THE UNIVERSITY OF AUCKLAND

SECOND SEMESTER, 2007
Campus: City

COMPUTER SCIENCE

Algorithms and Data Structures

(Time allowed: ONE hour)

NOTE: Attempt all questions!

Put the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

Use of calculators is NOT permitted.

Section:	**A**	**B**	Total
Possible marks:	30	20	50
Awarded marks:			

SURNAME:

FORENAME(S):

STUDENT ID:

Student Name: _____ Student ID: _____

Section A: Analysis of algorithms

1. (a) Prove directly from the definition that if a and b are positive real constants, then $an^2 + b\sqrt{n}$ is $\Theta(n^2)$,

[5 marks]

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2. For each question, circle the (unique) correct answer. Note: correct answers score +1 point, incorrect ones -0.25, so the expected score for completely random guessing is 0.

   (a) A certain quadratic time algorithm uses 500 elementary operations to process an input of size 10. What is the most likely number of elementary operations it will use if given an input of size 1000?

   A. 40 B. 500 C. 50000 D. 1000000 E. 5000000

   (b) I am thinking of a well-known algorithm (***-sort) that is not stable, in-place, and whose best case runtime is essentially the same as its worst. What is ***?

   A. merge B. insertion C. heap D. selection E. quick

   (c) If your life depended on correctly sorting data in a reasonable time and you had very little memory available in your application, your best choice would be ***-sort. What is ***?

   A. quick B. heap C. insertion D. selection E. merge

   (d) If, in a given computing environment, data moves are very expensive and comparisons are cheap, which sorting method is likely to be best for a medium-large file?

   A. selection B. heap C. insertion D. merge E. quick

   (e) The number of inversions in the permutation 135246 is

   A. 0 B. 1 C. 2 D. 3 E. 6

   (f) The average running time of quickselect when looking for the median of a list of size n is of order

   A. 1 B. n^2 C. $\log n$ D. $n \log n$ E. n

   (g) Consider the recurrence relation $T(n) = ***, T(1) = 1$. Which value of *** yields a solution that is in $O(\log n)$?

   A. $T(\lceil n/2 \rceil) + 1$ B. $T(n-1) + 1$ C. $T(\lceil n/2 \rceil) + n$
   D. $T(n-1) + n$ E. None of the above.

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(h) A certain sorting algorithm sorts n data items by assigning each item a random place in an array of size n. The algorithm then checks whether that array contains the items in the correct order. If it does not, then the process is restarted until the correct order is achieved. The expected running time is in:

A. $\Theta(n)$ B. $\Theta(n^2)$ C. $\Theta(n!)$ D. $\Theta(nn!)$ E. $\Theta(n^n)$

(i) You are presented with an unknown sorting algorithm. The algorithm appears to run at a constant speed on both random and sorted data sets, and does not markedly slow down as they increase in size. Any two items which have the same search key have their relative order unchanged. Which of the following is the algorithm most likely to be?

A. Insertion sort B. Mergesort C. Quicksort D. Heapsort
E. Bubble sort

(j) A certain algorithm processes a list of size n by first inspecting each element, splitting the list into 3 sublists of equal size, then recursively processing the sublists. It gives rise to a recurrence for the running time $T(n)$ that looks like $T(n) = ***$. What is ***?

A. $T(n/3) + 3$ B. $T(n/3) + n$ C. $T(n/3) + 3n$ D. $3T(n/3) + 3n$
E. $3T(n/3) + n$

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3. For each question, circle the (unique) correct answer. Note: correct answers score +2 points, incorrect ones -0.5, so the expected score for completely random guessing is 0.

   (a) Consider the following code loop.

   ```
   for (int i = 0; i < n; i++)
     for (int j = i - 2; j < i; j++)
       for (int k = 1; k < n; k*= 2)
         System.out.println(i);
   ```

   Its running time is in

   A. $\Theta(\log n)$ B. $\Theta(n)$ C. $\Theta(n \log n)$ D. $\Theta(n^2 \log n)$ E. $\Theta(n^3)$

   (b) Consider the binary search tree (BST) formed by inserting the integers $3, 5, 2, 1, 6, 7$ in order into an initially empty BST. Its internal path length is

   A. 3 B. 6 C. 9 D. 12 E. 18

   (c) When hashing with separate chaining is used, and the elements $1, 2, 3, 4, 5, 11, 12, 13, 21, 22, 31$ are inserted into an initially empty hash table of size 5 using the hash function $h(x) = 2x + 1 \mod 5$, what is the maximum chain size required?

   A. 2 B. 0 C. 1 D. 4 E. 3

   (d) Suppose that we use standard open addressing (probe left on collision) on a hash table of size 5 with hash function $h(x) = 2x+1 \mod 5$. If we insert the elements $1, 2, 3, 4, 5, 11, 12, 13, 21, 22, 31$ in that order, what is the final address of the element 31?

   A. 0 B. 1 C. 2 D. 3 E. 4

   (e) Which of the following inputs will make the standard implementation of quicksort (always choose the leftmost element as the pivot) do the most comparisons?

   A. 1,3,5,6,4,2 B. 6,5,4,3,2,1 C. 1,6,2,5,3,4 D. 2,4,6,1,3,5 E. 1,3,5,2,4,6

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4. Consider the recurrence relation $t(n) = t(\lfloor \frac{n}{2} \rfloor) + t(\lceil \frac{n}{2} \rceil) + n$, with $t(1) = 0$.

   Assume that $n = 2^k$ and derive an exact closed formula for $t(n)$. What is the asymptotic growth rate of $t(n)$ for such values of n?

[5 marks]

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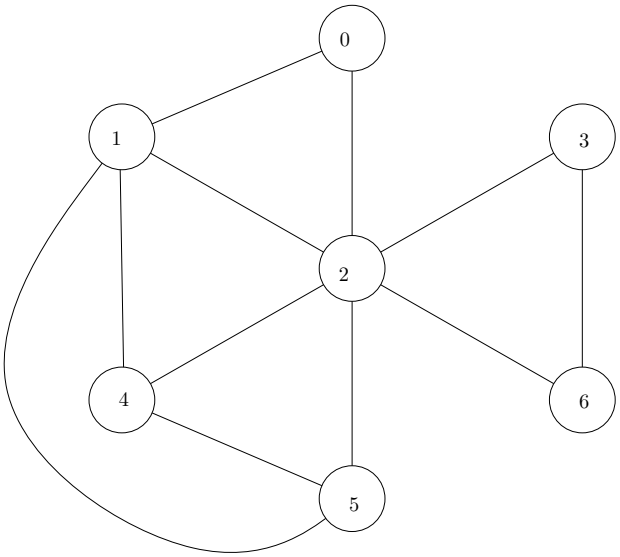
Section B: Graph algorithms

5. Consider the following graph.



(a) What is the order of the graph? [1 mark]

(b) What is the size of the graph? [1 mark]

(c) What is the diameter of the graph? [2 marks]

(d) What is the length of the longest path of the graph? [2 marks]

(e) What is the girth of the graph? [2 marks]

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(f) Write the adjacency matrix representation of this graph. [2 marks]

(g) Write the adjacency lists representation of this graph. [2 marks]

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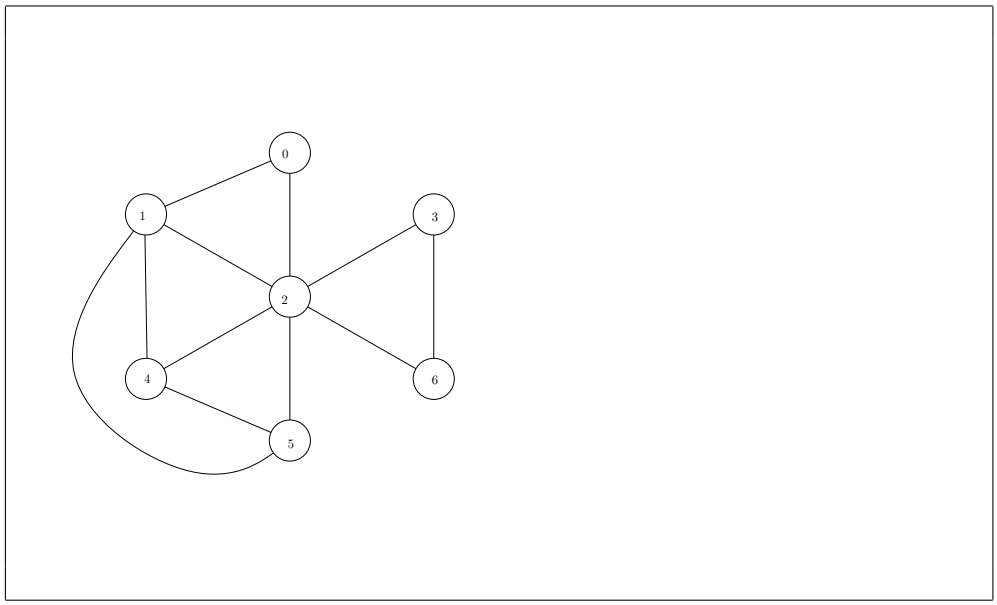
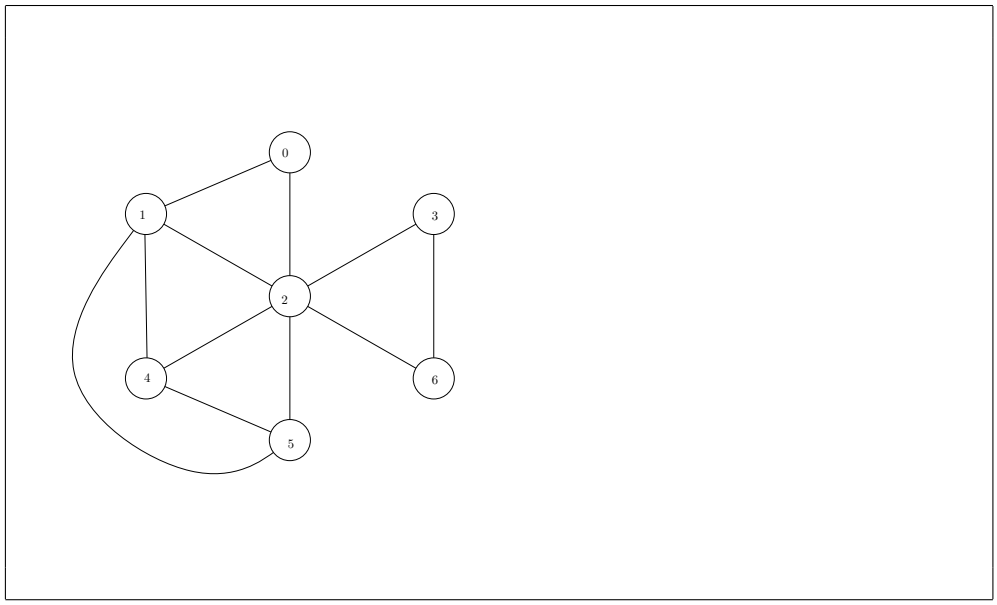
(h) Illustrate BFS on the graph starting from vertex 0 and draw the BFS tree, with the usual rule that when faced with a choice we take the lowest labeled vertex. [2 marks]



(i) Illustrate DFS on the graph and draw the DFS tree (with seen/done times) starting from vertex 0, with the usual rule that when faced with a choice we take the lowest labeled vertex. [2 marks]



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6. Give an example of a weakly connected ~~DAG~~ diagraph with 7 nodes, 9 arcs and 3 strongly connected components. Justify your answer. [4 marks]

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