THE UNIVERSITY OF AUCKLAND

SEMESTER TWO 2015
Campus: City

COMPUTER SCIENCE

Applied Algorithmics

(Time allowed: Fifty minutes)

NOTE: Attempt *all* questions. Put the answers in the space below the questions. Write clearly! You may continue your answers onto the "overflow" pages provided at the end of the test, if necessary.
Use of calculators is NOT permitted.
This test is worth 10% of your final grade for the course.

SURNAME:

FORENAME(S):

STUDENT ID:

1. Consider an instance of the Stable Matching Problem in which the preferences of the blue nodes B_1, B_2, B_3 are respectively $P_1 > P_2 > P_3, P_2 > P_3 > P_1, P_3 > P_1 > P_2$, while the preferences of the pink nodes P_1, P_2, P_3 are respectively $B_3 > B_2 > B_1, B_2 > B_1 > B_3, B_1 > B_3 > B_2$.

   (i) Show the execution of the Gale-Shapley algorithm (with blue nodes as proposers). List every engagement made and every engagement broken, and the final output. [5 marks]

   (ii) Does there exist a solution to the problem that has each B_i matched with P_i? Explain why or why not. [3 marks]

   (iii) Explain why in any solution, B_2 must be matched with P_2. [2 marks]

2. Suppose that I have a new algorithm for multiplication of matrices that works by dividing each of the $n \times n$ matrices x and y into 3 parts of as equal size as possible, and computing the product xy by means of 23 multiplications of the parts, plus 147 matrix additions and subtractions.

   (i) Write down a recurrence describing the worst-case running time of this algorithm on an instance of size n. [5 marks]

   (ii) Is this algorithm likely to be faster than the standard matrix multiplication algorithm when used on large inputs? Give full explanation. [5 marks]

3. Recall that an *Egyptian fraction* is a rational number of the form $1/n$, where n is a positive integer. Consider the problem of writing a positive rational number less than 1 as a sum of *different* Egyptian fractions. For example, $2/3 = 1/2 + 1/6$ is a valid representation, but $2/3 = 1/3 + 1/3$ is not valid.

   (i) State a greedy algorithm for this problem. Show that it always finds a solution to the problem. You must show why the algorithm terminates. [5 marks]

   (ii) Carry out the algorithm on the rational number $14/15$. [3 marks]

   (iii) Show that the greedy algorithm does not always find the optimal solution, if we seek to minimize the number of summands. Hint: consider numbers of the form $(a + 1)/na$. [2 marks]

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