THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2017 — MID-SEMESTER TEST
Version 1
Campus: City

COMPUTER SCIENCE

Computational Science

(Time allowed: 50 minutes)

NOTE: No calculators allowed

Each correct answer gains 1 mark

Each question has exactly 1 correct answer

There are 25 questions in total

1. What is the smallest value of $x \geq 0$ for which the condition number of evaluating the function $f(x) = e^{2x}$ is greater than or equal to 1000?

   (a) 500
   (b) 1000
   (c) e^{2000}
   (d) $\frac{1}{2}\log(1000)$

2. What are the values of a and b after one iteration of the bisection method where the function is $f(x) = -x^3 + 6x - 2$ and the initial values are $a = -1$ and $b = 1$?

   (a) $a = -1$ and $b = 1$
   (b) $a = -1$ and $b = 0$
   (c) $a = -2$ and $b = 1$
   (d) $a = 0$ and $b = 1$

3. An advantage of the bisection method over Newton's method for finding the roots of a function is:

   (a) Bisection is more accurate.
   (b) Bisection converges faster.
   (c) Bisection does not require a derivative.
   (d) Bisection has two starting points.

4. Gaussian elimination reduces a square matrix to the product of

   (a) two orthogonal matrices.
   (b) two orthogonal matrices and a diagonal matrix.
   (c) two triangular matrices.
   (d) and orthogonal and a triangular matrix.

5. Let $\mathbf{A}$ be an 8×6 matrix of floating point numbers. In general, the approximation based on the SVD of $\mathbf{A}$, $\widehat{\mathbf{A}_2}$ requires how many floating point numbers to store?

   (a) 2×48
   (b) 2×14
   (c) 2×49
   (d) 2×15

6. Suppose that the singular values of matrix $\mathbf{A}$ are 56, 14, 10, 6 and 2. What is the condition number of A?

   (a) 88
   (b) 112
   (c) 56
   (d) 28

7. Let the first column of matrix $\mathbf{A}$ be $A_1 = [-1\ 2\ 3]^T$ and the first principal component of $\mathbf{A}$ be $\mathbf{u} = \frac{1}{\sqrt{6}}\,[1\,1\,2]^T$. When we project $\mathbf{A}$ into the principal component space, what is the position of A_1 along the first principal component?

   (a) 1

   (b) $\frac{1}{\sqrt{6}}\,[-1\ 2\ 6]$

   (c) $\frac{1}{\sqrt{14}}$

   (d) $\frac{7}{\sqrt{6}}$

8. The system of linear equations $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ can be solved according to the least squares criterion by solving

   (a) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\mathbf{u} = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\mathbf{u} = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$

9. In the QR-decomposition of $\mathbf{A}$, $\mathbf{R}$ is found by

   (a) the Gram-Schimdt process

   (b) $\mathbf{R} = \mathbf{AQ}$

   (c) $\mathbf{R} = \mathbf{Q^T A}$

   (d) $\mathbf{R} = \mathbf{QA}$

10. Suppose that the columns of a matrix $\mathbf{A}$ are mutually orthogonal but not necessarily normalised. Then the strongest statement we can make about $\mathbf{A^T A}$ is that it is

   (a) lower triangular

   (b) upper triangular

   (c) diagonal

   (d) the identity

11. Numerical instability may arise in forming the pseudo-inverse of $\mathbf{A}$, $\mathbf{A}^+$, due to
    (a) dividing by very small numbers.
    (b) setting $\mathbf{A}^+ = \mathbf{V}\mathbf{D}^+\mathbf{U}^\mathsf{T}$.
    (c) multiplying by very large numbers.
    (d) finding the SVD.

12. The end product of the process of translation of RNA molecules is
    (a) more RNA.
    (b) proteins.
    (c) DNA.
    (d) codons.

13. Suppose we want to sample the values $\{A, B, C, D\}$ with probabilities $\{0.15, 0.20, 0.35, 0.30\}$ respectively using the method discussed in lectures for sampling from a finite discrete set. If the sampled uniform random variate is $u = 0.7843$, which value is drawn?
    (a) A
    (b) B
    (c) C
    (d) D

14. If $\mathbf{A}$ is upper triangular, the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ is easy to solve because:
    (a) $\mathbf{A}^+ = \mathbf{A}^{-1}$ so $\mathbf{x} = \mathbf{A}^+\mathbf{b}$.
    (b) the singular value decomposition of $\mathbf{A}$ is easy to compute.
    (c) back-substitution can be used.
    (d) $\mathbf{A}^T = \mathbf{A}^{-1}$ so $\mathbf{x} = \mathbf{A}^T\mathbf{b}$.

15. Let $\mathbf{A}$ be a 3×6 matrix and let the non-zero eigenvalues of $\mathbf{A}^\top\mathbf{A}$ be 4 and 9. What are the singular values of $\mathbf{A}$?
    (a) 2 and 3.
    (b) 3 and 6.
    (c) 4 and 9.
    (d) There is not enough information to say.

16. If A and B are independent events,
    (a) $P(A|B) = P(A, B)P(B)$.
    (b) $P(A, B) = \frac{P(A|B)}{P(B)}$.
    (c) $P(A|B) = P(A)P(B)$.
    (d) $P(A, B) = P(A)P(B)$.

17. Let $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}$ be the transition matrix of a Markov chain X_i which takes states 1 and 2 corresponding to the row and column indices. What is $\Pr(X_2 = 1 | X_0 = 2)$?
    (a) 0.7
    (b) 0.6
    (c) 0.3
    (d) 0.36

18. In a Poisson process with arrivals at rate 5, what is the distribution and mean of the times between arrivals?
    (a) Poisson with mean 5
    (b) Poisson with mean 0.2
    (c) Exponential with mean 0.2
    (d) Exponential with mean 5

19. Suppose 80% of houses have a balcony, 60% of houses have a garden, and 40% of houses have a balcony and a garden. What is the probability that a house has a garden given that it has a balcony?
    (a) 0.4
    (b) 0.5
    (c) 0.667
    (d) 0.75

20. Suppose we make 3 independent observations $D = \{x_1, x_2, x_3\}$ of an exponentially distributed random variable. Given that the the exponential distribution has density $f(x) = \lambda \exp(-\lambda x)$, what is the correct expression for the likelihood $L(\lambda; D)$ and which equation do we need to solve to find the maximum likelihood estimate?
    (a) $L(\lambda; 0) = \sum_{i=1}^{3} \lambda \exp(-\lambda x_i)$, solve $\frac{dL}{d\lambda} = 0$
    (b) $L(\lambda; D) = \prod_{i=1}^{3} \lambda \exp(-\lambda x_i)$, solve $L = 0$
    (c) $L(\lambda; D) = \prod_{i=1}^{3} \lambda \exp(-\lambda x_i)$, solve $\frac{dL}{d\lambda} = 0$
    (d) $L(\lambda; D) = \sum_{i=1}^{3} \lambda \exp(-\lambda x_i)$, solve $\log(L) = 0$

21. Suppose you draw Bernoulli random variables with parameter p until the first success. Let Y be the number of Bernoulli random variables drawn. What is the distribution of Y?
    (a) Binomial with parameters n and p.
    (b) Geometric with parameter p.
    (c) Bernoulli with parameter np.
    (d) Poisson with parameter np.

22. Suppose we observe three Poisson processes over 1.5 time units. The rates of the three processes are 1, 2, and 4, respectively. What is the expected number of events we observe?
    (a) 7
    (b) 1.5
    (c) 3
    (d) 10.5

23. Two traits or sequences are said to be homologs if they

    (a) share a common ancestry
    (b) share a common parent
    (c) share a common appearance
    (d) share a common function

24. Consider a statistical model with a single parameter θ for data D. For a fixed value of θ, the value of the prior is 2, the value of the likelihood is 0.02 and $P(D) = 0.1$. What is the value of the posterior $P(\theta|D)$ at that fixed value of θ?

    (a) 0.04
    (b) 0.1
    (c) 0.01
    (d) 0.4

25. If matches are scored 3, mismatches are scored -1 and there is a linear gap penalty of -2, what is the score of the alignment below?

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ACTG-ATT
A--GGTTT
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    (a) 1
    (b) 3
    (c) 5
    (d) 7

26. **This question is for information only, it is not marked. Feel free not to answer.**
    What mark do you predict you will get on this test?

    (a) 0-40%
    (b) 41-60%
    (c) 61-80%
    (d) 81-100%

27. **This question is for information only, it is not marked. Feel free not to answer.**
    Overall, do you think this course is

    (a) way too easy
    (b) a bit too easy
    (c) about right
    (d) a bit too hard
    (e) way too hard