COMPSCI 320SC 2024 Midterm Test

Put the answers in the space below the questions. Write clearly and *show all your work*!

Marks for each question are shown below and just before each answer area.

This 60 minute test is worth 10% of your final grade for the course.

Question #:	1	2	3	4	Total
Possible marks:	5	5	5	5	20
Awarded marks:					

1. (a) Write a formal definition for the big-Theta notation, $f(n) = \Theta(g(n))$. ***(2 marks)***

   (b) Let p be a nonnegative function in $\Theta(n)$, let q be a nonnegative function in $O(n^3)$, let r be a nonnegative function in $\Omega(n^2)$, and let $s = pq + r$. In other words, $s(n) = p(n) \cdot q(n) + r(n)$. Give an example of a function that might be s, and of a function that cannot be s. What else can you say about function s? ***(3 marks)***

2. For the following questions, use the following definition of Integer Programming.

Definition: *Let A be an $m \times n$ matrix with integral coefficients and $b \in Z^m$ as input. Assuming column vectors, solving:*

$$\exists x \{x \in \{0, 1, 2, \ldots\}^n \text{ and } Ax = b\}$$

is called an instance of the ***Integer Programming*** *problem.*

(a) With vectors $y = (1, -3, 4)$ and $z = (5, 1, 2)$ determine the dot product $y \cdot z$. ***(1 mark)***

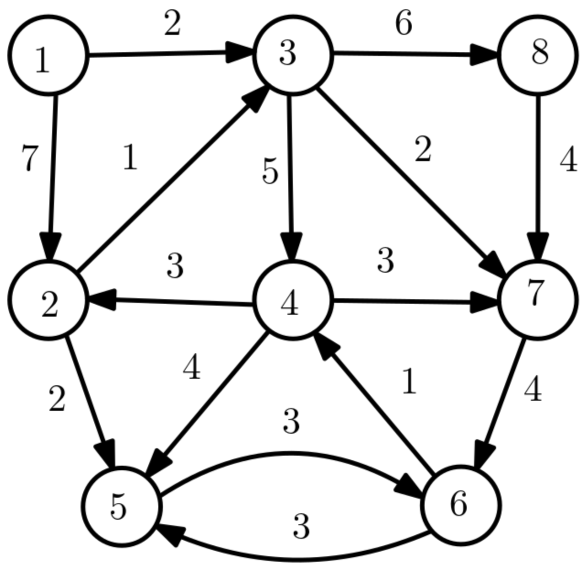
(b) With $M = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$ compute M^4 efficiently. ***(1 mark)***

(c) Compute the matrix times vector product $\begin{bmatrix} 5 & 3 & 0 \\ -1 & 2 & 1 \\ 7 & 0 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$. (***1 mark)***

(d) Let A be an adjacency matrix for a digraph G of order n and b a vector of n non-negative integers as input to an Integer Programming instance. If there is a solution, what do we know about G with respect to b? Briefly explain your answer in terms of vertex labels. (***2 marks)***

3. Apply greedy algorithms for the following two problems that were presented in lectures.

   (a) Compute a minimum weighted arborescence of the following digraph rooted at node 1.



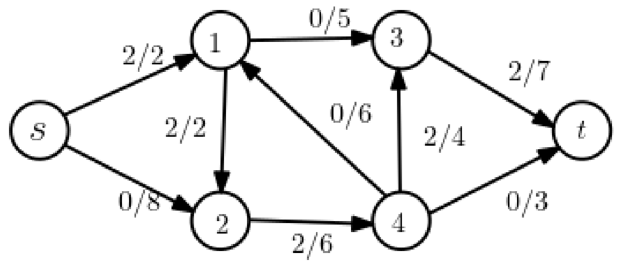
(*3 marks*)

(b) Suppose we have a coded message with six symbols $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta)$ of frequencies 0.40, 0.25, 0.10, 0.10, 0.10, and 0.05, respectively. We want to produce a Huffman code with the least average bits per letter (ABL). Compute an optimal prefix code and its ABL. ***(2 marks)***

(c) How many different optimal prefix codes are there for part (b)? ***(1 bonus marks)***

4. Consider the following $s - t$ network with "partial flow / capacities" listed.

G



(a) Draw the residual digraph. ***(2 marks)***

(b) Find an augmenting path with the largest bottleneck available. ***(1 mark)***

(c) Compute the maximum flow and give a minimum cut as a certificate. (***2 marks)***