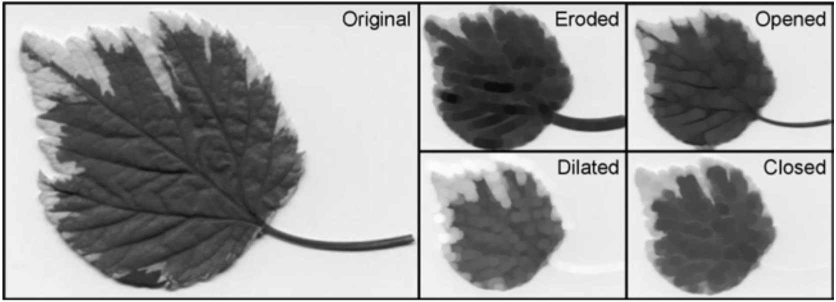
Closing $f \bullet s = (f \oplus s) \ominus s$

With an appropriate structuring element, these operations can smooth the image.

- However, it is the surface of grey levels that is smoothed, rather than the contours of shapes in a binary image.

Greyscale Opening / Closing

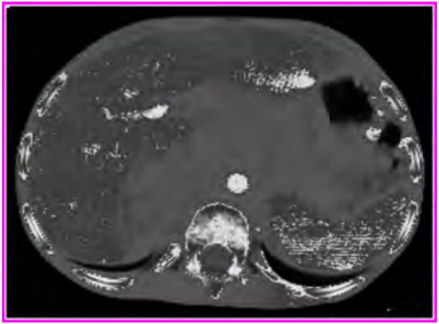
- Opening tends to smooth away small-scale **bright** details in an image.
- Closing tends to smooth away the small-scale **dark** details in an image.

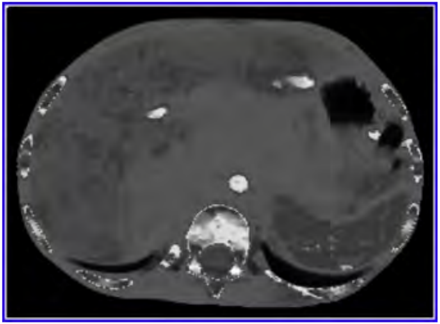


http://rsb.info.nih.gov/ij/plugins/images/gray-morphology.jpg

Greyscale Opening

Greyscale image → Opened image (3×3 square flat filter)





Smoothing out bright small-scale noisy details of the initial image.

http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Erosion - Opening / Dilation - Closing

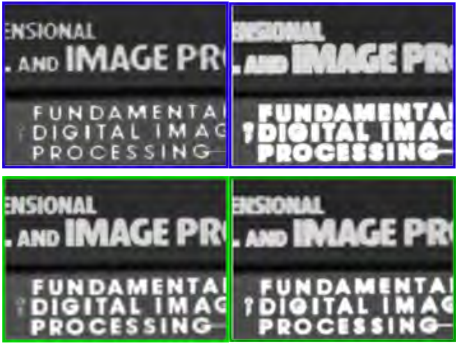
The pairs differ like the corresponding binary operations.

- **Erosion** shrinks bright features and enlarges dark features.
- **Opening** removes small bright features, but does not enlarge dark ones.
- Similar considerations apply to **dilation** and **closing**.

Greyscale image

3×3 square flat SE

Erosion / Opening Dilation / Closing



http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Smoothing / Top-hat Transform

Morphological smoothing: an opening-closing iteration.

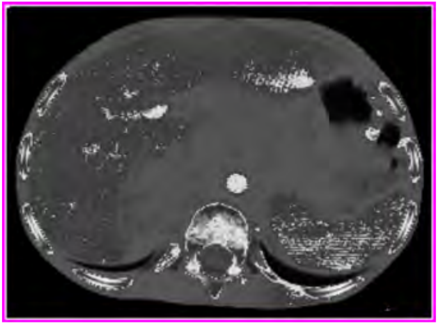
- Removes small-scale bright and dark details.
- Resembles extreme forms of the median filter.

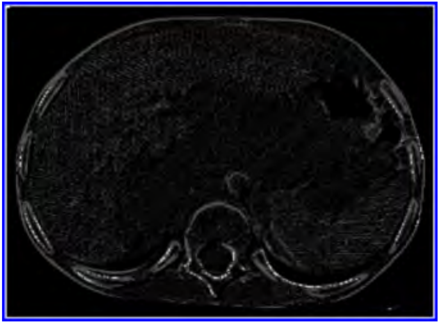
Top-hat transform: $g = f - (f \circ s)$.

- Opening $f \circ s$ removes small-scale bright details.
- Top-hat transform selects only these details.
- Therefore the top-hat transform acts as a detector of peaks and ridges of the grey level surface.

Top-hat Transform

Greyscale image → Top-hat transform (3×3 square flat SE)



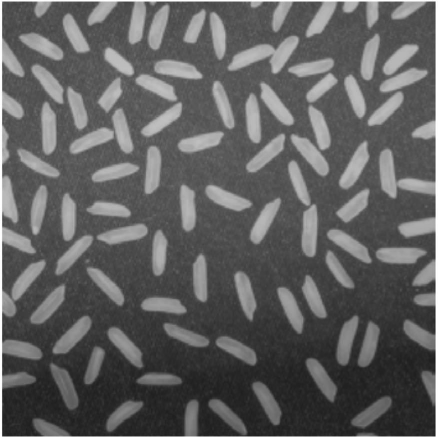


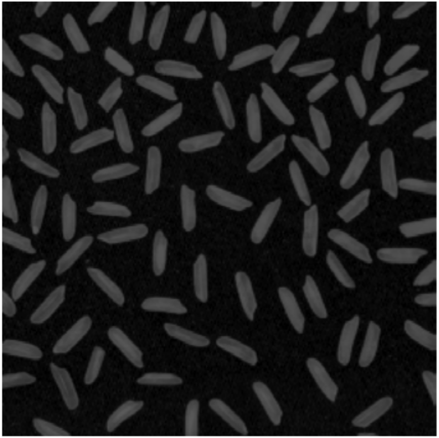
The dual of the top-hat transform, $(f \bullet s) - f$, detects pits and valleys in the grey level surface.

www.inf.u-szeged.hu/ ssip/1996/morpho/morphology.html

Top-hat Transform

Correcting uneven illumination if the background is dark with a 12 pixel wide disk-shaped SE:





http://www.mathworks.fr/help/toolbox/images/ref/imtophat.html