Modern Data Communications: Analog and Digital Signals, Compression, Data Integrity

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July-August 2011

C. Calude thanks to

Nevil Brownlee and Ulrich Speidel for stimulating discussions and critical comments.

Goals

- Understand digital and analog signals
- Understand codes and encoding schemes
- Understand compression, its applications and limits
- Understand codes for error detection and correction

References

1. B. A. Forouzan. *Data Communications and Networking*, McGraw Hill, 4th edition, New York, 2007.
2. W. A. Shay. *Understanding Data Communications and Networks*, 3rd edition, Brooks/Cole, Pacific Grove, CA, 2004. **(course textbok)**

Pictures

All pictures included and not explicitly attributed have been taken from the instructor's documents accompanying Forouzan and Shay textbooks.

Factors determining data transmission

- cost of a connection
- amount of information transmitted per unit of time (bit rate)
- immunity to outside interference (noise)
- security (susceptibility to unauthorised "listening", modification, interruption, or channel usage)
- logistics (organising the wiring, power, and other physical requirements of a data connection)
- mobility (moving the station)

Analog and digital signals

Connected devices have to "understand" each other to be able to communicate.

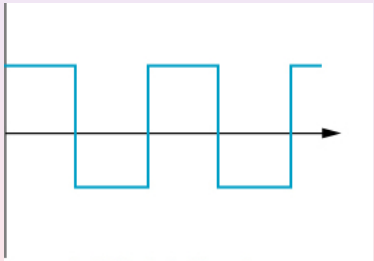
Communication standards assure that communicating devices represent and send information in a "compatible way".

There are two types of ways to transmit data:

- via **digital signals**, which can be represented either electronically (by sequences of specified voltage levels) or optically,
- via **analog signals**, which are formed by continuously varying voltage levels.

Digital signals 1

Digital signals are graphically represented as a square wave: the horizontal axis represents time and the vertical axis represents the voltage level.



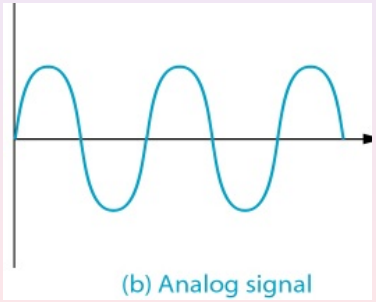
Digital signals 2

The alternating high and low voltage levels may be symbolically represented by 0s and 1s. This is the simplest way to represent a binary string (bit-string).

Each 0 or 1 is called a **bit**. Various codes combine bits to represent information stored in a computer.

Analog signals

PCs often communicate via modems over telephone lines using **analog signals** which are formed by continuously varying voltage levels:



How signals travel?

There are three types of transmission media, each with many variations:

- *conductive metal*, like copper or iron, that carries both digital and analog signals; coaxial cable and twisted wire pairs are examples,
- *transparent glass strand or optical fibre* that transmits data using light waves,
- *no physical connection* that transmits data using electromagnetic waves (as those used in TV or radio broadcast).

How is information coded?

Whether the medium uses light, electricity, or microwaves, we must answer perhaps the most basic of all communications questions:

How is information coded in a format suitable for transmission?

Bits

Regardless of implementation, all switches are in one of two states: open or closed, symbolically, 0 and 1.

Bits can store only two distinct pieces of information. Grouping them, allows for many combinations:

- two bits allow $2^2 = 4$ unique combinations: $00, 01, 10, 11$
- three bits allow for $2^3 = 8$ combinations,
- ten bits allow for $2^{10} = 1,024$ combinations,
- fifty bits allow for $2^{50} = 1,125,899,906,842,624$ combinations,
- n bits allow for 2^n combinations.

From bits to codes

Grouping bits allows one to associate certain combinations with specific items such as characters, numbers, pictures. Loosely speaking, this association is called a **code**. Not every association is a code as we shall soon learn.

A difficult problem in communications is to establish communications between devices that operate with different codes. There are standards, but not all standards are compatible!

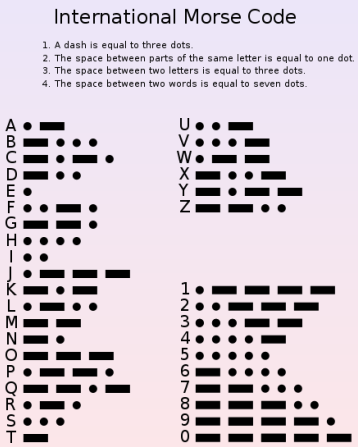
> *The nice thing about standards is that you have so many to choose from.*
> *– Tanenbaum, Computer Networks (2nd Ed.), 1988*

Early codes: Morse 1

Originally created for Morse's electric telegraph in 1838, by the American inventor Samuel Morse, the **Morse code** was also extensively used for early radio communication beginning in the 1890s.

The telegraph required a human operator at each end. The sender would tap out messages in Morse code which would be transmitted down the telegraph wire to a human decoder translating them back into ordinary characters.

Early codes: Morse 2



Early codes: Morse 3

Morse code is a **variable-length code**:

- letter codes have different lengths; the letter E code is a single dot (1000), the letter H code has four dots (1010101000);
- the code (0000000) for an inter-word gap (the 'space' character) is of length 7;

Reason: more frequent letters are assigned shorter codes, so messages can be sent quickly.

Early codes: Baudot code 1

The **Baudot code**—also known as **International Telegraph Alphabet No 2 (ITA2)**—is named after its French inventor Émile Baudot. ITA2 is a fix-length code using 5 bits for each character (digits and letters). This code was developed around 1874.

With 5-bit codes we can name $2^5 = 32$ different objects, but we have 36 letters and digits (plus special characters) to code!

For example, the letter Q and digit 1 have the same code: 10111. In fact each digit's code duplicates that of some letter.

Early codes: Baudot code 2

00	01	02	03	04	05	06	07
NUL	E 3	LF	A –	SP	S '	I 8	U 7
08	09	0A	0B	0C	0D	0E	0F
CR	D ENQ	R 4	J BEL	N ,	F !	C :	K (
10	11	12	13	14	15	16	17
T 5	Z +	L)	W 2	H £	Y 6	P 0	Q 1
18	19	1A	1B	1C	1D	1E	1F
O 9	B ?	G &	FIGS	M .	X /	V ;	LTRS
Letters			Figures			Control Chars.	

Early codes: Baudot code 3

Do you think we have got a problem?
More precisely, how can we tell a digit from a letter?

Answer: using the same principle that allows a keyboard key to represent two different characters. On the keyboard we use the Shift key; the Baudot code uses the extra information

11111 (shift down) and 11011 (shift up)

to determine how to interpret a 5-bit code. Upon receiving a shift down, the receiver decodes all codes as letters till a shift up is received, and so on.

Early codes: Baudot code 4

Here is an example.
ABC123, is coded from left to right as follows:

11111 00011 11001 01110 11011 10111 10011 00001

Early codes: BCD, BCDIC, ASCII codes

- BCD stands for **binary-coded decimal**, a code developed by IBM for its mainframe computers using 6-bit codes;
- BCDIC stands for **binary-coded decimal interchange code**, an expansion of BCD including codes also for non-numeric data;
- ASCII (pronounced ['æski]) stands for the **American Standard Code for Information Interchange**; it is a 7-bit code that assigns a unique combination to every keyboard character and to some special functions.

ASCII code (decimal, binary, hexadecimal) 1

Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL (null)	32	20	040		Space	64	40	100	@	@	96	60	140	`	`
1	1	001	SOH (start of heading)	33	21	041	!	!	65	41	101	A	A	97	61	141	a	a
2	2	002	STX (start of text)	34	22	042	"	"	66	42	102	B	B	98	62	142	b	b
3	3	003	ETX (end of text)	35	23	043	#	#	67	43	103	C	C	99	63	143	c	c
4	4	004	EOT (end of transmission)	36	24	044	$	$	68	44	104	D	D	100	64	144	d	d
5	5	005	ENQ (enquiry)	37	25	045	%	%	69	45	105	E	E	101	65	145	e	e
6	6	006	ACK (acknowledge)	38	26	046	&	&	70	46	106	F	F	102	66	146	f	f
7	7	007	BEL (bell)	39	27	047	'	'	71	47	107	G	G	103	67	147	g	g
8	8	010	BS (backspace)	40	28	050	(	(	72	48	110	H	H	104	68	150	h	h
9	9	011	TAB (horizontal tab)	41	29	051	)	)	73	49	111	I	I	105	69	151	i	i
10	A	012	LF (NL line feed, new line)	42	2A	052	*	*	74	4A	112	J	J	106	6A	152	j	j
11	B	013	VT (vertical tab)	43	2B	053	+	+	75	4B	113	K	K	107	6B	153	k	k
12	C	014	FF (NP form feed, new page)	44	2C	054	,	,	76	4C	114	L	L	108	6C	154	l	l
13	D	015	CR (carriage return)	45	2D	055	-	-	77	4D	115	M	M	109	6D	155	m	m
14	E	016	SO (shift out)	46	2E	056	.	.	78	4E	116	N	N	110	6E	156	n	n
15	F	017	SI (shift in)	47	2F	057	/	/	79	4F	117	O	O	111	6F	157	o	o
16	10	020	DLE (data link escape)	48	30	060	0	0	80	50	120	P	P	112	70	160	p	p
17	11	021	DC1 (device control 1)	49	31	061	1	1	81	51	121	Q	Q	113	71	161	q	q
18	12	022	DC2 (device control 2)	50	32	062	2	2	82	52	122	R	R	114	72	162	r	r
19	13	023	DC3 (device control 3)	51	33	063	3	3	83	53	123	S	S	115	73	163	s	s
20	14	024	DC4 (device control 4)	52	34	064	4	4	84	54	124	T	T	116	74	164	t	t
21	15	025	NAK (negative acknowledge)	53	35	065	5	5	85	55	125	U	U	117	75	165	u	u
22	16	026	SYN (synchronous idle)	54	36	066	6	6	86	56	126	V	V	118	76	166	v	v
23	17	027	ETB (end of trans. block)	55	37	067	7	7	87	57	127	W	W	119	77	167	w	w
24	18	030	CAN (cancel)	56	38	070	8	8	88	58	130	X	X	120	78	170	x	x
25	19	031	EM (end of medium)	57	39	071	9	9	89	59	131	Y	Y	121	79	171	y	y
26	1A	032	SUB (substitute)	58	3A	072	:	:	90	5A	132	Z	Z	122	7A	172	z	z
27	1B	033	ESC (escape)	59	3B	073	;	;	91	5B	133	[	[	123	7B	173	{	{
28	1C	034	FS (file separator)	60	3C	074	<	<	92	5C	134	\	\	124	7C	174			|
29	1D	035	GS (group separator)	61	3D	075	=	=	93	5D	135	]	]	125	7D	175	}	}
30	1E	036	RS (record separator)	62	3E	076	>	>	94	5E	136	^	^	126	7E	176	~	~
31	1F	037	US (unit separator)	63	3F	077	?	?	95	5F	137	_	_	127	7F	177	&#127;	DEL

Source: www.LookupTables.com

ASCII code (decimal, binary, hexadecimal) 2

Each code corresponds to a printable or unprintable character.

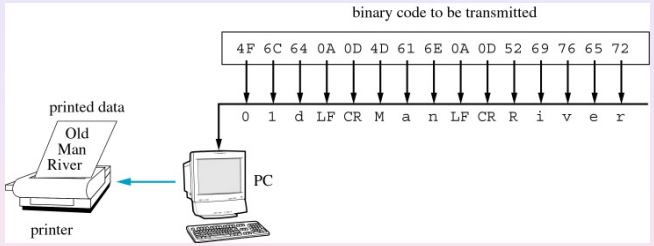
Printable characters include letters, digits, and special punctuation (commas, brackets, question marks).

Unprintable characters are special functions (e.g. line feed, tab, carriage return, BEL, DC1/XON/ctrl-Q, DC3/XOFF/ctrl-S).

Standard ASCII has 128 different characters.

Extended ASCII codes (e.g. ISO-8859-1, Mac OS Roman, ...) have an additional 128 characters.

ASCII code 3



If codes are sent with the leftmost first, as the printer receives each code, it analyses and takes some action: for 4F, 6C and 64 it prints 0, 1, and d. The next two codes, OA and OD, denote unprintable characters (LF = line feed, CR = carriage return). When OA is received, nothing is printed, but the mechanism to advance to the next line is activated.

What is a code? 1

We can now ask the important question:

What is a code?

Here is an example. The character "=" is represented by the binary code word "0111101". Why do we need the leading zero? Surely "111101" means the same thing because both "09" and "9" mean nine?

All ASCII codewords have the same length. This ensures that an important property—called the prefix property—holds true for the ASCII code.

What is a code? 2

A **code** is the assignment of a unique string of characters (a **codeword**) to each character in an alphabet.

A code in which the codewords contain only zeroes and ones is called a **binary code**.

The encoding of a string of characters from an alphabet (the cleartext) is the concatenation of the codewords corresponding to the characters of the cleartext, in order, from left to right. A code is **uniquely decodable** if the encoding of every possible cleartext using that code is unique.

What is a code? 3

For example, here are two possible binary codes for the alphabet $\{a, c, j, l, p, s, v\}$:

	code 1	code 2
a	1	010
c	01	01
j	001	001
l	0001	10
p	00001	0
s	000001	1
v	0000001	101

What is a code? 4

Both code 1 and code 2 satisfy the definition of a code. However,

- code 1 is uniquely decodable, but
- code 2 is not uniquely decodable; for example, the encodings of the cleartexts "pascal" and "java" are both

001010101010

Prefix codes 1

A **prefix code** is a code with the "prefix property":

no codeword is a (proper) prefix of any other codeword in the set.

The code $\{0, 10, 11\}$ has the prefix property; the code $\{0, 1, 10, 11\}$ does not, because "1" is a prefix of both "10" and "11".

Code 1 is a prefix code, but code 2 is not.

Why is the prefix property important?

Because prefix codes are uniquely decodable.

Prefix codes 2

Every fixed-length code is a prefix code.

There can be no prefixes in the code table, because no codeword is any longer or shorter than any other.

Therefore, ASCII is a prefix code.

Prefix codes 3

Given a sequence of lengths, can we construct a prefix code whose codewords have exactly those lengths?

Kraft's theorem. *A prefix code exists for codewords lengths* $l_1, l_2, \ldots, l_N$ *if and only if*

$$2^{-l_1} + 2^{-l_2} + \cdots + 2^{-l_N} \leq 1.$$

Arrange the lengths in increasing order so (after relabelling) $l_1 \leq l_2 \leq \ldots \leq l_N$. Take as the first codeword, $w_1 = 0^{l_1}$, i.e. $00\ldots0$ for l_1 times. If $w_1, w_2, \ldots, w_i$ have been constructed, choose w_{i+1} to be the first string (lexicographically) of length l_{i+1} such that no w_j is a prefix of it. The above inequality guarantees that w_{i+1} always exists.

Prefix codes 4

Here is an example. Consider the lengths $3, 2, 4$ which satisfy the condition in Kraft theorem:

$$2^{-3} + 2^{-2} + 2^{-4} = \frac{7}{16} < 1.$$

We arrange the lengths in increasing order $2, 3, 4$, and we have:
$w_1 =$ **00**
$w_2 =$ first string in the set 000,001,010,011,100, 101, 110,111 such that **00** is not its prefix $=$ **010**
$w_3 =$ first string in the set 0000,0001,0010,0011,0100, 0101, 0110,0111, 1000,1001,1010,1011,1100, 1101, 1110,1111 such that its prefixes do not include both **00** and **010** $=$ **0110**

Prefix codes 5

Kraft's theorem does not assert that any code which satisfies the inequality therein must be a prefix code. For example, the code

$$\{00, 010, 0100\}$$

satisfies the inequality but is not a prefix code.

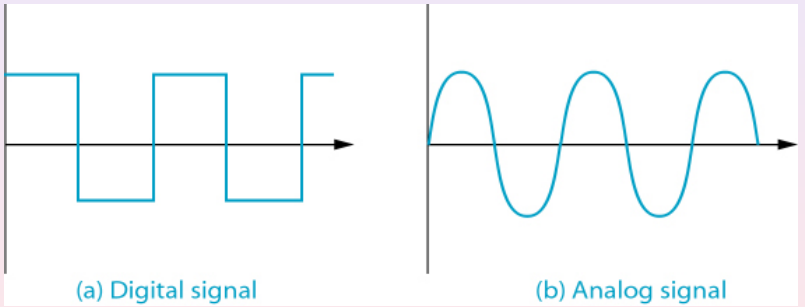
However,

> *Any uniquely decodable code can be replaced by a prefix code without changing any of the lengths of the codewords.*

Questions

1. How does symbolic data relate to electrical signals, microwaves, and light waves?
2. What does a 0 or a 1 actually look like as it travels through a wire, optical fibre, or space?
3. How many bits can a signal transmit per unit of time?
4. What effect does electrical interference (noise) have on data transmission?

Analog vs. digital signals



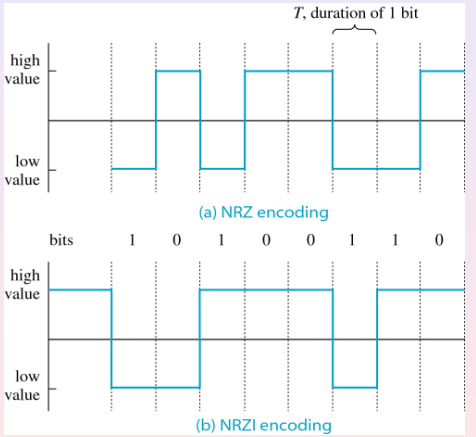
Encoding Bits in Analog Signals 1

Because digital signals can alternate between two constant values—say "high voltage" and "low voltage"—we simply associate 0 with one value and 1 with the other. The actual values are irrelevant.

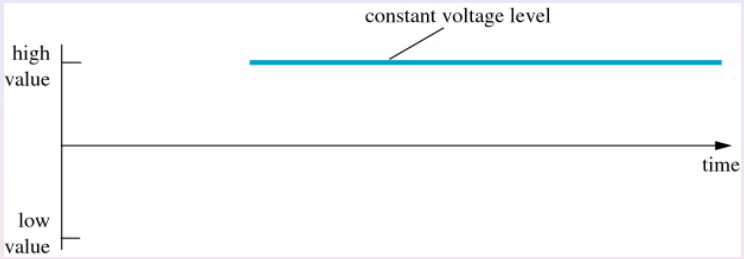
Non-Return to Zero (NRZ): a 0 is transmitted by raising the voltage level to high, and a 1 is transmitted using a low voltage. Alternating between high and low voltage allows for the transmission of any string of 0s and 1s.

Non-Return-to-Zero-Inverted Encoding (NRZI): a 0 is encoded as no change in the level. However a 1 is encoded depending on the current state of the line. If the current state is 0 [low] the 1 will be encoded as a high, if the current state is 1 [high] the 1 will be encoded as a low. Used in USB.

Example: NRZ and NRZI



Synchronisation Problem



What is being transmitted in NRZ? In NRZI? A string of 0s, in either case. But... how many?

NRZ and NRZI require a *clock signal* as well as a *data signal*. Sending the clock signal requires an additional connection with the same latency as the data signal, otherwise the clock will be *skewed* and the data will be decoded incorrectly.

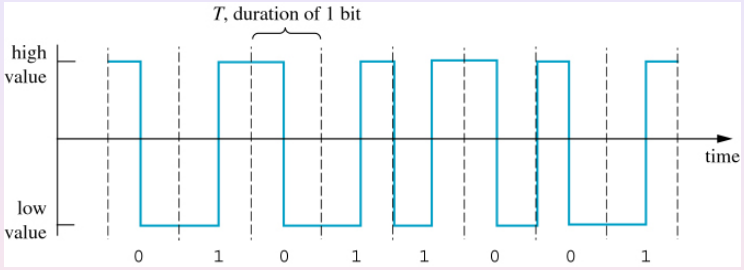
Self-synchronising Encoding 1

The **Manchester code** uses signal changes to keep the sending/receiving devices synchronised. It encodes 0 and 1 by changing the voltage:

0 is represented by a change from high to low and
1 is represented by a change from low to high.

Note: the signal will never be held constant longer than a single bit interval, no matter what data is being transmitted.

Analog signals 5



A Manchester signal can change levels twice every T seconds, and must change at least once. An NRZ or NRZI signal can change level at most once every T seconds: this is half the bandwidth of a Manchester signal. (Are there more efficient encodings of a clock signal?)

Modulation and Demodulation

The process of adding a data signal (e.g. a digital bitstream from a PC) to an analog *carrier signal* is called **modulation**. The process of extracting the data from a modulated signal is called **demodulation**.

Frequency modulation is used in FM radio transmission. The analog audio signal (typically limited to 15 kHz) modulates the analog carrier signal (e.g. 101.4 MHz for National FM in the Auckland region).

A **modem**, short for modulator/demodulator, is a device that does both conversions – for digital data and an analog carrier.

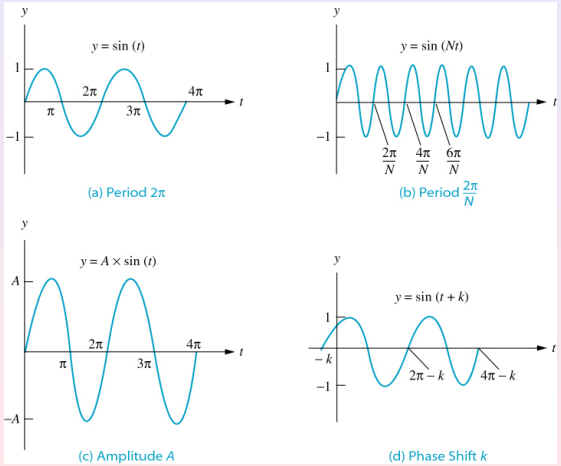
Understanding analog signals 1

A sine wave is the simplest analog signal. There are three ways to adjust a sine wave:

1. changing its frequency,
2. changing its amplitude,
3. changing its phase.

See `http://www.ltscotland.org.uk/5to14/resources/science/sound/index.asp`.

Understanding analog signals 2



Understanding analog signals 3

1. the **period**, p, is the time it takes to complete a pattern,
2. the **frequency**, f, is the number of times the signal oscillates per unit of time ((hertz) Hz if time is measured in seconds); $f = 1/p$,
3. the **amplitude** is the range in which the signal oscillates; the **peak amplitude** is the absolute value of signal's highest intensity),
4. the **phase** shift describes the position of the signal relative to time zero; it is obtained by adding or subtracting k from the sine argument ($\sin(t + k)$, $\sin(t - k)$).

Understanding analog signals 4

Here are some numerical examples with reference to the previous picture:

In (a) the period for $y = sin(t)$ is $p = 2\pi$,

In (b) the period for $y = sin(Nt)$ is $p = 2\pi/N$,

In (b) the frequency is $f = 1/p = N/2\pi$ Hz,

In (a), (b), and (d) the amplitude is $[-1, 1]$; and the peak amplitude is 1. In (c), the amplitude is $[-A, A]$ with a peak of A.

Understanding analog signals 5

A sine function can be written in the form:

$$s(t) = A\sin(2\pi ft + k),$$

where $s(t)$ is the instantaneous amplitude at time t, A is the peak amplitude, f is the frequency, and k is the phase shift.

These three characteristics fully describe a sine function.

Understanding analog signals 6

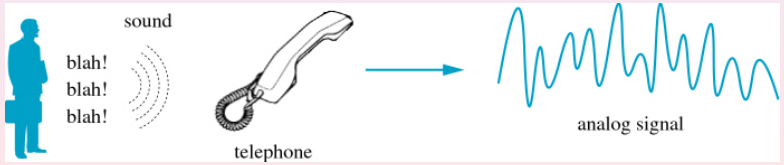
For electrical signals, the peak amplitude is measured in volts (V). The frequency is measured in hertz (Hz).

Unit	Equivalent	Unit	Equivalent
second (s)	1 s	hertz (Hz)	1Hz
millisecond (ms)	10^{-3} s	kilohertz (kHz)	10^3 Hz
microsecond (μs)	10^{-6} s	megahertz (MHz)	10^6 Hz
nanosecond (ns)	10^{-9} s	gigahertz (GHz)	10^9 Hz
picosecond (ps)	10^{-12} s	terahertz (THz)	10^{12} Hz

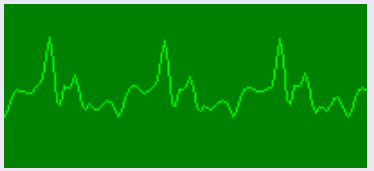
Understanding analog signals 7

Sound can be *transduced* by a microphone, or by our ears, into an electrical (analog) signal. The audio signal from a microphone has the following relationships to our aural perceptions:

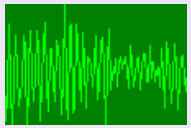
- the amplitude of the audio signal correlates with our perception of volume, and
- the frequency correlates with our perception of pitch.



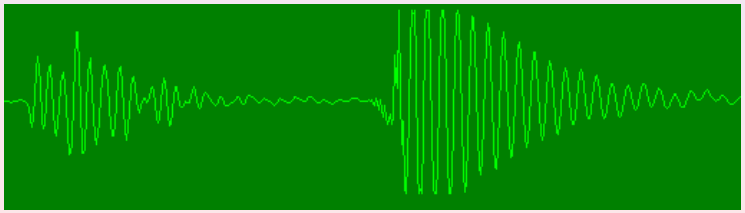
Understanding analog signals 8



Trumpet



White noise



Tongue click

Understanding analog signals 9

A single sine function is not useful for data communications.

We need to change one or more of its characteristics—amplitude, frequency and phase shift—to encode a signal.

How do we decode a signal from a sine wave carrier that has been modulated by shifts in amplitude, frequency and/or phase?

Answer: *Fourier Analysis*. The French mathematician Jean Baptiste Fourier proved that any periodic function can be expressed as an infinite sum of sine and cosine functions of varying amplitudes, frequencies and phase shifts—a **Fourier series**.

Understanding analog signals 10

Any periodic signal $x(t)$ with period P can be expressed as a Fourier series:

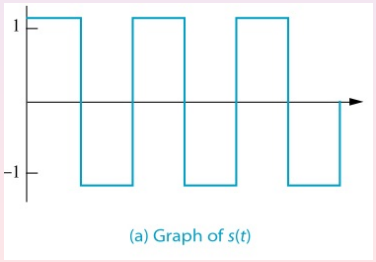
$$x(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty} \left[a_i \cos \frac{2\pi i t}{P} + b_i \sin \frac{2\pi i t}{P} \right]$$

The coefficients $a_1, a_2 \ldots, b_1, b_2, \ldots$ are uniquely determined by this equation, and are called the *Fourier transform* of $x(t)$.

Understanding analog signals 11

For example, consider a unit-amplitude square wave $s(t)$ with period 2π:

$$s(t) = \begin{cases} 1, & \text{if } t \in [0,\pi) \cup [2\pi,3\pi) \cup [4\pi,5\pi) \cup \ldots, \\ -1, & \text{if } t \in [\pi,2\pi) \cup [3\pi,4\pi) \cup [5\pi,6\pi) \cup \ldots, \end{cases}$$



(a) Graph of $s(t)$

Understanding analog signals 12

Because the square wave $s(t)$ is periodic, it can be written as a Fourier series:

$$s(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty} \left[a_i \cos \frac{2\pi i t}{P} + b_i \sin \frac{2\pi i t}{P} \right]$$

The values of P and the coefficients a_i and b_i are not complicated expressions (but would be hard to guess ;-):

$$P = 2\pi, \quad a_i = 0, \quad b_i = \begin{cases} 0, & \text{if } i \text{ is even}, \\ \frac{4}{\pi i}, & \text{if } i \text{ is odd}. \end{cases}$$

So a square wave is

$$s(t) = \sum_{i=1,3,5,\ldots} \frac{4}{\pi i} \sin(it).$$

Understanding analog signals 13

Let's look at the Fourier analysis of the square wave more carefully:

$$s(t) = \sum_{i=1,3,5,\ldots} \frac{4}{\pi i} \sin(it) = \frac{4}{\pi} \sin(t) + \frac{4}{3\pi} \sin(3t) + \cdots$$

Each term in this series can be expressed in our general form for sine waves:

$$\frac{4}{i\pi} \sin(it) = A \sin(2\pi f t i + k),$$

where the amplitude, frequency and phase shift are

$$A = \frac{4}{i\pi}, \quad f = \frac{1}{2\pi}, \quad k = 0.$$

Understanding analog signals 14

A square wave is thus a sum of sine functions with frequencies $f, 3f, 5f, \ldots$ and amplitudes $\frac{4}{\pi}, \frac{4}{3\pi}, \frac{4}{5\pi}, \ldots$.

$$s(t) = \frac{4}{\pi} \sin(2\pi f t) + \frac{4}{3\pi} \sin[2\pi(3f)t] + \frac{4}{5\pi} \sin[2\pi(5f)t] + \cdots$$

The term with frequency f is called the **fundamental frequency**; the term with frequency $3f$ is called the **third harmonic**; the term with frequency $5f$ is called the **fifth harmonic**; etc. These are all **odd harmonics**.

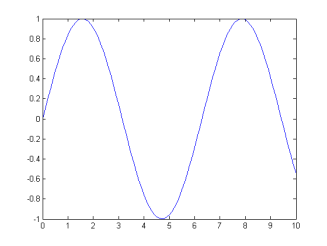
We can compute a finite number n of Fourier coefficients efficiently, in $O(n \log n)$ floating-point multiplications and additions, using an algorithm called the *fast Fourier transform*.

Understanding analog signals 15

The Fourier series expansion for a square-wave is made up of a sum of odd harmonics. We show this graphically using MATLAB®.

We start by forming a time vector running from 0 to 10 in steps of 0.1, and take the sine of all the points. Let's plot this fundamental frequency.

```
t = 0:.1:10;
y = sin(t);
plot(t,y);
```

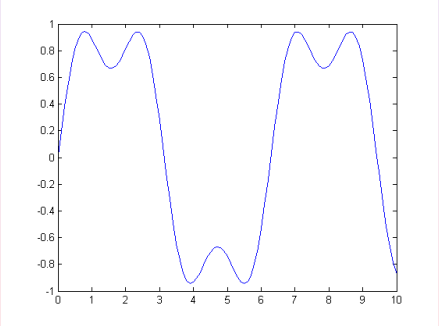


Text and graphics on this series of slides is from http://www.mathworks.com/products/matlab/demos.html?file=/products/demos/shipping/matlab/xfourier.html

Understanding analog signals 16

Now add the third harmonic to the fundamental, and plot it.

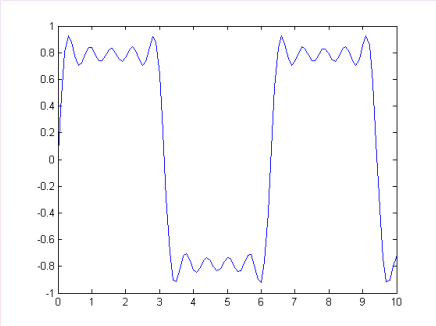
```
y = sin(t) + sin(3*t)/3; plot(t,y);
```



Understanding analog signals 17

Now use the first, third, fifth, seventh, and ninth harmonics.

```
y = sin(t) + sin(3*t)/3 + sin(5*t)/5 +
sin(7*t)/7 + sin(9*t)/9; plot(t,y);
```

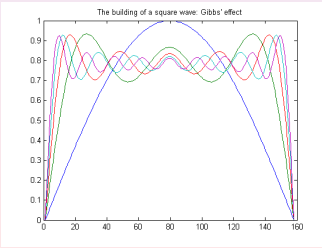


Understanding analog signals 18

For a finale, we will go from the fundamental to the 19th harmonic, creating vectors of successively more harmonics, and saving all intermediate steps as the rows of a matrix.

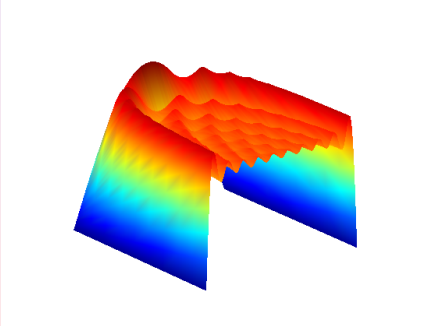
These vectors are plotted on the same figure to show the evolution of the square wave. Note that Gibbs' effect says that it will never really get there.

```
t = 0:.02:3.14; y =
zeros(10,length(t));
x = zeros(size(t));
for k=1:2:19
    x = x + sin(k*t)/k;
    y((k+1)/2,:)  = x;
end
plot(y(1:2:9,:)')
title('The building of a
square wave:  Gibbs'' effect')
```



Here is a 3-D surface representing the gradual transformation of a sine wave into a square wave.

```
surf(y); shading interp axis off ij
```



The fast Fourier transform can be used in digitised versions of analog signal-processing devices.

- **Filters** attenuate certain frequencies while allowing others to pass. The Bass control on a stereo system is an adjustable *lowpass filter* which limits the amount of low-frequency sound in the output. The Treble control is an adjustable *highpass filter*. Stereo equalisers have many adjustable *bandpass filters*.
- **Tuners** extract one modulated signal from a signal which has been modulated by many signals. Tuners are necessary in radio and TV receivers, to select one station or channel from all of the ones that are currently being broadcast.

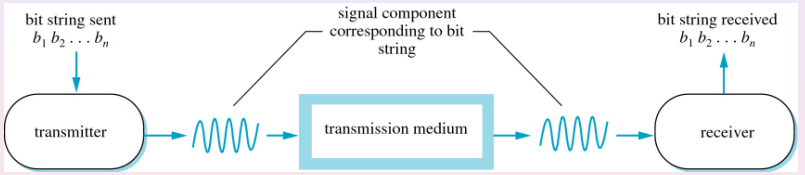
The **bit rate** describes the information-carrying capacity of a digital channel, and is measured in bits per second (b/s).

The range of frequencies in a channel is called its *bandwidth*.

Roughly:

a higher-bandwidth channel has a higher bit rate.

(We will develop a more refined understanding, in a moment...)



Sending data via signals

Bit rate 3

The bit rate $R = f_s n$ is the product of the frequency f_s at which symbols are sent, and the number of bits per symbol n.

Note that this equation is trivial, by a dimensional analysis:

$$\frac{\text{bits}}{\text{second}} = \frac{\text{bits} \cdot \text{symbol}}{\text{second} \cdot \text{symbol}} \quad (1)$$

$$= \frac{\text{bits}}{\text{symbol}} \cdot \frac{\text{symbols}}{\text{second}} \quad (2)$$

As a shorthand for symbols/second, we write *baud* – after Émile Baudot, the inventor of the Baudot code. The abbreviation is Bd, and its units are symbols per second.

Bit rate 4

Nyquist theorem. *In a distortion-free transmission, the baud rate f_s is at most twice the maximum frequency f of the medium.*

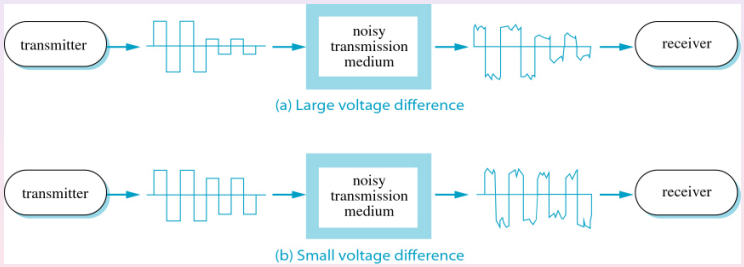
Since the baud rate f_s is upper-bounded by $2f$, the bit rate R is at most $2fn$ when each symbol carries n bits.

Telephone connections have a maximum frequency f of 3300 Hz.

bits/symbol (n)	number of symbols (2^n)	max bits/second (R)
1	2	6,600
2	4	13,200
3	8	19,800
4	16	26,400

Noisy channels 1

Many channels are **noisy**. Note the voltage difference between the first and last part of the transmitted signal



Sending data via signals

Noisy channels 2

The **signal-to-noise (S/N) ratio or SNR** is measured by the ratio S/N, where S is the signal power and N is the noise power. A clear signal has a large SNR, and a distorted (noisy) signal has a low SNR. Since the measured values of S/N vary hugely, we typically report their logarithms rather than their absolute values:

$$\text{SNR}_{\text{dB}} = \log_{10}(S/N) \text{ bels}$$

where the unit of measurement is called the **bel**. The more familiar **decibel** is defined by

$$1 \text{ dB} = 0.1 \text{ bel}.$$

Decibels can also be used to measure sound intensity, relative to some baseline level (N). Typically, a 3 dB change is barely perceptible. http://en.wikipedia.org/wiki/Weber%E2%80%93Fechner_law.

Noisy channels 3

If our SNR is reported as 25 dB, this is 2.5 bels, i.e. we have

$$\text{SNR}_{\text{dB}} = \log_{10}(S/N) = 2.5 \text{ bels}$$

This implies

$$S/N = 10^{2.5}$$

or

$$S = 10^{2.5} \times N = 100\sqrt{10} \times N \approx 316N$$

Noisy channels 4

The American scientist Claude Shannon, inventor of the classical theory of information, refined Nyquist's theorem by taking into account the channel's noise:

Shannon's theorem. *In a noisy transmission,*

$$\text{bit rate (in b/s)} \leq \text{bandwidth (in Hz)} \times \log_2(1 + S/N)$$

The quantity $\log_2(1 + S/N)$ is the maximum number of bits that can be transmitted per cycle on this channel.

Note that decreasing the signal-to-noise ratio on any channel will decrease its information-carrying capacity. Somewhat surprisingly, a channel with any non-zero signal strength has some capacity (because its SNR must be greater than zero).

Noisy channels 5

Telephones carry audio frequencies in the range 300 Hz to 3300 Hz: this is a bandwidth of 3000 Hz. A good telephone connection has a signal-to-noise ratio of 35 dB. So,

$$3.5 \text{ bels} = \log_{10}(S/N) \text{ bels},$$
$$S = 10^{3.5} \times N \approx 3162N.$$

Using Shannon's theorem we get:

$$\begin{aligned} \text{bit rate} &\leq \text{bandwidth} \times \log_2(1 + S/N) \\ &= 3000 \times \log_2(1 + 3162) \text{ b/s} \\ &\approx 3000 \times 11.63 \text{ b/s} \\ &\approx 34880 \text{ b/s} \end{aligned}$$

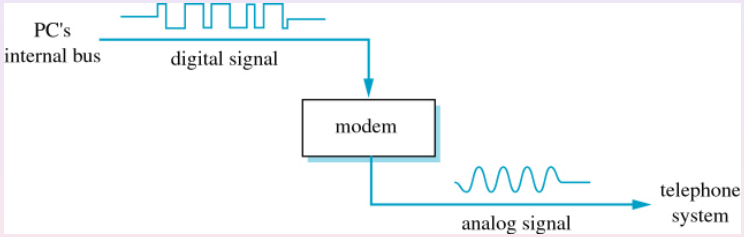
Noisy channels 6

How can a 56 kb/s modem possibly achieve its maximum data rate on a 3000 Hz analog phone line with a S/N of 35 dB? (Did Shannon make a mistake?)

Answer: Modern telephone systems use 64 kb/s digital signalling on their long-distance connections, and the downlink on a V.90 modem is able to interpret these digital signals when it receives data from your ISP. Your downlink is a channel with an 8 kBd signalling rate and 8 bits/symbol; the bandwidth of this channel is 4 kHz, not the 3 kHz of an analog voice connection. The uplink on a V.90 modem uses the 3 kHz analog voice channel, transmitting data at 33.6 kb/s if your phone line isn't noisy, and transmitting at a lower bitrate if you have a noisy line.

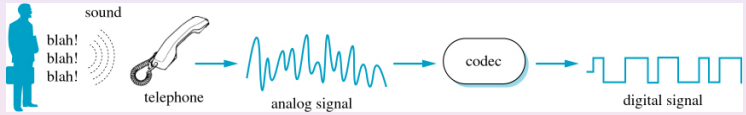
V.92 modems can upload at 56 kb/s, but as far as I know, no NZ ISP provides a V.92 dialup service.

Digital to analog conversion 1



Computer data transmitted over telephone lines

Digital to analog conversion 2



Voice information transmitted through a digital connection, using a **codec** (coder/decoder)

Digital to analog conversion 3

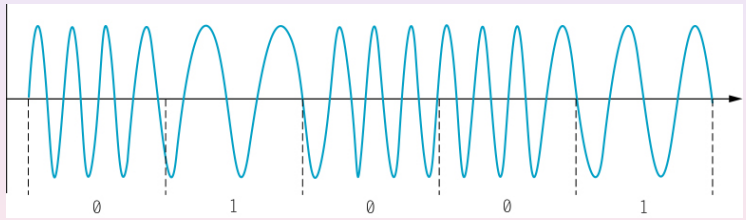
There are three main ways to encode a digital signal as an analog signal, corresponding to the three parameters in a sine wave. We can modulate

1. by frequency, for example by **frequency shift keying (FSK)**,
2. by amplitude, for example by **amplitude shift keying (ASK)**, or
3. by phase modulation, for example **phase shift keying (PSK)**.

We can also use combined methods, such as **quadrature amplitude modulation (QAM)** which modulates both amplitude and phase.

Digital to analog conversion 4

FSK or **frequency modulation (FM)** assigns a digital 0 to one analog frequency and a 1 to another.

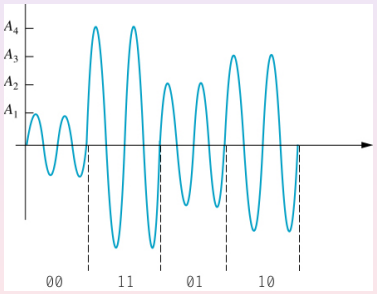


FSK, at one bit per symbol.

Note: we might use n-bit symbols, where each symbol is assigned one frequency in a set of 2^n frequencies.

Digital to analog conversion 5

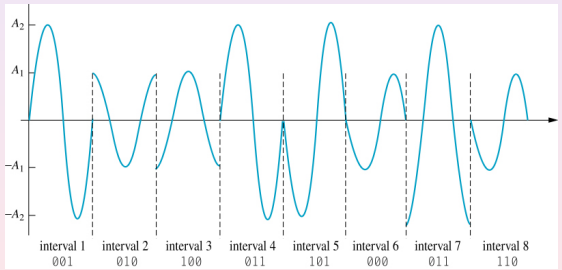
ASK or **amplitude modulation (AM)** assigns a digital '0' to one analog amplitude, a '1' to another amplitude, ... and (for AM at n bits per symbol) a '$2^n - 1$' to yet-another amplitude.



ASK, at two bits per symbol.

Digital to analog conversion 6

PSK or **phase modulation (PM)** assigns each bit-string of a fixed length to one analog phase shift. **Quadrature amplitude modulation (QAM)** is a combination of amplitude and phase shift.

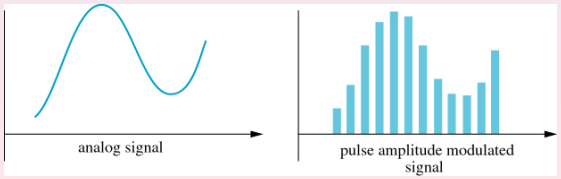


QAM with two amplitudes, four phases, and three bits per symbol.

Analog to digital conversion 1

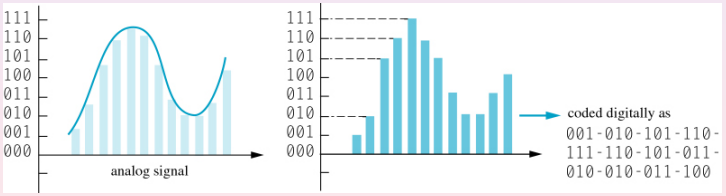
In principle, any digital to analog encoding can be decoded. However some analog-to-digital decodings are easier than others.

In the most obvious analog-to-digital decoding, we start by sampling an analog signal at regular intervals. When our samples are analog, we are modifying the analog signal by "flattening" all its high-frequency components. This modification process is called **pulse amplitude modulation (PAM)**.



Analog to digital conversion 2

PAM signals may seem digital, but they have analog amplitudes. In **pulse code modulation (PCM)**, we define a set of 2^n amplitudes, where n is the number of bits per symbol. The process of "rounding" an analog amplitude to its nearest available ("digital") approximation is called **quantisation**.



Digital telephone signalling (DS0, E0, J0) in North America, Europe, and Japan, is PCM with 8 bits per symbol at 8 kbaud.

Why compression? 1

Digital media, by using sophisticated compression algorithms, can have significant performance benefits over analog media.

Here is an example. In the (obsolescent) PAL broadcast standard for New Zealand television, the bandwidth is approximately 7 MHz. The SNR is roughly 20 dB, depending greatly on the location, design, and orientation of your antenna.

If we sample a PAL signal at the Nyquist rate of twice its bandwidth (14 MBd), and if we use the trivial digital encoding of 8 bits/sample for three channels (Red, Green, and Blue), then we will have a data rate of $3 \cdot 8 \cdot 14 = 336$ Mb/s, or 42 MB/s.

If we record all of these bytes for two hours ($= 2 \cdot 60 \cdot 60 = 7200$ seconds), we will have 302400 MB, or 302.4 GB. A digital video disk (DVD-5) can hold about 4.7 GB ...

Why compression? 2

A trivial form of data compression, for any analog signal, is to cut its bandwidth – we can use a low-pass filter to limit its high-frequency information. Analog PAL broadcasting, and VHS tapes, use this technique to compress the chrominance (colour) information in studio-quality TV recordings. The broadcast chroma is only about 2 MHz at 20 dB.

We could digitally sample the chroma signal (using PCM) at a rate of $4 \log_2(1 + 10^{20/2}) = 27$ Mb/s ≈ 3 MB/s. The luminance signal is about 9 MB/s, for a total of 12 MB/s.

This is more than a 3:1 compression of our naive 42 MB/s encoding. However we need to get a 40:1 compression, down to about 1 MB/s, in order to record 90 minutes of video on a DVD.

How do you reduce bits and still keep enough information?

Guiding principle of compression: Discard any information (such as high-frequency chroma) that is unimportant; retain any information that is essential to the "meaning".

Here is a simple example. Assume that you wish to email a large file consisting entirely of strings of capital letters. If the file has n characters each stored as an 8-bit ASCII code, then we need $8n$ bits.

However, we don't need all ASCII codes to code strings of capital letters: they use only 26 characters. We can make our own code with only 5-bit codewords ($2^5 = 32 > 26$), code the file using this coding scheme, send the encoded file via email, and finally decode it at the other end.

Big deal? The size of the file has decreased by $8n - 5n = 3n$, i.e. a 37.5% reduction.

Huffmann code 1

ASCII code is an 8-bit code which gives no preference in coding characters. However, some characters appear more frequently than others. A **frequency-dependent code** varies the lengths of codewords on frequency: more frequent characters have shorter codewords. An example of frequency-dependent code is the **Huffmann code code**.

Huffmann code 2

To illustrate assume that we have five characters, A-E, whose frequencies are as follows:

Letter	Frequency (%)
A	25
B	15
C	10
D	20
E	30

Huffmann code 3

To construct the Huffmann codeword for the string we follow the following algorithm:

1. To each character we associate a binary tree consisting of just one node. To each tree we assign the tree's *weight*. Initially, the weight of nodes is exactly its frequency: 0.25 for A, 0.15 for B, etc.
2. Calculate the two lightest-weight trees (choose any if there are more than two). Merge the two chosen trees into a single tree with a new root node whose left and right sub-trees are the two we chose. The weight of the new tree is the sum of the weights of the merged trees.
3. Repeat the procedure till one tree is left.

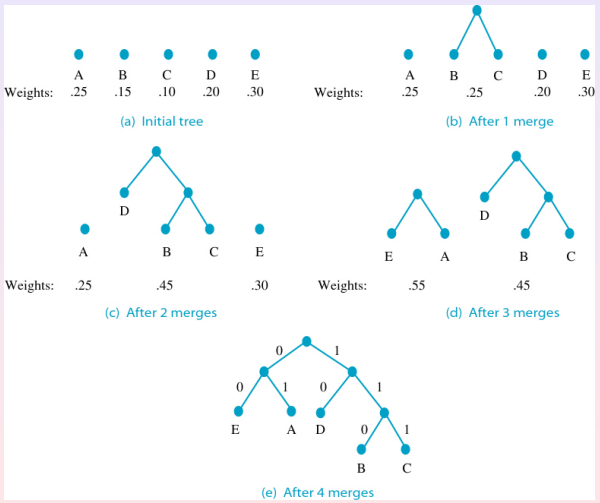
Huffmann code 4

When completed, each of the original nodes is a leaf in the final tree. Arcs are labelled with 0 and 1 as follows: we assign a 0 each time a left child pointer is followed and a 1 for each right child pointer.

As with any binary tree, there is a unique path from the root to any leaf. The path to any leaf defines the Huffmann code of the character on labelling the leaf.

Let us apply this procedure to the character strings A,B,C,D,E.

Huffmann code 5



The Huffmann code is the following:

Letter	Frequency (%)	Code
A	25	01
B	15	110
C	10	111
D	20	10
E	30	00

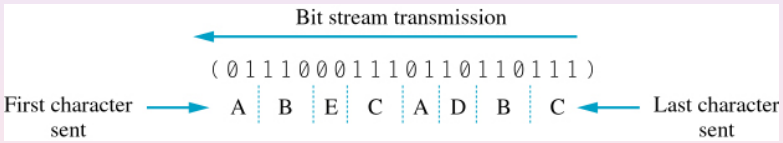
As one can see, the algorithm for constructing the Huffmann tree is very "tolerant", it gives a lot of flexibility. In the above example one could, for instance, choose putting D on the right-hand side of the merged tree in stage (c) and swap A and E in stage (d). The result will be the Huffmann code:

Letter	Frequency (%)	Code
A	25	00
B	15	100
C	10	101
D	20	11
E	30	01

Huffmann code properties:

- There are many Huffmann codes which can be associated to the same characters and weights. So, we should better speak of *a* Huffmann code instead of *the* Huffmann code.
- All Huffmann codes are, in general, variable-length, frequency-dependent, prefix codes. Consequently, Huffmann codes are uniquely decodable.

Let us decode the codeword 01110001110110110111:



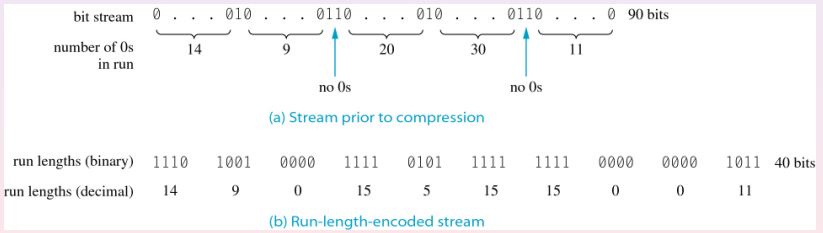
Run-length encoding 1

Huffmann codes are useful only in case we know the frequency of characters. Many items that travel the communications media, including binary files, fax data, and video signals, do not fall into this category.

Run-length encoding analyses bit-strings by looking for long runs of 0 or 1. Instead of sending all bits, *it sends only how many of them are in the run*. Fax transmission is well-served by this technique as potentially a large part of a page is white space, corresponding to a long run of 0s.

Run-length encoding 2

The sender transmits only the length of each run as a fixed-length integer; the receiver gets each length and generates the proper number of bits in the run, inserting the other bit in between. Here is an example:



This technique works well when there are many long 0 runs. Note that the symbol "15" indicates a run of length 15 or more, so the run of thirty 0s is encoded as "15", "15", "0".

Run-length encoding 3

What about long runs of different bits or even characters?

Solution: send the actual character along with the run length. For example,

HHHHHHHUFFFFFFFFFFFFFFYYYYYY
YYYYYYYYYYYYYYDGGGGGGGGGGG

can be sent as 7,H,1,U,14,F,20,Y,1,D,11,G.

Relative encoding 1

A single video image may contain little repetition, but *there is a lot of repetition over several images*.

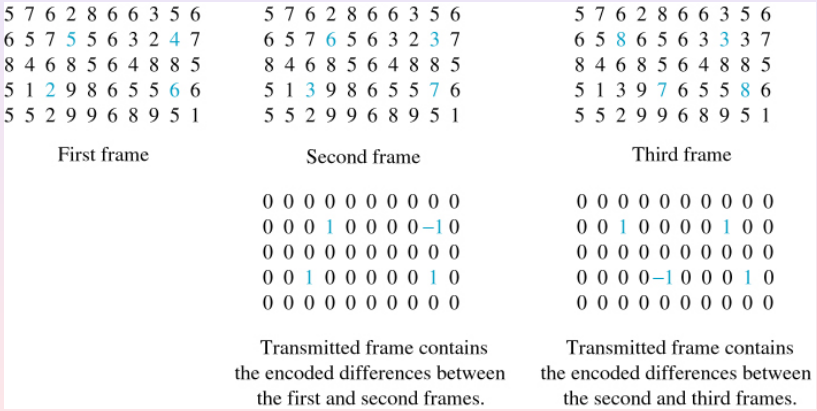
Reason: a) a USA TV signal sends 30 pictures per second, b) each picture generally varies only slightly from the previous one.

The **relative encoding** or **differential encoding** works as follows:

- the first picture is sent and stored in a receiver's buffer;
- the second picture is compared with the first one and the encoding of differences are sent in a frame format; the receiver gets the frame and applies the differences to create the second picture;
- the second picture is stored in a receiver's buffer and the process continues.

Relative encoding 2

Here is an example of relative encoding:



First frame	Second frame	Third frame
5 7 6 2 8 6 6 3 5 6	5 7 6 2 8 6 6 3 5 6	5 7 6 2 8 6 6 3 5 6
6 5 7 5 5 6 3 2 4 7	6 5 7 6 5 6 3 2 3 7	6 5 8 6 5 6 3 3 3 7
8 4 6 8 5 6 4 8 8 5	8 4 6 8 5 6 4 8 8 5	8 4 6 8 5 6 4 8 8 5
5 1 2 9 8 6 5 5 6 6	5 1 3 9 8 6 5 5 7 6	5 1 3 9 7 6 5 5 8 6
5 5 2 9 9 6 8 9 5 1	5 5 2 9 9 6 8 9 5 1	5 5 2 9 9 6 8 9 5 1
	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
	0 0 0 1 0 0 0 0 −1 0	0 0 1 0 0 0 0 1 0 0
	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
	0 0 1 0 0 0 0 0 1 0	0 0 0 0 −1 0 0 0 1 0
	0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0
	Transmitted frame contains the encoded differences between the first and second frames.	Transmitted frame contains the encoded differences between the second and third frames.

Lempel-Ziv compression

The **Lempel-Ziv compression** method looks ahead in the input, finding the longest previously-transmitted string which is a prefix of the input. It transmits a reference to that previous string, thereby avoiding sending the same string more than once.

This technique—replacing an input string by an encoded reference to a prior string—is widely used:

1. UNIX compress command,
2. gzip on UNIX,
3. V.42bis compression standard for modems,
4. GIF (Graphics Interchange Format).

JPEG 1

Pixels are represented using 8 bits which can distinguish 256 shades of gray, ranging from white to black.

For colour pictures we can use one byte to represent each of the three primary colours. The intensity of each colour can be adjusted according to the 8-bit value to produce the desired colour. With $3 \times 8 = 24$ bits we can describe $2^{24} = 16,777,216$ different colours.

An alternative method uses three codes as follows: one codeword Y for *luminance* (brightness), and two codewords (U, V) for *chrominance*. The U value is the Blue intensity minus Y, and V is the Red intensity minus Y.

In 4:2:2 Y'UV coding, the luminance is scaled non-linearly (gamma-corrected) into a Y' signal at four bits per pixel. The U and V components are just two bits per pixel, because the human eye is less sensitive to chroma than to luma.

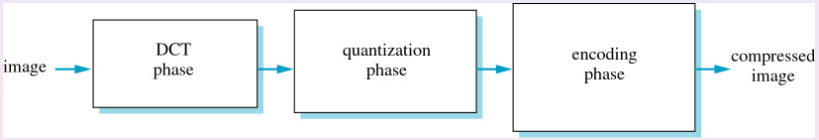
JPEG 3

JPEG is an acronym for the Joint Photographic Experts Group and **JPEG compression** is a **lossy** data compression method. This means that the image obtained after decompression may not coincide with the original image. Lossy compression is acceptable for images because of the inherent limitations of the human optical system.

There are three phases in a JPEG compression:

1. the discrete cosine transform (DCT),
2. quantisation,
3. encoding phase.

JPEG 4



JPEG three phases

JPEG 5

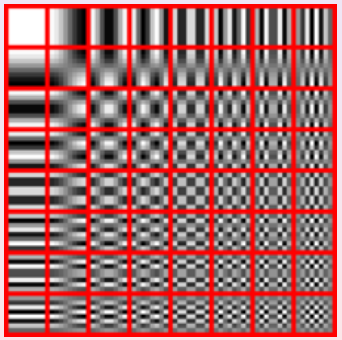
In the DCT phase, each component of the image is "tiled" into sections of 8×8 pixels each; dummy data fills incomplete blocks.

Usually, there are three components in a JPEG image, corresponding to the (Y', U, V) we discussed earlier. Other colour-spaces and grey-scale (a single 8-bit component) may be used.

The chroma components are usually downsampled, i.e. encoded at lower spatial resolution than the luma.

JPEG 6

Then, each tile in each component is converted to frequency space using a two-dimensional forward "discrete cosine transform", using the basis functions shown below.



picture source: http://en.wikipedia.org/wiki/JPEG

JPEG 7

The human eye can see small differences in brightness over a relatively large area, but is insensitive to brightness variation at high frequency.

JPEG attenuates the high frequency signal components by dividing each component G_{ij} in the frequency domain by a constant Q_{ij}, where the value of the constant is larger for the higher frequency components (i.e. the ones with larger i,j values). After division, the component is rounded to the nearest integer. This is the **quantisation phase**, in which the main lossy operation takes place.

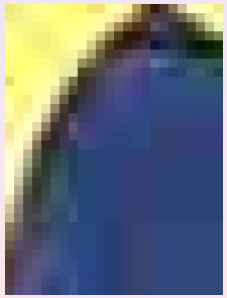
As a result, in the **encoding** phase, many of the higher frequency components are rounded to zero, and many of the rest become small positive or negative numbers; they take many fewer bits to store.

JPEG 8

The compression ratio of JPEG depends greatly on the divisors used during the quantisation phase. These are controlled by a "Quality" parameter.

At 10:1 compression, it is difficult to distinguish the compressed image from the original one.

At 100:1 compression, a JPEG-compressed image is usually still recognisable but has many visual artifacts, especially near sharp edges.



http://en.wikipedia.org/wiki/JPEG

GIF

GIF (Graphics Interchange Format) compresses by: a) reducing the number of colours to 256, and b) trying to cover the range of colours in an image as closely as possible.

It replaces each 24-bit pixel value with an 8-bit index to a table entry containing the colour that matches the original "most closely". In the end, a variation of the Lempel-Ziv encoding is applied to the resulting bit values.

GIF files are lossy if the number of colours exceeds 256 and lossless otherwise.

MPEG and MP3 1

The **Moving Picture Experts Group (MPEG)** is a working group of ISO/IEC charged with the development of video and audio encoding standards.

The video codecs from MPEG use the discrete cosine transform techniques of the JPEG image compressor, and they also take advantage of redundancy between successive frames of video for "inter-frame compression". Differences between successive frames can be encoded very compactly, when the motion-prediction techniques are successful.

Note: MPEG holds patents, and charges license fees. In June 2002, China formed a working group to develop an alternative set of audio and video codecs. The group was successful, see `http://www.avs.org.cn`, however the H.263 (MPEG-2) and H.264 (MPEG-4 Part 10) codecs are still commonly used in China.

MPEG and MP3 2

MPEG has standardised the following compression formats and ancillary standards:

- MPEG-1: video and audio compression standard; it includes the popular Layer 3 (MP3) audio compression format
- MPEG-2: transport, video and audio standards for broadcast-quality television
- MPEG-4: expands MPEG-1 to support video/audio "objects", 3D content, low bit-rate encoding and support for Digital Rights Management

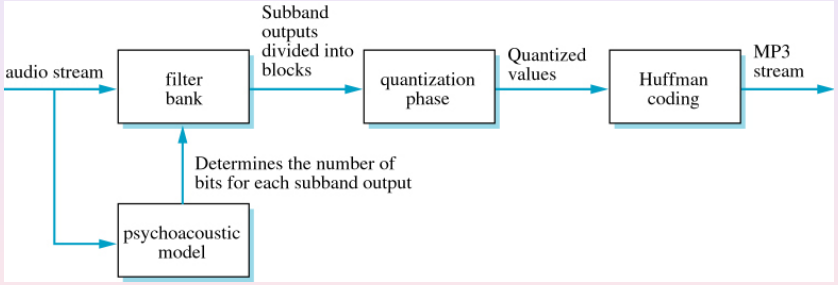
MPEG-1 Audio Layer 3, better known as **MP3**, is a popular digital audio encoding, lossy compression format, and algorithm.

MP3 is based on the **psycho-acoustic model**, **auditory mask,** and **filter bank**.

Psycho-acoustics is the study of the human auditory system to learn what we can hear and what sounds we can distinguish. In general we can hear sounds in the 20 Hz to 20 kHz range, but sounds with close frequencies (for example, 2000 Hz and 2001 Hz) cannot be distinguished.

The auditory mask is the following phenomenon: if a sound with a certain frequency is strong, then we may be unable to hear a weaker sound with a similar frequency.

The filter bank is a collection of filters, each of which creates a stream representing signal components of a specified ranges. There is one filter for each of the many frequency ranges. Together they decompose the signal into **sub-bands**.



MP3 encoding

The MPEG standards define audio and video codecs, as well as file formats.

The MP4 file format (MPEG-4 Part 14) is based on Apple's QuickTime container format. The file header indicates what codecs are used for video and audio, and it also gives values for important paramaters such as the video frame rate, horizontal and vertical resolution (i.e. number of pixels per line, and number of lines per frame), and colourspace.

MPEG4 2

The video codecs defined in MPEG-4 are all extensions of JPEG (or closely-related DCT-based techniques). Each video frame can be individually JPEG-encoded, however usually each JPEG-encoded frame (an "I-frame") is followed by a few frames which have been encoded, much more compactly, using codewords which refer to 8x8 image blocks in the preceding (or following) I-frame.

These video compressors are especially successful in video with stationary backgrounds, but are not as effective in scenes with a lot of differently-moving objects – such as the leaves on a tree, on a windy day. When compression ratios are high for video with high-frequency information, JPEG-like artifacts become apparent e.g. sharp edges are sometimes rendered at very low resolution (as 8x8 blocks).

Why check integrity?

Data can be corrupted during transmission—many factors can alter or even wipe out parts of data.

Reliable systems must have mechanisms for detecting and correcting **errors**.

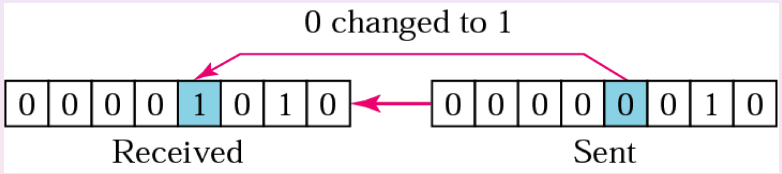
The capability to detect when a transmission has been changed is called **error detection**. In some cases a message with errors is discarded and sent again, but not always this is possible (e.g. in real-time viewing). In the later case the error has to be fixed (in real-time) and the mechanism doing this job is called **error correction**.

Types of errors

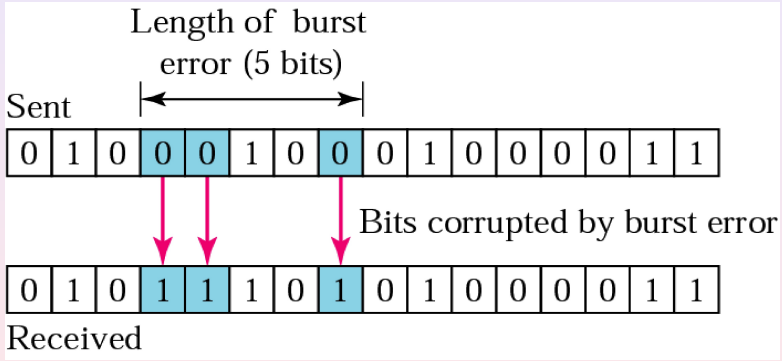
There are two types of errors:

1. single-bit error, when only one bit in the data has changed,
2. burst error, when two or more bits in the data have changed.

Single-error bit



Burst error

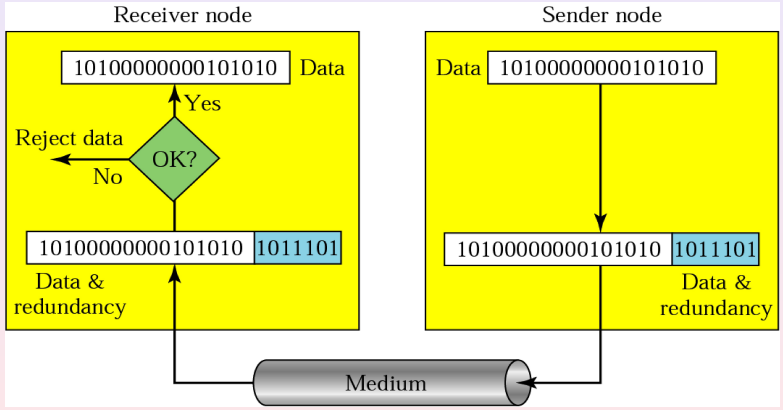


Redundancy 1

One way to check errors is by sending every data unit twice. A bit-by-bit comparison between the two versions is likely to identify all errors. The system is reasonably accurate (assuming that the errors are randomly distributed), but inefficient. Transmission time is doubled, and storage costs have increased significantly (because the transmitter and receiver must both retain a complete copy of the message).

A better approach is to add a fixed extra information to each segment (packet) of the message—this technique is called **redundancy**. There is no additional information in these error-checking bits, so they can be discarded after the check is completed. Note: natural languages are redundant, for example it is possible to understand most English sentences if vowels are omitted: FCTSSTRNGRTHNFCTN.

Redundancy 2



Redundancy 3

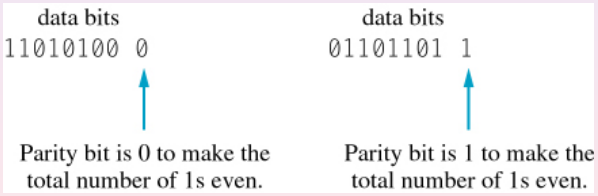
There are tree main types of redundancy checks:

1. parity check,
2. checksums,
3. cyclic redundancy check (CRC).

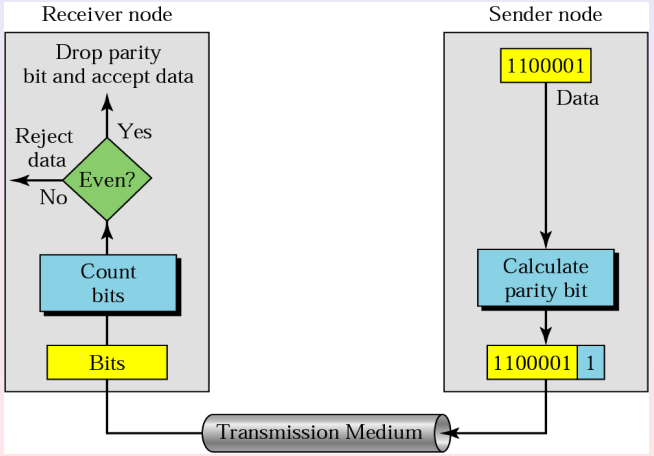
Low-density parity-check (LDPC) codes were the first to allow data transmission rates close to the theoretical maximum, the Shannon Limit.

Parity check: one-dimensional 1

A redundant bit, called **parity bit**, is added to every data unit so that the total number of 1s in the unit (including the parity bit) becomes even (or odd).



Parity check: one-dimensional 2



Parity check: one-dimensional 3

A simple parity check will detect all single-bit errors.

It can detect multiple-bit errors only if the total number of errors is odd.

Question: If the bit error rate (BER) is 0.1%, and errors are equiprobable at each bit in a 999-bit message with a single parity bit, what is the probability of an undetected error?

Parity check: one-dimensional 4

Answer: find someone who knows a little probability theory.

They should calculate the probability of no error as $(99.9\%)^{1000} \approx 37\%$, the probability of a single error as $1000(0.1\%)(99.9\%)^{999} \approx 37\%$, the probability of a double error as $\binom{1000}{2}(0.1\%)^2(99.9\%)^{998} \approx 18\%$, the probability of a triple error as $\binom{1000}{3}(0.1\%)^3(99.9\%)^{997} \approx 6\%$, and the probability of a quadruple error as 2%, and the probability of more than four errors as less than 1%.

The chance of an undetected error is thus about 20% for 999-bit messages with a single parity bit, on a channel with a BER of 0.1%.

Would you be happy with a digital communication system that introduces errors in 20% of your messages?

Parity check: two-dimensional 1

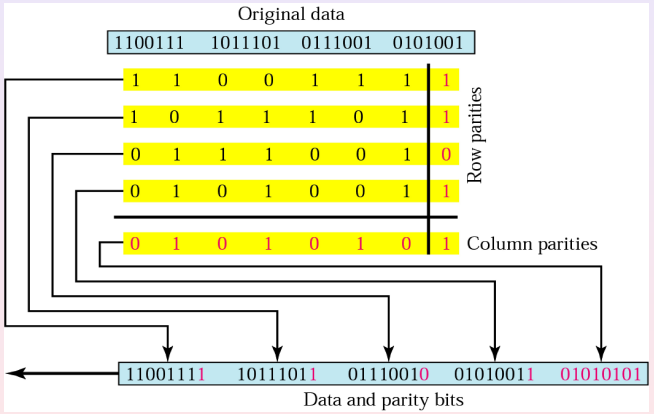
In a **two-dimensional parity check** a block of bits is organised in a table and parity is checked on both dimensions.

- First one calculates the parity bit for each data unit.
- Second one calculates the parity bit for each column and one creates a new row of 8 bits—the parity bits for the whole block.

A two-dimensional parity check significantly increases the likelihood that a burst error will be detected.

Definition. If a message $m_1 m_2 \dots m_n$ contains a single **burst error** of length B, then its bit-errors are confined to a single subsequence $m_i \dots m_{i+B-1}$ of length B.

Parity check: two-dimensional 2



Checksums 1

The method divides all data bits into 32-bit groups and treats each as an integer value. The sum of all these values gives the **checksum**. Any overflow that requires more than 32 bits is ignored.

An extra 32 bits representing in binary the checksum is then appended to the data before transmission.

The receiver divides the data bits into 32-bit groups and performs the same calculation. If the calculated checksum is different to the value received in the checksum field, then an error has occurred.

Checksums 2

A checksum is more sensitive to errors than a single-bit parity code, but it does not detect all possible errors. For example, a checksum is insensitive to any errors which *simultaneously* increases (by any value c) one of the 32-bit groups while also decreasing another 32-bit group by the same value c.

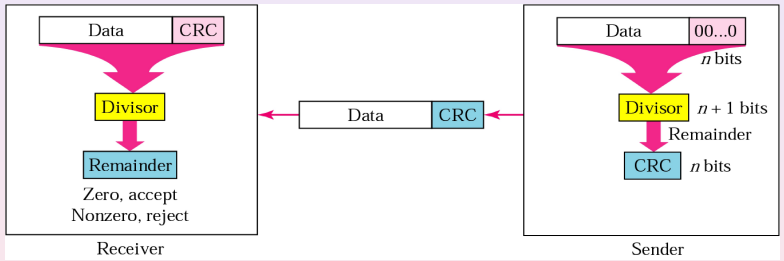
Cyclic redundancy check 1

The most powerful (and elaborate) redundancy checking technique is the **cyclic redundancy check (CRC)**.

CRC is based on binary division: a string of redundant bits—called the **CRC** or the **CRC remainder**—is appended to the end of the data unit such that the resulting data unit is exactly divisible by a second, predetermined binary number.

The receiver divides the incoming data by the same number. If the remainder is zero, the unit of data is accepted; otherwise it is rejected.

Cyclic redundancy check 2



Error correction

Error-correcting codes include so much redundant information with the unit block that the receiver is able to deduce, with high likelihood, not only how many bits are in error, but also which bits are incorrect. After these bits are inverted, all errors are corrected in the received message.

These codes are used when the error rate is high (e.g. on a WiFi channel). Detecting an error and than re-transmitting the message is very inefficient on a noisy channel, because every retransmission is likely to have errors.

Note that we cannot absolutely guarantee error correction, unless (somehow) we can place an upper bound on the total number of bit-errors in a message. In particular, if all of the error-check bits may be simultaneously in error, then any error in the data bits might not be corrected—and it may not even be detected.

Forward error correction 1

We examine the simplest case, i.e. single-bit errors. A single additional bit can detect single-bit errors. Is it enough for correction?

To correct a single-bit error in an ASCII character we must determine which of the 7 bits has changed. There are **eight** possible situations: no error, error on the first bit, error on the second bit, . . . , error on the seventh bit. Apparently we need **three** bits to code the above eight cases. But what if an error occurs in the redundancy bits themselves?

Forward error correction 2

Clearly, the number of redundancy (or, error control) bits required to correct n bits of data **cannot be constant, it depends on** n.

To calculate the number of redundancy bits r required to correct a n bits of data we note that:

- with n bits of data and r bits of redundancy we get a code of length $n + r$,
- r must be able to handle at least $n + r + 1$ different states, one for no error, $n + r$ for each possible position,
- therefore $2^r \geq n + r + 1$.

Forward error correction 3

In fact we can choose r to be the smallest integer such that $2^r \geq n + r + 1$. Here are some examples:

Number of data bits (n)	Minimum number of redundancy bits (r)	Total bits ($n + r$)
1	2	3
2	3	5
3	3	6
4	3	7
5	4	9
6	4	10
7	4	11

The Hamming code 1

The **Hamming code** is a practical solution which detects and corrects all single-bit errors in data units of any length.

A Hamming code for 7-bit ASCII code has 4 redundancy bits. These bits can be added in arbitrary positions, but it is simplest if we do this on positions 1, 2, 4, and 8. Data bits are marked with **d** and parity bits are denoted by r_8, r_4, r_2, r_1.

11	10	9	8	7	6	5	4	3	2	1
d	**d**	**d**	r_8	**d**	**d**	**d**	r_4	**d**	r_2	r_1

The Hamming code 2

In a Hamming code for bit-strings of length $2^n - 1$, each r bit is the parity bit for a specific combination of data bits, where the combinations follow a binary pattern shown below:

r_1: parity on bits 1, 3, 5, 7, 9, 11, 13, 15, ..., $2^n - 1$. That is, check 1 bit, skip 1 bit, check 1 bit, skip 1 bit, etc.

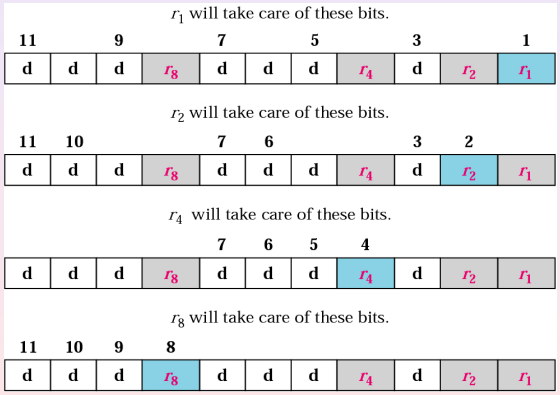
r_2: parity on bits 2, 3, 6, 7, 10, 11, 14, 15, ... That is, skip 1 bit, check 2 bits, skip 2 bits, check 2 bits, skip 2 bits, etc.

r_4: parity on bits 4, 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, ... That is, skip 3 bits, check 4 bits, skip 4 bits, check 4 bits, skip 4 bits, etc.

... $r_{2^{n-1}}$: parity on bits 2^{n-1} through $2^n - 1$. That is, skip $2^{n-1} - 1$ bits, and check 2^{n-1} bits.

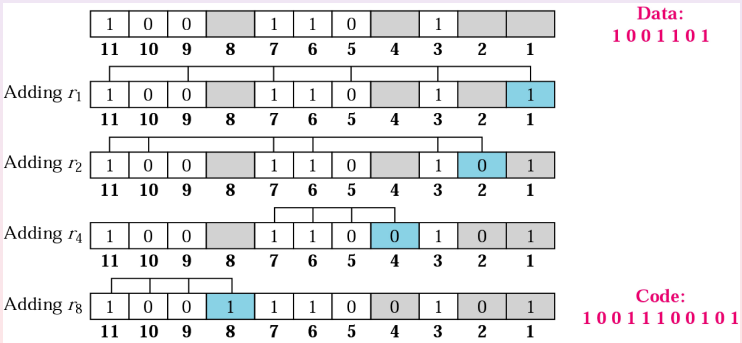
The Hamming code 3

Each data bit may be included in more than one calculation.



The Hamming code 4

This picture describes the calculation of r bits for the ASCII value of 1001101: start with data and repeatedly calculate, one by one, the parity bits r_1, r_2, r_4, r_8:

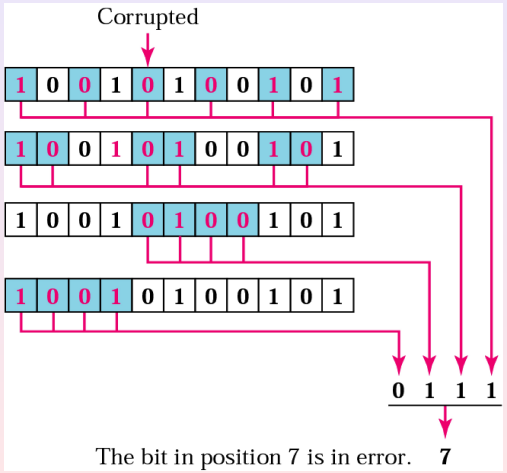


The Hamming code 5

Imagine that instead of the string 10011100101 the string 1001**0**100101 was received (the 7th bit has been changed from 1 to 0).

The receiver recalculates 4 new parity bits using the same sets of bits used by the sender plus the relevant parity r bit for each set. Then it assembles the new parity values into a binary number in the order used by the sender, r_8, r_4, r_2, r_1. This gives the location of the error bit. The sender can now reverse the value of the corrupted bit!

The Hamming code 6



Multiple-bit error correction 1

The basic Hamming code cannot correct multiple-bit errors, but can easily be adapted to cover the case where bit-errors occur in bursts.

Burst errors are common in satellite transmissions, due to sunspots and other transient phenomena which, occasionally, greatly increase the error rate for a brief period of time.

Scratches on CDs and DVDs introduce burst errors into the data read from these disks.

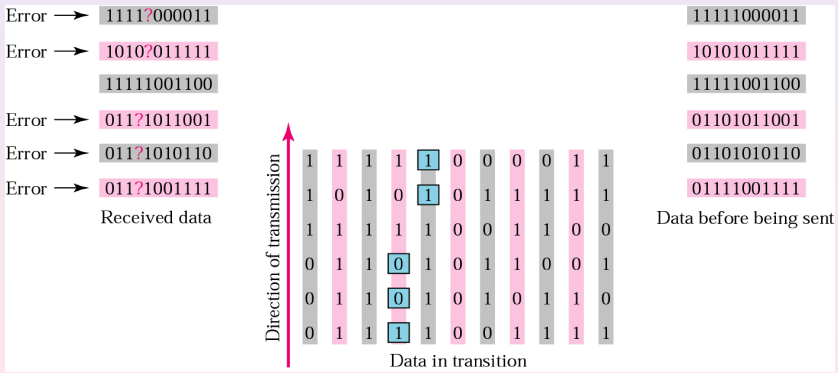
Multiple-bit error correction 2

To protect a message of length N against a single burst error of length $B \leq C$, we can break it up into groups (columns) of length C, then transmit it in transposed order with a Hamming code for each row.

No burst error of length $B < C$ can introduce more than one bit error in a row, so the row-wise Hamming codes are sufficient to correct a single burst error.

On the next slide, we show an example of the technique with $N = 36$ data bits. These bits are organised into columns of length $C = 6$, where the 6 data bits in each row are protected by a 5-bit Hamming code. A burst error of length 5 has affected two columns in our example, but has introduced at most one error in each row – so this error can be corrected by the Hamming code for the affected rows.

Multiple-bit error correction 3



Multiple-bit error correction 4

Hamming codes are pretty easy to understand, and they are pretty easy to implement – but they are not an effective way to protect very long messages against errors in arbitrary positions (i.e. multiple bit errors that aren't in bursts).

Much more powerful error-correcting codes were developed by Bose, Chaudhari, and Hocquenghem in 1959-60. These are the BCH codes. The Reed-Solomon codes (also developed in 1960) are a very important sub-class of the BCH codes. Efficient decoding algorithms, suitable for special-purpose hardware implementation, were developed in 1969; these were used in satellite communications.

Nowadays, Reed-Solomon error-correction is used routinely in compact disks and DVDs.

Multiple-bit error correction 5

A Hamming code uses $r = \log n$ bits to guard n data bits, so we can send only 2^n different messages in a Hamming-protected message of length $n + r$.

This might, or might not, be the appropriate ratio of error-checking (redundant) symbols to information-carrying symbols for this channel. Let's develop a little more theory...

Multiple-bit error correction 6

The **Hamming distance** between two m-bit strings is the number of bits on which the two strings differ.

Given a finite set of codewords one can compute its *minimum distance*, the smallest value of the distance between two codewords in the set.

If d is the minimum distance of a finite set of codewords, then the method can detect any error affecting fewer than d bits (such a change would create an invalid codeword) and correct any error affecting fewer than $d/2$ bits.

Multiple-bit error correction 7

For example, if $d = 10$, and 4 bits of a codeword were damaged, then the string cannot possibly be a valid codeword: at least 10 bits must be changed to create another valid codeword.

If the receiver assumes that any error will affect fewer than 5 bits, then she needs only to find the closest valid codeword to the received damaged one to conclude that it is the correct codeword. Indeed, any other codeword would have had at least 6 bits damaged to resemble the received string.

When designing a code that can correct $d/2$ or fewer errors in a message, we must select codewords which are at least Hamming distance d from each other. It's an interesting combinatorial puzzle... randomly-chosen codebooks do pretty well (but would require decoders to use large – and therefore slow and/or expensive – lookup tables)...

Error correction on the Internet is performed at multiple levels

- Each Ethernet frame carries a CRC-32 checksum. The receiver discards frames if their checksums do not match.
- The IPv4 header contains a header checksum of the contents of the header (excluding the checksum field). Packets with checksums that don't match are discarded.
- The checksum was omitted from the IPv6 header, because most current link layer protocols have error detection.
- UDP has an optional checksum. Packets with wrong checksums are discarded.
- TCP has a checksum of the payload, TCP header (excluding the checksum field) and source- and destination addresses of the IP header. Packets found to have incorrect checksums are discarded and will eventually be retransmitted (when the sender receives three identical ACKs, or times out).

DEF CON, Las Vegas, Nevada

[Started 19 years ago DEF CON is a 15,000-person, four-day convention where anyone with $150—in cash only—can learn the latest tricks and trade of computer hacking, lock picking and security breaching. The following letter comes from `http://tinyurl.com/3zcm3fc`.]

Hi John,
Great talking with you!
You are about to enter one the most hostile environments in the world. Here are some safety tips to keep in mind ...
- Your hotel key card can be scanned by touch, so keep it deep in your wallet.
- Do not use the ATM machines anywhere near either conference. Bring cash and a low balance credit card with just enough to get you through the week.

DEF CON, Las Vegas, Nevada

- Turn off Fire Sharing, Bluetooth and Wi-Fi on all devices. Don't use the Wi-Fi network unless you are a security expert; we have wired lines for you to use.
- Don't accept gifts, unless you know the person very well - a USB device for instance.
- Make sure you have strong passwords on ALL your devices. Don't send passwords "in the clear," make sure they are encrypted. Change your passwords immediately after leaving Vegas.
- Don't leave a device out of sight, even for a moment.
- People are watching you at all times, especially if you are new to the scene.
- Talk quietly. Conduct confidential phone calls off site ...

That is it for now.