COMPSCI 320SC 2014 Midterm Test

Attempt *all* questions. (Use of calculators is NOT permitted.)

Put the answers in the space below the questions. Write clearly and *show all your work*!

Marks for each question are shown below and just before each answer area.

This 50 minute test is worth 10% of your final grade for the course.

Question #:	1	2	3	4	Total
Possible marks:	5	5	5	5	20
Awarded marks:					

University ID: _____

Student Name: _____

Student Signature: _____

Time Finished: _____

1. Recall Huffman's greedy algorithm for constructing an optimal prefix code. Consider the behavior of this algorithm on the sample instance with characters $\{a, e, i, o, u\}$ and frequencies $f_a = .32$, $f_e = .25$, $f_i = .20$, $f_o = .18$ and $f_u = .05$.

   (a) Draw the Huffman prefix tree for this instance. (***3 marks***)

   See textbook Figure 4.16(c).

   (b) What is the average number of bits per letter (ABL) for this code? (***2 marks***)

   $(.32 + .25 + .20) \cdot 2 + (.05 + .18) \cdot 3$

2. Consider the following divide-and-conquer "algorithm":

```
function printer(int n)
    if n == 0 return
    for j = 1 to 3 do
        printline "hello world"
        printer(⌊n/3⌋)
```

Let $T(n)$ denote the number of lines of output generated by a call of printer(n).

(a) Provide a recurrence equation for $T(n)$. (***2 marks***)

$$T(n) = \begin{cases} 0 & \text{if } n = 0 \\ 3 \cdot T(\lfloor n/3 \rfloor) + 3 & \end{cases}$$

(b) Solve the recurrence exactly for n being a power of 3 (that is, $n = 3^k$ for integer $k \geq 0$). (***3 marks***)

$$T(3^k) = \sum_{i=1}^{k+1} 3^i = \frac{3^{k+2} - 3}{2}$$

3. Suppose you are given an edge-weighted connected graph G of order n and size $m \leq n + 10$. You may assume all the edge weights are distinct. Note that for *any* cycle C in the graph G we know the largest edge weight of C is not used in *some* minimum spanning tree.

   (a) Give a greedy algorithm that finds a minimum spanning tree in time $O(n)$. (***3 marks***)

   We run BFS on the graph at most 11 times to find a cycle C. Each time we remove the largest edge weight from C from G. Eventually, in a constant number of iterations, we get a spanning tree.

   (b) Explain why the algorithm you gave in part (a) is correct and that its running time is linear. (***2 marks***)

   From the hint we know any largest weighted edge in a cycle is not part of a minimum spanning tree so we can eliminate those. BFS can be run in time $O(n + m)$ and we need to do this at most 11 times.

4. Answer each of the following statments as True or False.

(a) The complete bipartite graph $K_{2,5}$ is planar. (***1 mark***)

True

(b) The worst case and average case complexity of the `Median-of-Medians` algorithm for finding the k-th smallest are the same. (***1 mark***)

True, $\Theta(n)$

(c) The running time of the Gale-Shapley algorithm for finding a stable matching is in $\Omega(n^2)$, where there are n men and women. (***1 mark***)

True

(d) If the Perrin number $P(n)$ is divisible by n then n is a prime number. (***1 mark***)

False

(e) The complex number $e^{\pi i} = -1$. (***1 mark***)

True