THE UNIVERSITY OF AUCKLAND

SECOND SEMESTER, 2006 Campus: City

COMPUTER SCIENCE

Algorithms and Data Structures

(Time allowed: TWO hours)

NOTE: Attempt *all* questions!

Put the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

Use of calculators is NOT permitted.

Section:	Α	B	C	Total
Possible marks:	25	30	45	100
Awarded marks:				

SURNAME:

FORENAME(S):

STUDENT ID:

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_Student ID: ____

Section A: Analysis of algorithms

- 1. Write only your answer in the box provided. Use the overflow sheet for your working, which will not be marked.
 - (a) Suppose that f and g are functions from \mathbb{N} to \mathbb{N} . Explain exactly as in the definition what "f is O(g)" means.

[1 mark]

(b) Algorithm A has quadratic running time and takes 500 microseconds to process 1000 data items. About how long will it take to process 100000 items?

[1 mark]

(c) Algorithm *B* has running time that is $\Theta(n \log n)$ and takes 300 microseconds to process 2^{10} data items. About how long will it take to process 2^{20} items?

[1 mark]

(d) Algorithm C takes exponential time $\Theta(2^n)$ and can solve a problem of size 20 in 1 day. What size problem can it solve in 32 days?

[1 mark]

(e) Write down (but do not solve) a recurrence relation describing the running time T(n) of an algorithm that makes a single pass through an array, splits it into two equal parts, and then calls itself recursively on the two subarrays.

[1 mark]

right child of the root?

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(f) List all sorting methods covered in lecture and textbook that are stable, comparison-based, and in-place.

[1 mark]

(g) I am thinking of a well-known algorithm (***-sort) that is in-place, not stable, and runs in quadratic time on every input. What is ***?

[1 mark]

(h) Shellsort is a generalization and improvement of which quadratic sorting algorithm?

[1 mark]

[1 mark]

(j) Name 2 collision resolution methods commonly used with open addressing hashing. Which is more likely to satisfy the uniform hashing hypothesis?

(i) After the items 2, 6, 1, 5, 4, 3 are inserted into an initially empty binary max-heap, what is the

[1 mark]

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2. Recall that selection sort makes n(n-1)/2 comparisons to sort a file of size n while mergesort uses $n \lg n$. Is it ever worthwhile to use a hybrid mergesort/selection sort algorithm that stops the mergesort recursion for files below a certain cutoff size and uses selection sort directly on these files (as we normally do with quicksort/insertion sort)? Explain.

[3 marks]

- 3. For each of the following scenarios, explain which sorting algorithm you would recommend. Give brief explanation. [12 marks]
 - (a) A researcher has a large database of 10^6 records of people born in the 20th century. The researcher wants to study the distribution of given names according to the year of birth, so wants to produce a list ordered first by name, and within each name by year of birth.
 - (b) You have been sentenced to hard labour in prison and your job today involves moving some very heavy rocks so they are in a line from smallest to largest. You have to decide the method before you see the initial configuration of the rocks.
 - (c) We have a list of 10^4 random integers between 0 and 100 which must be sorted by computer.
 - (d) We have a list of 10^4 random integers between 0 and 10^9 which must be sorted by computer.
 - (e) We have a database of customers of size about 10⁶ that has about 100 additions and deletions of records each day. We want to update it every night so it is sorted by name, then by date of birth, then by number of years the person has been a customer.
 - (f) We have to design an electronic device for use in an outdoor game with many players (each player will have an identical one). A crucial part of its job is to sort numbers for use in an algorithm to find the nearest pair of players. We can use very little memory and the game should be fair to all players and not be held up by processing delays.

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Section B: Graph algorithms

4. Consider the following graph.



(a) What is the order of the graph?

[1 mark]

[1 mark]

[1 mark]

(b) What is the size of the graph?

(c) What is the degree of vertex a of the graph?

(d) What is the diameter of the graph?

(e) What is the girth of the graph?

[2 marks]

[2 marks]

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(f) What is the radius of the graph?

[2 marks]

(g) Write the adjacency matrix representation of this graph.

[2 marks]

(h) Write the adjacency lists representation of this graph.

[2 marks]

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(i) Illustrate BFS on the graph starting from vertex *d* and draw the BFS tree (on the side), with the usual rule that when faced with a choice we take the lowest labeled vertex. [4 marks]

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(j) Illustrate DFS on the graph and draw the DFS tree (with seen/done times) starting from vertex d, with the usual rule that when faced with a choice we take the lowest labeled vertex. [4 marks]



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5. Consider the following edge-weighted digraph.



Illustrate Dijkstra's single-source shortest paths algorithm starting from node *a* by filling out the remaining entries of the following table. [6 marks]



QUESTION/ANSWER SHEET

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6. Give an example of a edge-weighted graph with 6 vertices, 8 edges and exactly three different minimum spanning trees of cost 18. Justify your answer. [3 marks]

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Section C: Automata and Formal Languages

7. Give a NFA with 4 states (which is not a DFA) that accepts the language $aa|b^*$. [5 marks]

8. Convert the NFA from Question 7 to a DFA.

[6 marks]

9. Minimize (or prove it already is minimum) the DFA from Question 8.

[7 marks]

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10. Give a regular expression that represents all character strings over $\Sigma = \{0, 1\}$ that represent the set of even binary numbers $\{0, 10, 100, 110, 1000, \ldots\}$. (Note that we do not want strings with leading zeros, except for the one representing 0.) [5 marks]

11. Build a NFA, using the technique given in class, that accepts the closure of the language accepted by the following automaton.







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12. Build a NFA, using the technique given in class, that accepts the language represented by concatenation of $L(M_1)$ and $L(M_2)$.



[7 marks]

13. Let L_1 and L_2 be two languages accepted by DFA M_1 and M_2 . Explain how you would build a DFA M that accepts the language $L_2 \setminus L_1$.

[8 marks]

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