THE UNIVERSITY OF AUCKLAND

SEMESTER TWO 2020
Campus: City

COMPUTER SCIENCE

Applied Algorithmics

(Time allowed: Fifty minutes)

NOTE:

1. Attempt *all* questions.
2. This test is worth 10% of your final grade for the course.
3. By completing this assessment, I agree to the following declaration:

   I understand that the University expects all students to complete coursework with integrity and honesty. I promise to complete this online assessment with the same academic integrity standards and values. Any identified form of poor academic practice or academic misconduct will be followed up and may result in disciplinary action.
   As a member of the University's student body, I will complete this assessment in a fair, honest, responsible and trustworthy manner. This means that:

   - I declare that this assessment is my own work.
   - I will not seek out any unauthorized help in completing this assessment.
   - I declare that this work has not been submitted for academic credit in another University of Auckland course, or elsewhere.
   - I am aware that the University of Auckland may use Turnitin or any other plagiarism detecting methods to check my content.
   - I will not discuss the content of this assessment with anyone else in any form, including Canvas, Piazza, Facebook, Twitter or any other social media / online platform within the assessment period.
   - I will not reproduce the content of this assessment anywhere in any form.

1. Provide your fullname, student ID and signature here to indicate that you acknowledge your understanding of the University Academic Integrity policies, as highlighted in the coversheet. [0 marks]

2. Consider an instance of the Stable Matching Problem in which the preferences of the blue nodes B_1, B_2, B_3 are

$$B_1 : P_1 > P_2 > P_3$$
$$B_2 : P_2 > P_1 > P_3$$
$$B_3 : P_1 > P_2 > P_3$$

while the preferences of the pink nodes P_1, P_2, P_3 are

$$P_1 : B_3 > B_2 > B_1$$
$$P_2 : B_2 > B_1 > B_3$$
$$P_3 : B_1 > B_3 > B_2.$$

   (i) Explain why $(B_1, P_1), (B_2, P_2), (B_3, P_3)$ is not a solution. [2 marks]

   (ii) Explain why in any solution, B_2 must be matched with P_2. [2 marks]

   (iii) Show the execution of the Gale-Shapley algorithm (with blue nodes as proposers). List every engagement made and every engagement broken, and the final output. [6 marks]

3. Suppose that we have a new algorithm for multiplication of matrices of dimension n (and hence size $m = n^2$) that works by dividing each of the $n \times n$ matrices x and y into 9 submatrices of as equal size as possible, and computing the product xy by means of 26 multiplications of the submatrices, plus 216 matrix additions and subtractions.

   (i) Write down a recurrence describing the worst-case running time of this algorithm on an instance of size $m = n^2$. [5 marks]

   (ii) Is this algorithm likely to be faster than the standard matrix multiplication algorithm when used on large inputs? Give full explanation. [5 marks]

4. Define a *simple fraction* to be a rational number of the form $1/n$, where n is a positive integer. Consider the problem of writing a positive rational number less than 1 as a sum of *different* simple fractions. For example, $2/3 = 1/2 + 1/6$ is a valid representation, but $2/3 = 1/3 + 1/3$ is not valid.

   (i) State a greedy algorithm for this problem. Show that it always finds a solution to the problem. You must show why the algorithm terminates. [5 marks]

   (ii) Carry out the algorithm on the rational number $3/8$. [2 marks]

   (iii) Show that the greedy algorithm does not always find the optimal solution, if we seek to minimize the number of summands. Hint: consider numbers of the form $(a + 1)/na$. [3 marks]