THE UNIVERSITY OF AUCKLAND

SECOND SEMESTER, 2006
Campus: City

COMPUTER SCIENCE

Algorithms and Data Structures

(Time allowed: TWO hours)

NOTE: Attempt *all* questions!

Put the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

Use of calculators is NOT permitted.

Section:	**A**	**B**	**C**	Total
Possible marks:	25	30	45	100
Awarded marks:				

SURNAME:

FORENAME(S):

STUDENT ID:

Student Name: _____ Student ID: _____

Section A: Analysis of algorithms

1.

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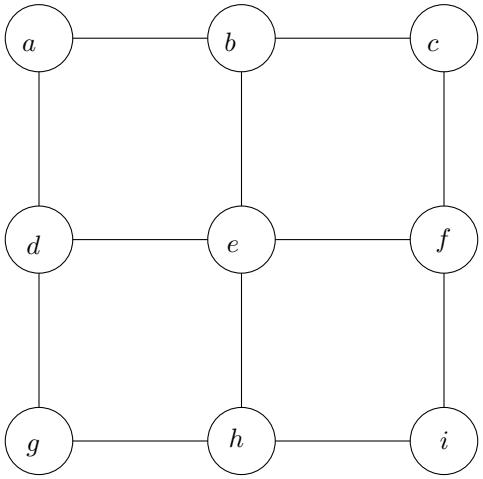
Section B: Graph algorithms

2. Consider the following graph.



(a) What is the order of the graph? [1 mark]

9

(b) What is the size of the graph? [1 mark]

12

(c) What is the degree of vertex a of the graph? [1 mark]

2

(d) What is the diameter of the graph? [2 marks]

4

(e) What is the girth of the graph? [2 marks]

4

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(f) What is the radius of the graph? [2 marks]

2

(g) Write the adjacency matrix representation of this graph. [2 marks]

9 – optional order of graph
0 1 0 1 0 0 0 0 0
1 0 1 0 1 0 0 0 0
0 1 0 0 0 1 0 0 0
1 0 0 0 1 0 1 0 0
0 1 0 1 0 1 0 1 0
0 0 1 0 1 0 0 0 1
0 0 0 1 0 0 0 1 0
0 0 0 0 1 0 1 0 1
0 0 0 0 0 1 0 1 0

(h) Write the adjacency lists representation of this graph. [2 marks]

a: b d
b: a c e
c: b f
d: a e g
e: b d f h
f: c e i
g: d h
h: e g i
i: f h

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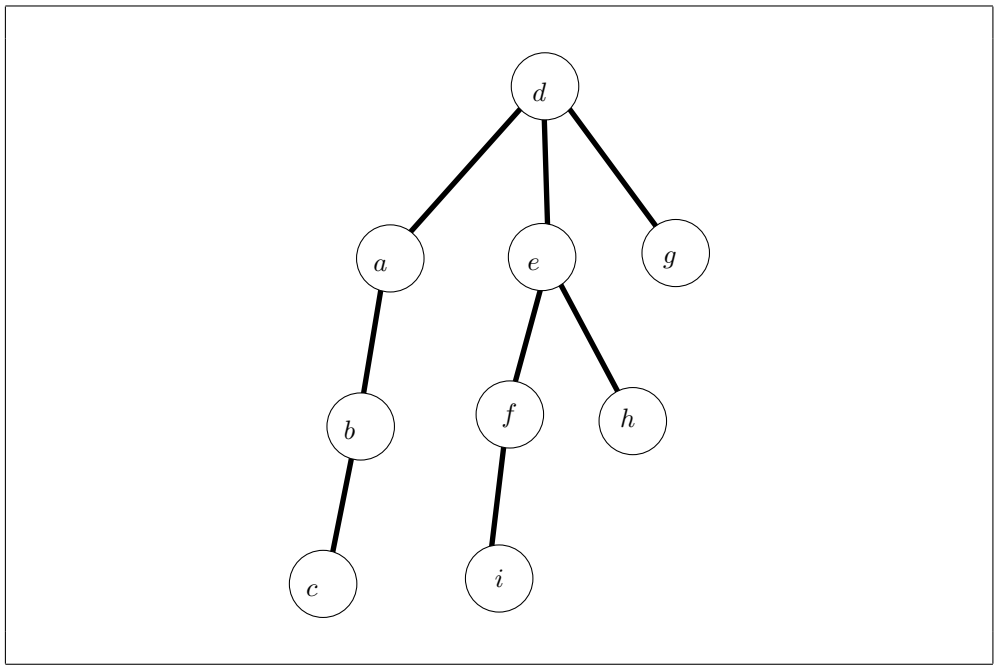
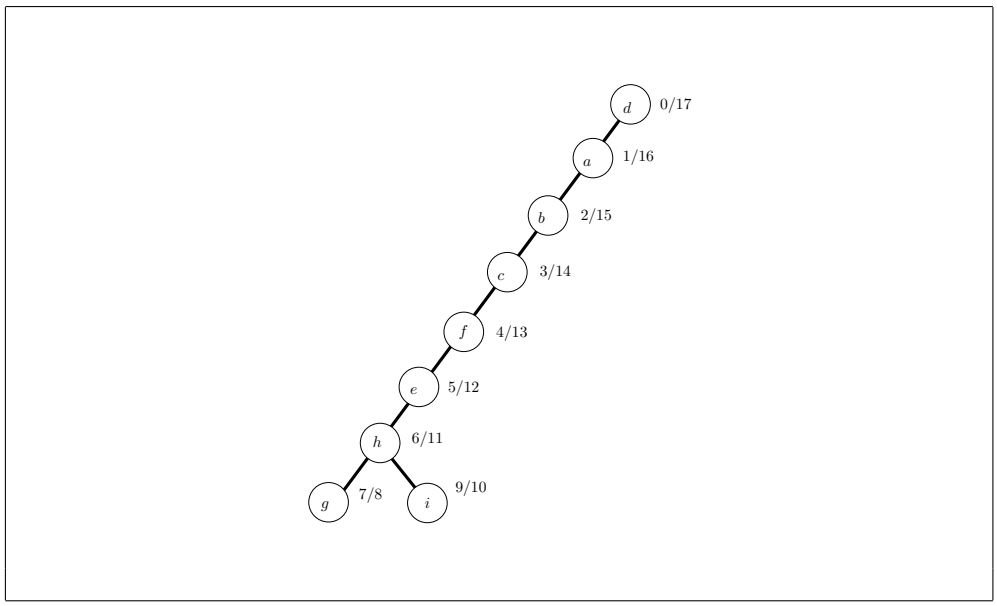
(i) Illustrate BFS on the graph starting from vertex d and draw the BFS tree (on the side), with the usual rule that when faced with a choice we take the lowest labeled vertex. [4 marks]



(j) Illustrate DFS on the graph and draw the DFS tree (with seen/done times) starting from vertex d, with the usual rule that when faced with a choice we take the lowest labeled vertex. [4 marks]



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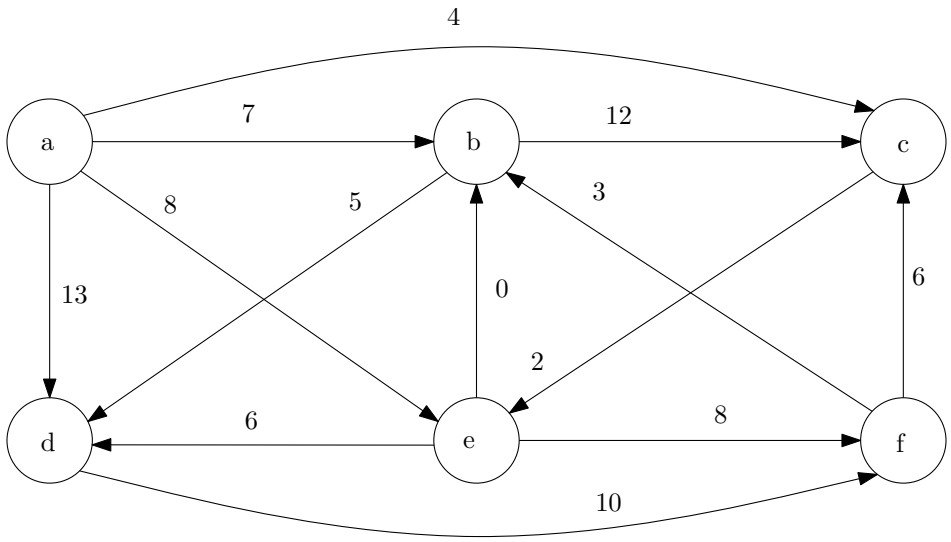
3. Consider the following edge-weighted digraph.



Illustrate Dijkstra's single-source shortest paths algorithm starting from node a by filling out the remaining entries of the following table. [6 marks]

Black nodes S	Distance to nodes (from node a) a	b	c	d	e	f
a	0	7	4	13	8	∞
a, c	0	7	4	13	**6**	∞
a, c, e	0	**6**	4	**12**	6	**14**
a, c, e, b	0	6	4	**11**	6	14
a, c, e, b, d	0	6	4	11	6	14
a, c, e, b, d, f	0	6	4	11	6	14

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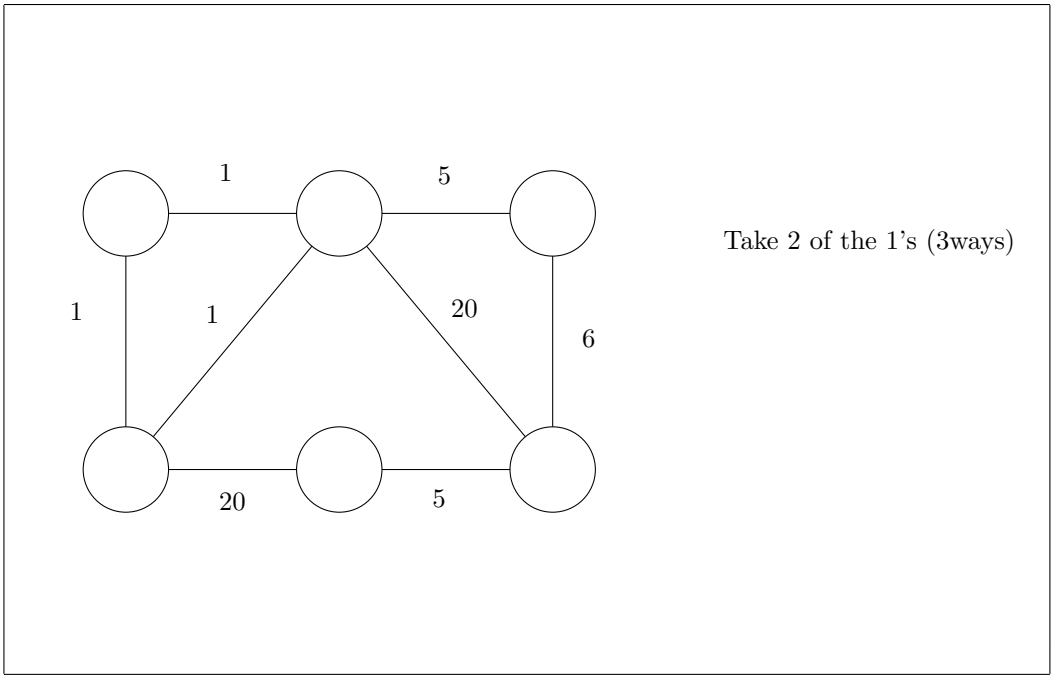
4. Give an example of a edge-weighted graph with 6 vertices, 8 edges and exactly three different minimum spanning trees of cost 18. Justify your answer. [3 marks]



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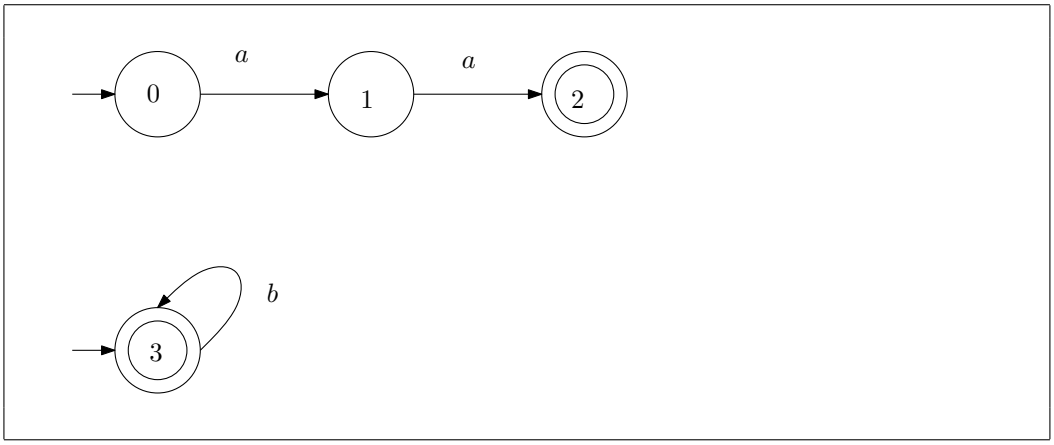
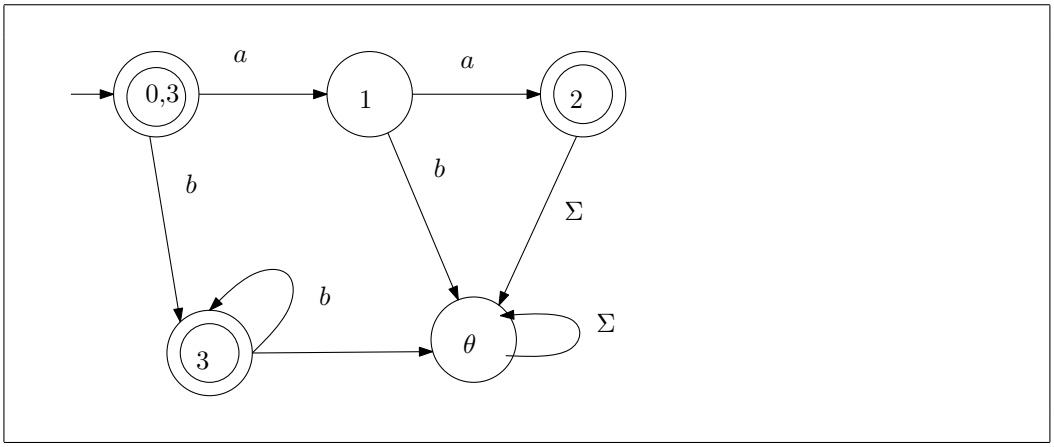
Section C: Automata and Formal Languages

5. Give a NFA with 4 states (which isn't a DFA) that accepts the language $aa|b^*$. [5 marks]



6. Convert the NFA from Question 5 to a DFA. [6 marks]



7. Minimize (or prove it already is minimum) the DFA from Question 6. [7 marks]

It is minimum. Suffices to distinguish the three accepting states and two non-accepting states. The string a distinguishes state 1 from the dead state θ. The string aa distinguishes state $0, 3$ from the other accepting states. Lastly, string b distinguishes state 3 from state 2.

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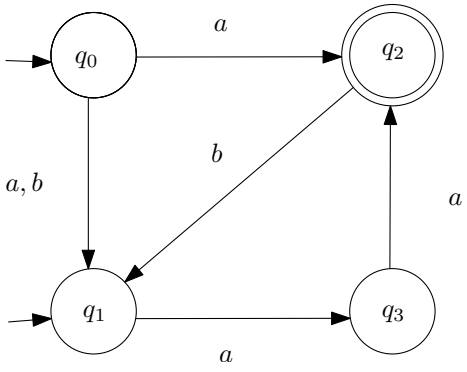
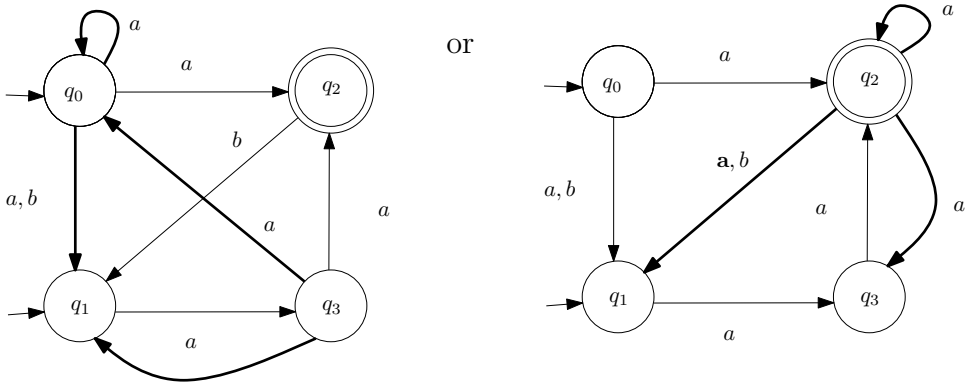
8. Give a regular expression that represents all character strings over $\Sigma = \{0, 1\}$ that represent the set of even binary numbers $\{0, 10, 100, 110, 1000, \ldots\}$. (Note that we do not want strings with leading zeros, except for the one representing 0.) [5 marks]

$0 \mid 1(0 \mid 1)^*0$

9. Build a NFA, using the technique given in class, that accepts the closure of the language accepted by the following automaton.



[7 marks]



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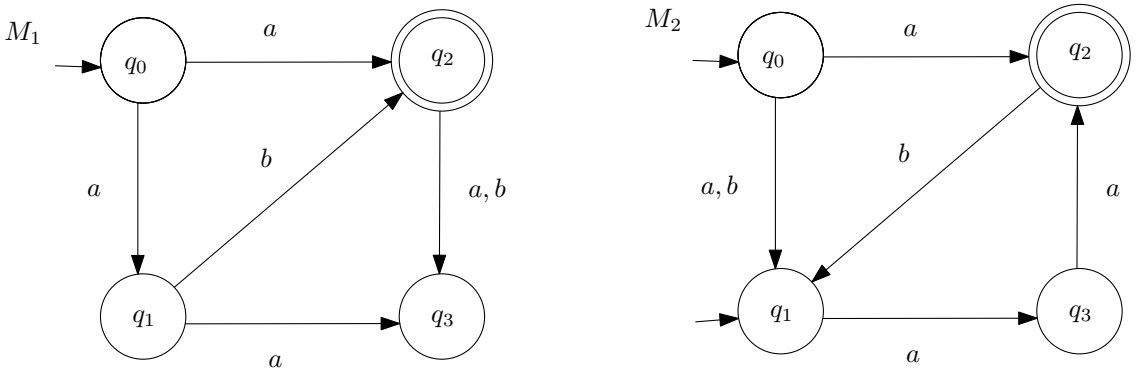
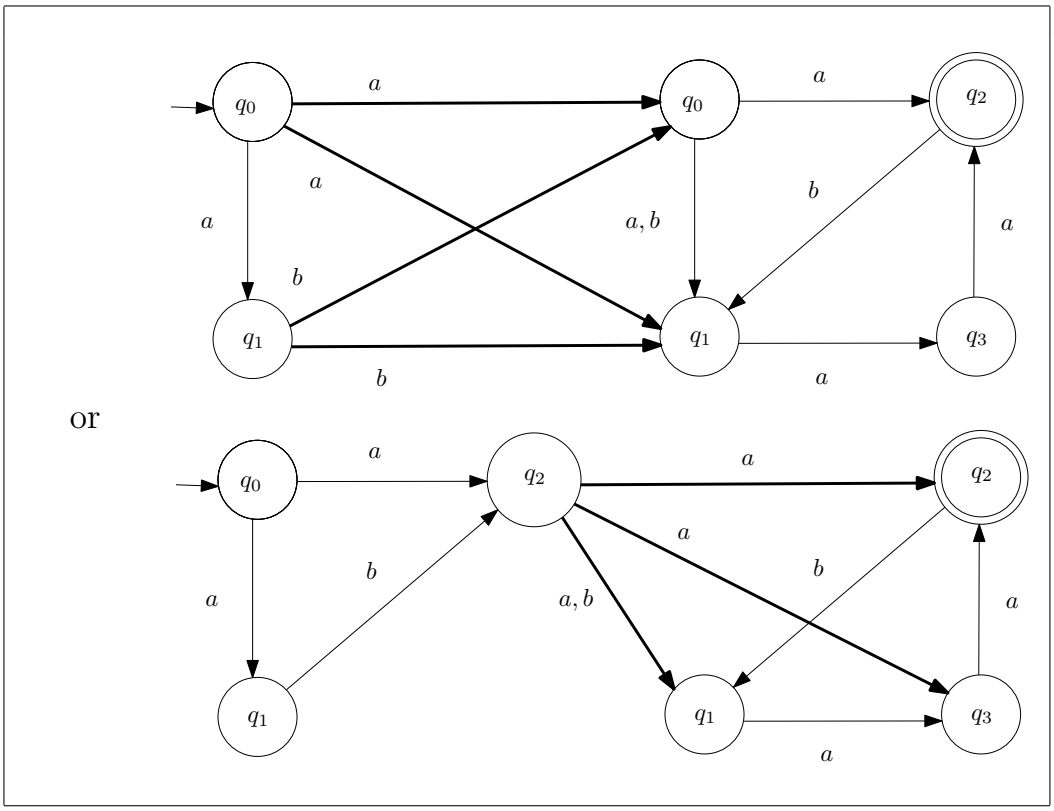
10. Build a NFA, using the technique given in class, that accepts the language represented by concatenation of $L(M_1)$ and $L(M_2)$.



[7 marks]



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11. Let L_1 and L_2 be two languages accepted by DFA M_1 and M_2. Explain how you would build a DFA M that accepts the language $L_2 \setminus L_1$. [8 marks]

$$L_2 \setminus L_1 = L_2 \cap \overline{L_1} = \overline{\overline{L_2} \cup L_1}$$

1. Build DFA M_3 for $\overline{L_2}$. (swap accept/reject states)
2. Build NFA M_4 for $\overline{L_2} \cup L_1$.
3. Convert M_4 to DFA M_5.
4. Build DFA M from M_5. (swap accept/reject states)