

COMPSCI 320SC 2018 Midterm Test

Attempt *all* questions. (Use of calculators is NOT permitted.)

Put the answers in the space below the questions. Write clearly and *show all your work!*

Marks for each question are shown below and just before each answer area.

This 50 minute test is worth 10% of your final grade for the course.

Question #:	1	2	3	Total
<i>Possible marks:</i>	10	10	10	30
<i>Awarded marks:</i>				

University ID: _____

Student Name: _____

Student Signature: _____

Time Finished: _____

1. Consider a stable-matching problem with the following preference lists:

R	1	2	3	4	S	1	2	3	4
Ahi	Wini	Xuan	Yasi	Zina	Wini	Cy	Ahi	Bob	Dax
Bob	Xuan	Wini	Yasi	Zina	Xuan	Dax	Bob	Ahi	Cy
Cy	Wini	Xuan	Yasi	Zina	Yasi	Bob	Dax	Cy	Ahi
Dax	Wini	Xuan	Yasi	Zina	Zina	Ahi	Cy	Dax	Bob

Recall that a matching is unstable if there exists any unmatched pair who prefer each other to their current partners. Also recall that the Gale-Shapley algorithm iterates over all currently-unengaged $r \in R$, with each r proposing to their highest-preference $s \in S$ until they find an s who is either unengaged, or who prefers them to their current fiancé f .

- (a) What matching is produced by the Gale-Shapley algorithm for these preferences? Show your working: you will get 0 marks if you exhibit a matching without briefly explaining how you found it. (4 marks)

- (b) Consider the following method **prefer(s,r,f)** of determining whether a given $s \in S$ prefers a potential match $r \in R$ to their current fiancé $f \in R$: scan s 's preference list, returning **True** if r appears earlier in the list than f . What is the worst-case (big-Oh) runtime of the Gale-Shapley algorithm if this implementation of **prefer()** is used, and if there are $n = |R| = |S|$ elements in each set? Explain your reasoning briefly: you will get no marks for writing down a big-Oh expression with no explanation. (3 marks)

- (c) The Gale-Shapley algorithm is unsuitable for use in some applications, because it is biased toward its first set R . Is this bias apparent for the given input? Explain briefly. To receive full marks, you must show a second stable matching for this input which is clearly more favourable to elements in S . **(3 marks)**

Here is another copy of the input:

R	1	2	3	4
Ahi	Wini	Xuan	Yasi	Zina
Bob	Xuan	Wini	Yasi	Zina
Cy	Wini	Xuan	Yasi	Zina
Dax	Wini	Xuan	Yasi	Zina

S	1	2	3	4
Wini	Cy	Ahi	Bob	Dax
Xuan	Dax	Bob	Ahi	Cy
Yasi	Bob	Dax	Cy	Ahi
Zina	Ahi	Cy	Dax	Bob

2. Consider the following divide-and-conquer “algorithm”:

```
function foo(int  $n$ )  
  for  $i = 1$  to  $n$  do  
     $j = 1$   
    while  $j * j < n$  do  
      println “bar”  
       $j = j + 1$   
  if  $n > 0$  then  
    for  $k = 1$  to 7 do  
      foo( $\lfloor n/4 \rfloor$ )
```

Let $T(n)$ denote the number of lines of output generated by a call of foo(n).

(a) What are the values of $T(4)$, $T(12)$ and $T(16)$? (3 marks)

(b) Provide a recurrence equation for $T(n)$. (4 marks)

(c) Solve the recurrence asymptotically for general n .

You may want to make use of the following ‘master recurrence theorem’:

Assume $T(n) = a \cdot T(n/b) + \Theta(n^c)$ is the total time for a divide-and-conquer algorithm then:

$$T(n) \in \begin{cases} \Theta(n^c) & \text{if } a < b^c \\ \Theta(n^c \log n) & \text{if } a = b^c \\ \Theta(n^{\log_b a}) & \text{if } a > b^c \end{cases}$$

(3 marks)

3. (a) Explain Karatsuba's integer multiplication algorithm and indicate why it runs in time $O(n^{\lg 3}) = O(n^{1.585})$, where n is the number of bits of each of the two input integers.
Hint: $bc + ad = ac + bd - (a - b)(c - d)$. **(5 marks)**

- (b) We are given a collection C of $n > 1$ car parts, and also a mechanic expert $\mathcal{E}$ ddy that can decide whether two parts belong to the same brand of car (e.g. Ford, Holden, Nissan, Toyota, ...). Describe a divide-and-conquer algorithm that decides if at least $\lfloor n/2 \rfloor + 1$ of the parts in C are the same brand by asking at most $O(n \lg n)$ questions to $\mathcal{E}$ ddy. You can only make an inquiry to the mechanic with two parts at a time and $\mathcal{E}$ ddy just says 'yes/no' and does not indicate any brand names. **(5 marks)**